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THE THERMODYNAMICS OF PRODUCT DIFFERENTIATION

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INTRODUCTION

THE ASSUMPTION of homogeneous products which is widely employed in models of perfect competition is not generally applicable. Products are differentiated and it is reasonable to assume that consumers have preferences distributed across the characteristic space. A firm thus has an incentive to differentiate products in order to act as a monopolist and insulate its own market to mitigate the possibility of unbridled price competition. Thus, an increase in the degree of product differentiation can lead to a greater degree of market power; albeit over an restricted market.

Given the importance and significance of product differentiation it would seem timely to develop formal models to trace the extent to which products become differentiated and analyse the extent to which product diversity is optimal. Of significant importance are the economic applications, if any of these models. Hotelling began this problem with his linear city model. In this paper we extend the basic Hotelling model of product differentiation. The motivation, it will be seen, comes from an analysis of a physical analogy. The model will demonstrate that the import of the parallel from physics is useful in analysing the problem.

HOTELLING'S LINEAR CITY

Some of the fundamentals of Hotelling's model are presented here. We note firstly that Hotelling's model of product differentiation is an abstraction of reality and involves a number of simplifying (and perhaps) unrealistic assumptions. He envisaged a bounded linear city (street or market) along which customers were uniformly distributed. Two firms supply the market, have a constant and common marginal cost of production of c and face an inelastic demand. Consumers face a linear transportation cost of £t per unit distance travelled along the linear market. Thus the cost to a customer located at x; of one unit of firm i's product (whose location is a) is given by the delivered price; that is:

$$p_i + t|x_i - a|.$$

No customer has any preference for either firm except on the grounds of price plus transportation cost. In general there will be many causes leading particular classes of buyers to prefer one seller to another, but the ensemble of such considerations is symbolised in the transportation cost. In this paper we draw attention to the location game of the two forms; that is, if prices were fixed, where would the two firms optimally locate in order to maximise profits?

We consider one aspect of Hotelling's model in this paper - namely the location game of the firms, with given prices. If firms face a fixed common price £p Hotelling showed that a Nash equilibrium would prevail in which both firms would locate at the same point on the linear city - namely at the centre. Both firms thus maximise the pool of potential consumer surplus available to them by locating at the centre of the market. However distance along the linear market is just a figurative term for a product characteristic - it represents a dimension of characteristic space, as Hotelling noted. Thus firms tend to locate at the same point in characteristic space (given prices). Consumers are confronted with an excessive amount of similar products: this became known as Hotelling's Principle of Minimal Differentiation.

Physical Model

We sought to extend Hotelling's original model by generalising some of his assumptions, and, due to the complexity of the problem, chose a computational rather than an analytical approach. Firstly, we wanted to examine a characteristic space of higher dimension and so we took the modest generalisation of two dimensional space, and limited it by considering only a finite square area. Secondly, we wanted to examine the behaviour of the market when, instead of just two companies, we have several companies in competition, although we had to restrict ourselves to considering just three due to computational limitations. Finally we dropped the assumption of a uniform population distribution over characteristic space, and ultimately chose one that was quite simple but realistic. It consisted of two Gaussian peaks of the same variance but different heights, representing, in a geographical interpretation for example, two different cities:

$$\rho(x,y) = 3e^{-\frac{1}{2}\alpha((x-x_a)^2+(y-y_a)^2)} + e^{-\frac{1}{2}\alpha((x-x_b)^2+(y-y_b)^2)}$$

where (x_a, y_a) and (x_b, y_b) are the coordinates of the centres of the peaks. We expect a priori the compaies to locate at or near the peaks in order to maximise their shares of the market and to service the demands of the consumers there.

We assume that each consumer purchases one unit of each good, and that each acts independently of the other, the only criterion used being the amount of utility derived from the product being purchased. Like Hotelling, we assume linear transportation costs for simplicity, so that the utility a consumer at (x,y) derives from company k located at (x_k, y_k) is

$$U_k = s - p_k - t\sqrt{(x-x_k)^2 + (y-y_k)^2}$$

where s is the gross surplus, is the price charged by company k for its product, and t is the fixed linear transportation cost. Each consumer only purchases once, and thus wants to do so from whichever company maximises their utility. However, in the case where two companies offer the same maximum utility the question arises (even if only in the context of finding an algorithm for the computational experiment) as to which company the consumer actually buys from. Even if there is a difference in utility one might expect the consumer to be indifferent to it, if it is sufficiently small, and only to be swayed by it if it is significant compared to other randomising factors, such as convenience or habit.

The answer we chose is motivated by thermodynamics. A physical system typically has a range of behaviours available to it, each characterised by an energy, and the most stable configuration is the one of lowest energy. A thermodynamical system is one made up of many identical systems acting independently, or only loosely coupled, and such systems tend to emit energy in order to settle into their lowest energy, or ground state: for example the atoms in a laser that are excited by optical pumping will drop to their ground state by emitting light. However if a configuration exists with energy close to that of the ground state, then the system will not discriminate between them, and at equilibrium will be almost equally likely to be in either state. If the difference in energy between them is DE, then the ratio of the probabilities of occupancy of the ground state and the excited state, p_0 and p_1 respectively, is given by the Maxwell-Boltzmann statistic:

$$\frac{p_1}{p_0} = e^{-\frac{\Delta E}{kT}}$$

If $\Delta E >> kT$ then the system is almost certainly in the ground state, while for $\Delta E << kT$ the system is equally likely to be in either state. In fact T is the temperature of the system, and, in a sense, is a measure of how much random energy is in the system: the higher the temperature the more likely the system is to be in higher energy states, (or more simply, the hotter it will be).

Thus we assume that the consumers purchase according to the Maxwell - Boltzmann distribution, and so the number of consumers at (x,y) who purchase from company k is

$$\rho(x,y)\frac{e^{\frac{U_k(x,y)}{U_0}}}{\sum_i e^{\frac{U_i(x,y)}{U_0}}}$$

where U_0 is a critical utility differential, analogous to the temperature of a thermodynamical system. Notice that if U_0 is zero, each consumer simply chooses the cheapest company for them. To find the total demand D_k for each company we add up the the demand from the whole population by integration:

$$D_k = \iint \rho(x,y)\, \frac{e^{\frac{U_k(x,y)}{U_0}}}{\sum_i e^{\frac{U_i(x,y)}{U_0}}}\, dxdy$$

Note that:

$$\sum_j D_j = \iint \rho(x,y)\, \frac{e^{\frac{U_k(x,y)}{U_0}}}{\sum_i e^{\frac{U_i(x,y)}{U_0}}}\, dxdy$$

so the total demand is simply the total population, since each consumer buys one unit. Finally, to determine the equilibrium condition we again appeal to physical analogy and take it to be the maximisation of the following

$$I = \prod_i \Pi_j$$

where p_i is the profit of the ith company. For a fixed price, common to all companies, profit is proportional to demand, and so

$$I = \prod_i D_i$$

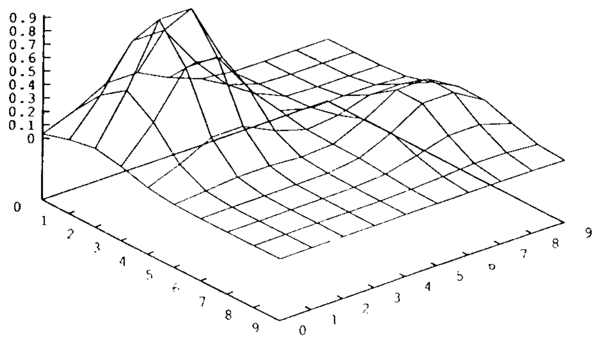
Experience guided us to use a Monte Carlo technique rather than a dynamical one to find the equilibrium configuration: we selected a random location for each company and calculated the quantity I above. That configuration was then given a corresponding weighting and the process was repeated until a significant portion of the total possible configuration space was sampled. Due to the length of time needed for each calculation of I we had to discretise the characteristic space into a square grid of one hundred points and use just three companies, each with a common fixed price for their goods, thus making it a pure location game.

The following code segment is the core of the program, choosing random-configurations, and calculating corresponding demands.

```
for(k=0;k<S;k++)
{for(i=0;i<3;i++)
      { I[i]=random()%100;
            comp[i][0]=point[I[i]][0];
            comp[i][1]=point[I[i]][1];
            comp[i][2]=point[I[i]][2];
                  demand[i]=0; }
      for(i=0;i<100;i++)
```

```
{ for(D=0,j=0;j<3;j++)
{ x=comp[j][1]-cons[i][1];
y=comp[j][2]-cons[i][2];
cos[j] = comp[j][0] +T*sqrt(x*x + y*y)/CO
D+=exp(-cost[i]); }
      for(j=0;j<3;j++) demand[j]+=cons[i][0]*exp(-
cost[i])/D; }
      for(D=1,i=0;i<3;i++) D*=demand[i];
      for(i=0;i<3;i++)
      prob[I[i]]+=D/100; }
```

The results are shown graphically below:



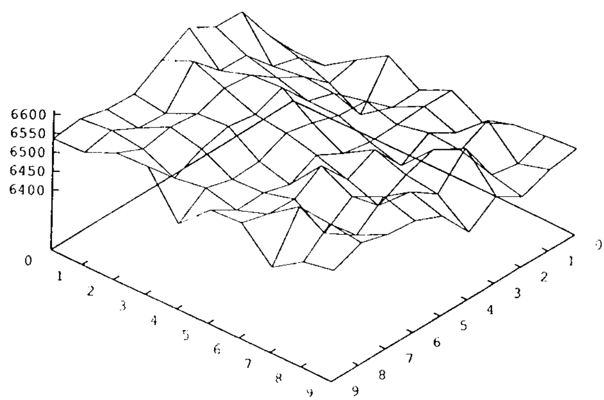


Figure 1 **Figure 2**

Fig. 1 shows the underlying population distribution, representing perhaps two cities, and fig. 2 shows the probability at each location of a company maximising its profits by locating there. This clearly shows that the companies are equally likely to locate at any position in characteristic space. This suggests, contrary to our initial expectations, that there is no 'equilibrium' in this game due to the interdependent reactions among the three companies.

Comment

The above model is simple, yet produces results that accord with conventional wisdom. The analogy to physics points to two features in particular that merit explicit mention. Firstly, economics cannot and should not exclude the influence of other disciplines of study. Social, political and other influences have a role to play in shaping the economic environment and the analogy to physics, while not realistic is somewhat illuminating. Secondly, the above analysis raises a methodological issue: namely that economics may be amenable (under restrictive assumptions) to scientific analysis and may make considerable progress if the concepts are suitably adapted to the economic context.

Conclusion

The concept of product differentiation is an important feature of the economic climate and the development of formal models attempt to make tractable the features and implications of product differentiation. While the model presented here is simple, we hope that is will empower individuals to view product differentiation from a new perspective and will prove useful in formulating more specific models,ones which better powers of prediction.

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