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IT'S A MATHS MATHS WORLD!

by Alan White

MATHEMATICAL ECONOMICS embodies various applications of mathematical techniques to economics, particularly economic theory. This branch of economics dates from the nineteenth century and has developed a rate of natural increase in recent decades. Mathematical economics is not so much a subject as an area of study within economics closely affiliated with economic theory. Its scope is changing constantly, since it acts as a port of entry for new analytical techniques imported from mathematics on their way into the main body of economic analysis. Many economists have discovered that the language and tools of mathematics are useful. Simultaneously many mathematicians have found that mathematical economic theory provides an important and interesting area for applying mathematical skills and that economics has given rise to some important new mathematical skills such as game theory.

This paper is divided into four sections. Section one traces in brief the history of mathematical economics. Given that the role of mathematics in economics has grown in importance, it is constructive to evaluate professional attitudes to this growth. Section two contains a synthesis of two such surveys. Section three will rationalise the current role of mathematics in economics, while section four shall draw some conclusions.

HISTORY

The history of mathematical economics consists of three broad and somewhat overlapping periods. The first (1837-1947) has been referred to as the calculus-based marginalist period. The early period of mathematical economics was one in which economics borrowed methodologies from the physical sciences and related mathematics to develop a formal theory based largely on calculus. Employment of total and partial derivatives and of the Lagrange multiplier was widespread. During this period the mathematical foundations of the modern theories of consumer and producer surplus, oligopoly and general equilibrium were developed. Names associated with this period include Cournot, Hicks, Walras and Pareto.

Although the calculus approach was never abandoned, the 1950s witnessed the advent of a new approach namely that of set theory and linear models in economic analysis. Using set theory meant greater generality in that the classical assumption of smooth (continuous) functions could be replaced by more general functions. Using linear models facilitated treatment of phenomena that could not be represented by smooth functions. Arrow, Debreu and McKenzie used set theory, topology and convexity to study the existence of equilibrium, culminating in Debreu's "The Theory of Value" in 1959. Input-output models which are linear models of inter-industry relations had been developed by Leontief and linear programming stemmed from the early works of Dantzig.

The current period of investigation which dates from the 1960s has essentially been one of integration, in which modern mathematical economics combines calculus, set theory and linear models. Research has been conducted on the economics of uncertainty, optimal growth and taxation. Gerard Debreu and Stephen Smale have investigated the properties of equilibrium while Scarf has studied the computational procedures for calculating equilibrium prices. This list is not exhaustive and is intended to whet the appetite and give an indication of the growing dominance of mathematics in economics. However, many have found this advent somewhat difficult to digest. This issue is addressed in section two.

Attitudes to Maths

Two independent (though very similar) surveys on attitudes to maths were conducted by Gruebel and Boland in the U.S. in 1986 and by Greenaway in the U.K. in 1989. The intention was to evaluate the professional attitudes of academics to the growth of the use of mathematics in economics. It can be taken as axiomatic that the use of mathematics in economics has increased both as a tool of scientific investigation and for training. Evidence of this growth is cited in the U.S. survey. Gruebel and Boland looked at economics journals - the principal outlet of academic economists and noted that in 1951 2.2% of the pages of the American Economic Review contained at least one equation while this percentage had risen to a significant 44% by 1978. Such a measure represents only one indicator of the rise of mathematics but is nonetheless indicative of its importance alluded to in section one.

Greenaway adopted much the same format as Gruebel and Boland in order to facilitate useful comparisons between the U.S. and U.K. results. Both surveys asked academics if they considered the amount of resources devoted to mathematical training at the undergraduate level to be adequate. In the U.K. the response was favourable (66%) though this percentage was lower in the U.S. Typically U.K. undergraduate courses in economics are more specialised but at the post-graduate level there is a less formal training element than in the U.S. Interestingly 60% of respondents in the British survey felt that too much journal space was devoted to mathematical articles (that is, articles whose arguments were predominantly mathematical) with a comparable figure for the American survey.

If economists feel that the composition of economics is, in some sense wrong then why does such a state of affairs exist? This point shall be dealt with later. 45%(U.K) and 31%(U.S.) of respondants felt that the mathematical and modelling skills of research economists have improved the reputation and prestige of the economics profession since the 1950s with a significant proportion uncertain (21% and 50% for the U.K. and U.S. respectively). A significant proportion felt that the development of mathematical skills did not prepare young economists for careers in industry or government. This is rather alarming when one remembers that over half of the respondants were pleased with the level of resources devoted to undergraduate training in mathematics. A resolution of such an anomaly may lie in the fact that the respondants of the survey had academic careers in mind when evaluating the role of mathematics.

In addition, British economists felt that possession of mathematical modelling skills were not a reliable screening device for high quality academics and yet felt that such individuals have less trouble in securing employment at universities. On a practical level academics may be judged on the basis of mathematical merit because it is a more accessible screening device, given its importance in current economic thought. Indeed a majority would feel that the acquisition of mathematical techniques leads to a higher rate of publication and that such economists have greater evidence of their productivity to offer potential employers.

Despite the importance of the technique of mathematical applications to the discipline of economic, little had been known about professional attitudes towards its growing importance. Greenaway, in particular did not set out to support a specific hypothesis but rather to uncover little known attitudes. The implications of the surveys are stark. U.K. and U.S. economists are happy with the resources devoted to teaching mathematics but feel that too much journal space is devoted to mathematical articles, that mathematical specialists can publish more easily and can obtain jobs in the profession with greater ease. All of this implies a perception that mathematical competence and ability is used as a screening device for entry into the profession.

Role of Mathematics in Economics

It is perhaps no small wonder that mathematical competence is a necessary condition for entry into the profession. After all, some of the best economists have been mathematicians. Marshall, Keynes and Fisher, to name a few, made their contributions to economics in fields outside of what would be regarded today as mathematical economics. Their respective contributions did not embody the formal introduction of mathematics per se, but rather they extended the frontiers of conventional economics by familiarising economists with simple mathematical techniques and notation. We must immediately beg the question; why is its role in economics more or less tacitly accepted despite the apparent discontent expressed in the two surveys ?

Mathematics is a rigorous and well-defined study of the structures, configurations, and interrelationships that characterise the world in which we live. It functions as an exacting language that articulates the essential characteristics of a wide range of situations so that the key aspects of those situations can be dispassionately examined. Modern mathematics is economical in the very best sense of the word, in that:

- It clearly states the bare-bone assumptions that underpin a relationship.

- It highlights the logical processes the characterise the relationship.

- It states any conclusions that are implied by the relationship in a clear and concise form.

Mathematics has proven most successful in making significant contributions of a non-mathematical variety. It is especially suited to abstract thought. An economist may extract from the economy relationships considered of most importance and the extent of the progress is then determined by the facility with which he can make deductions from the logical structure of his abstract theory and transform these into statements about the real world. Indeed mathematics removes the inherent ambiguities and value-loaded connotations associated with strictly verbal treatment. Furthermore when confined to a strictly verbal analysis, arguments become tangled in an intricate web of grammar, particularly when considering complex models involving several interrelationships. Professor Stephen Smale, recognising this difficulty remarks "....to write a paper with no technical mathematics is a real (but very constructive) confrontation with the problem of communication". It is this parsimonious and precise feature which has aided decision makers and policy makers in general.

While the success of mathematics in economics is less impressive than that in the natural sciences, its contribution cannot be overlooked. The introduction of prices and money necessitated the introduction of elementary mathematics while consideration of equilibrium supply equals demand conditions brought with it its reticent equation while analysis of multiple markets extends the range of consideration to solutions of systems of equations. In this regard, the use of difficult ideas of algebraic topology, such as Brouwer's Fixed Point Theorem have led many to believe that mathematics can obscure the economic phenomena underlying the existence of equilibrium. Indeed it may be felt that economics becomes too mathematically-loaded and that the mathematics-averse individual may (understandably) be repuled by such presentations. Nonetheless, economics by its very nature can attain a level of abstractness and obscurity independent of any mathematical orientation. Mathematics is employed as a means to an end , not an end in itself (at least as far as economics is concerned). Assuming that the economist is rational, (assuming economists are allowed to make assumptions) successful utilisation of mathematics will entail an agglomeration of the essential features of mathematics and the disposal of those aspects of peripheral interest to the economist. This will imply an economy on the use of mathematical skills and techniques in order to attain an optimal level of usage.

CONCLUSION

In light of the growing contribution that mathematics is making to economics, many economists have stopped to evaluate this rise to dominance. Has its use grown too much? It has been argued that its employment in economics has been advantageous in progressing the advancement of economics; such is its importance that it constitutes a barrier to entry into the economics profession. The person who ignores the contribution of mathematics to the study of economics not only bucks a strong trend in these areas, but also cannot take advantage of tools and a mode of analysis that have greatly enhanced the power and ability of decision-makers in those areas. Equations no longer raise eyebrows; non-mathematicians no longer call themselves economists.

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