# Domestic or global imbalances? Rising inequality and the fall in the US current account

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**Abstract**

This paper shows how the rise in individual income risk in the US since the 1980s could help explain the fall in its foreign asset position. The key to this result is endogenous financial deepening in an open economy where individuals can default on contracts, at the price of exclusion from financial trade. More volatile income makes default less attractive, and thus allows higher borrowing against future income. In a closed economy, this improves consumption smoothing across volatile income realisations, as shown by Krueger and Perri (2006). In an open economy, on the other hand, relaxed default constraints increase the economy's aggregate borrowing capacity. So a rise in income risk causes a rise in foreign debt, with only a minor effect on consumption insurance. My theoretical results show that, unlike with unconstrained complete markets, individual participation constraints guarantee a well-defined stationary equilibrium at a given world interest rate. Against the intuition from simple savings models, higher income risk is shown to decrease stationary excess demand for consumption and assets in a standard version of the model, but has little or no effect on consumption volatility, which depends mainly on world interest rates. A quantitative exercise shows that the observed rise in individual income risk in the US implies a significant fall in its demand for foreign assets. In a two country general equilibrium model, this is reinforced by a precautionary "savings glut" from increased income volatility in a less financially developed economy, calibrated to capture the evolution of individual inequality in China.

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# 1 Introduction

Over the past 25 years, the US economy has experienced a significant rise in both cross-sectional income inequality and the uncertainty of individual incomes. Simple economic models suggest that this should have increased individual savings at the same time as consumption inequality. But instead, as figure 1 shows, during this period of rising individual risks the US savings rate declined and the country's net foreign assets as a share of GDP fell by almost 30 percentage points, while consumption inequality increased only very little. This paper presents a simple model of an open economy where a rise in individual income risk lowers aggregate foreign assets, while leaving consumption inequality largely unchanged. The crucial assumption is that individuals have the option to default on financial contracts, at the price of permanent exclusion from financial trade. This limits the amount they can credibly borrow against the future during periods when their income is temporarily low. Higher income risk increases individuals' incentives to maintain financial market access, as this allows them to smooth consumption. A rise in risk thus makes default less attractive, relaxes incentive constraints and allows individuals to issue more claims against higher income in the future. In a closed economy, this increase in supply reduces the price of state-contingent assets and thus improves consumption insurance, as pointed out by Krueger and Perri (2006). In an open economy, on the other hand, the price response is muted, and the relaxed supply constraint increases aggregate issuance of claims to the rest of the world. So net foreign assets fall. For the limit case of a given world interest rate, corresponding to a zero price response, I show that consumption volatility can be unchanged as higher income risk relaxes default constraints and increases aggregate debt.

To derive these results, I first show that, unlike with unconstrained complete markets, the debt-constrained economy has a unique stationary equilibrium at a given interest rate that does not depend on initial conditions. So individual participation-constraints "close small open economies" (Schmidt-Grohé et al 2003). Using this feature, the paper's main theoretical result is to prove that stationary aggregate excess demand for consumption and assets declines with income risk in a standard limited commitment economy with two income values (Krueger and Perri (2005), Thomas and Worrall (2007)), and to give sufficient conditions for this to hold in the more general version of the model. The theory also shows how financial openness can significantly change the determinants of consumption risk, which, under certain conditions, depends solely on the level of world interest rates, and can thus become completely decoupled from income risk.

Using a calibrated version of the model, the quantitative section of the paper then shows that the rise in income risk experienced by the US economy over the last 25 years implies a significant reduction in the stationary demand for foreign assets. Furthermore, a two country general equilibrium model, calibrated to capture the observed rises in income heterogeneity in the US and China, shows that this effect can be augmented by a precautionary "savings glut" in emerging economies where insurance against increased income risks is unavailable. By increasing the borrowing capacity of US consumers at the same time as precautionary savings in less financially developed economies, the rise in individual income risk over the last 25 years thus leads to a reallocation of savings in line with the observed "global imbalances", including a fall in US net foreign assets on the order of one third of US GDP.

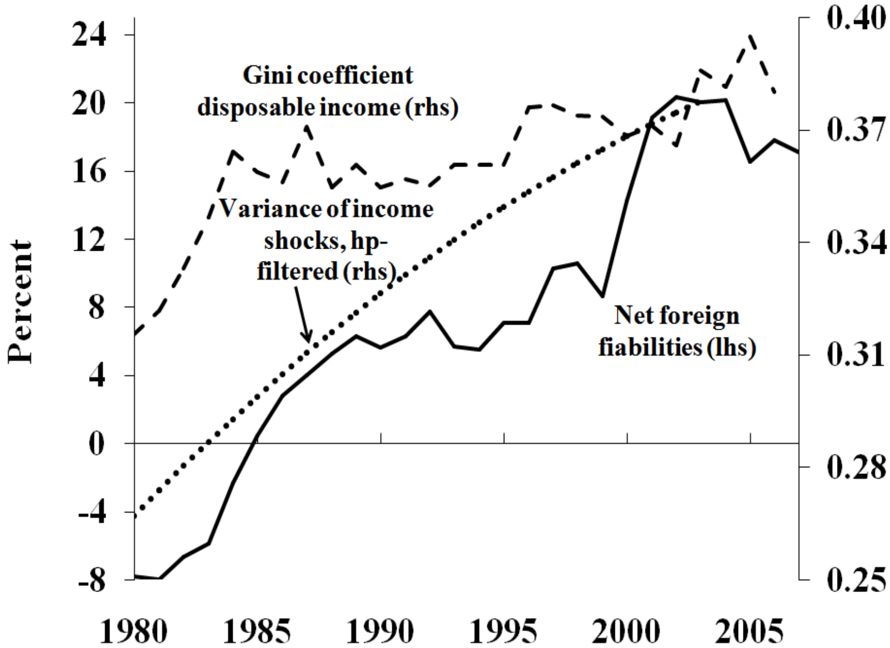


Figure 1: The series for US net foreign liabilities is taken from an updated and extended version of the dataset constructed by Lane and Milesi-Ferretti (2007). Both the Gini coefficient for US equivalent household disposable income (Heathcote et al 2010) and the hp-filtered total variance of estimated shocks to log household incomes (Krueger and Perri 2006), are based on data from the US Consumer Expenditure Survey.

To assess the sustainability of the recent rise in US indebtedness, several previous contributions to the debate on global imbalances have focused on changes in the structure of the world economy that might imply a permanently lower US net asset position. For example, in Mendoza et al (2007), capital account liberalisation in countries whose financial markets are less capable of insuring individual risks leads to a reallocation of their precautionary savings to the US. Fogli and Perri (2006), on the other hand, argue that the more pronounced "great moderation" in macro-volatility in the US could have reduced precautionary savings in the US more than in other countries. The present paper complements these studies by analysing how long-run savings and foreign asset holdings of the US economy may have been affected by the strong rise in individual, rather than aggregate, income risk over the last 25 years. Its contribution is to show how, in an economy where the depth of financial markets depends endogenously on the attractiveness of default, as opposed to the exogenous insurability of individual shocks in Mendoza et al (2007), an increase in risk can reduce asset holdings. And the quantitative general equilibrium analysis shows how this financial deepening effect on foreign asset demand might be important, and possibly augmented by a rise in precautionary savings in response to higher individual risks in financially less developed emerging economies. The analysis does not, however, predict a sustainable US current account deficit. Rather, in the long-run, net exports finance interest payments on the foreign liabilities that the US economy accumulated during a period of adjustment, leading to a balanced current account. This is similar to Fogli and Perri (2006), Mendoza et al (2009) and Jeanne et al (2009) who all focus on transitional deficits in response to a change in relative long-run equilibrium savings. The predictions are thus different to, for example, Caballero et al (2009), where a fall in the share of output that emerging economies can pledge as dividends leads to permanent net ressource flows to the United States. Also, since I abstract from aggregate risk, and the gains from international asset trade are thus exhausted entirely through unconditional riskless bonds, the model can not generate an "exorbitant privilege" (Gourinchas et al 2007), that allows the US to finance a permanent current account deficit by earning excess returns on its country-portfolio.

Beyond this contribution to the recent literature on global imbalances, the present study also has conceptual implications for the analysis of open economies that face a given world interest rate. This is because individual participation-constraints are shown to imply existence of a unique stationary equilibrium independent of initial conditions in this setting. So endogenous debt-constraints can "close small open economies" (Schmidt-Grohé et al 2003), without ad hoc assumptions on preferences.

Finally, this paper relates to the literature on the macroeconomic role of uninsurable individual income risks and, more particularly, the determinants of individual consumption inequality with participation-constrained complete markets, as in Alvarez and Jermann (2000), Kehoe and Levine (2001), Kocherlakota (1996), or Ligon et al (1996). The theoretical contribution is to show analytically that, in the limited commitment continuum economy with two income values (Krueger and Perri 2005), aggregate excess demand is decreasing in individual income risk. This goes against the intuition from traditional bufferstock savings models, but is the main theoretical result that explains falling foreign asset positions after a rise in the riskyness of individual incomes. And the theoretical section of this paper provides sufficient conditions for this to hold also in a more general version of the model. Finally, the paper shows that the relationship between income risk and consumption inequality in an open economy is fundamentally different to that in models without international asset trade such as Krueger and Perri (2006) or Heathcote et al (2008). Particularly, in the environment of this paper, world interest rates are the key driver of changes in consumption risk and inequality. So the evolution of the international economy is a key determinant of the domestic consumption distribution and risk sharing.

The rest of the paper is structured as follows: Section II describes the environment of an open economy with debt-constrained domestic financial markets. Section III derives the analytical results on the basis of the associated planner's problem. Section IV reports the quantitative results.

# 2 An open economy with debt-constrained domestic financial markets

This section presents a simple model of an open economy where domestic financial markets are constrained by individual default. It focuses on the impact of slow-moving changes in idiosyncratic risk on the long-run equilibrium levels of savings and consumption risk, and therefore abstracts from aggregate and transitional dynamics that are likely to dominate asset holdings in the short- and medium-run. The environment is thus similar to that in Krueger and Perri (2005), amended for international asset trade.

## 2.1 The general environment

The analysis focuses on a country that takes prices of goods and assets traded with the rest of the world as given. The country is populated by a large number of individuals of unit mass. Individuals are indexed by i, located on a unit-interval $i \in \mathbb{I} = [0,1]$. Time is discrete $t \in \{0, 1, 2, ..., \infty\}$ and a unique perishable endowment good is used for consumption.
The consumption endowment of agent i in period t, $z_{i,t}$, takes values in a finite set $Z$: $z_{i,t} \in Z = \{z_1 > z_2 > ... > z_N\}, N \geq 2$. Let $s_t : \mathbb{I} \longrightarrow Z \times R$ denote the state of the economy in period t, a measurable function that assigns income and asset values to all agents. Endowments follow a stochastic process described by a Markov transition matrix $F$. $F$ has strictly positive entries, is identical across agents, monotone (in the sense that the conditional expectation of an increasing function of tomorrow's income is itself an increasing function of today's income), and has a unique ergodic distribution $\Phi_Z : \mathbb{Z} \rightarrow [0, 1]$, where $\mathbb{Z}$ is the power set of $Z$. Thus, in the

long-run, aggregate income $Y = \int_{\mathbb{I}} z_i$ is constant, while individual income fluctuates.
To study the effect of changes in income risk, we need to define an increase in the riskyness of the endowment process. I follow Kehoe and Levine (2001) and Krueger and Perri (2006), and define a rise in risk as a spread to the income support $Z$ that leaves aggregate income unchanged. More formally, for some $j \in \{1, ..., N\}$ a rise in risk is a vector of changes in income values $dZ \in R^N$ such that $\sum_i \Phi_Z(z_i) dZ_i = 0$, and $dz_i < -\vartheta < 0, \forall i > j \; and \; dZ_i \geq 0, i = 1, ..., j$. Note, however, that this does not imply mean-preserving spreads to the conditional income distribution for all individuals. Rather, given persistence, $dZ$ raises (lowers) current and expected future income for today's high (low) income earners.
Agents live forever and order consumption sequences according to the utility function

$$U = E_{s_0} \sum_{t=0}^{\infty} \beta^t u(c_{i,t}) \tag{1}$$

where $E_{s_0}$ is the mathematical expectation conditional on $s_0$, $0 < \beta < 1$ discounts future utility, $c_{i,t}$ is consumption by agent i in period t, and $u : R^+ \to R$ is an increasing, strictly concave, continuously differentiable function that satifies Inada conditions and is identical for all agents in the economy.

## 2.2 Asset markets

I choose a specification of the economy similar to that by Alvarez and Jermann (2000), amended for the international setting. Agents engage in sequential trade of a complete set of state-contingent bonds domestically, but international asset trade is limited to non-contingent bonds.[1] Individual endowment realisations are verifiable and contractable, but asset contracts are not completelely enforceable: at any point, individuals can default on their contractual payments at the price of eternal exclusion from financial markets. Thus the total amount an agent can borrow today against any income state $z_j$ tomorrow is bounded by the option to default into financial autarky. There, consumption is forever equal to income. Given the markov structure of income, the value of default as a function of the vector of current income $z$ can be written as

$$W(z) = \sum_{t=0}^{\infty} (\beta F)^t U(z) = (I - \beta F)^{-1} U(z) \tag{2}$$

I denote holdings of bonds and Arrow-Debreu securities paying off in state $s_t$ by $b$ and $a(s_t)$ respectively. In any state $s_t$, $V(z(s_t), a(s_t), b_t)$ is the contract value as a function of income $z(s_t)$

[1] This is non-restrictive as there is no aggregate risk and the law of large numbers holds. It requires, however, no default on foreign debt on a country level. In a previous version of this paper I show that Broner and Ventura's (2006) result applies to my setting. So perfect secondary markets prevent governments from defaulting on agents' foreign liabilities.

and current asset holdings $\{a(s_t), b_t\}$.
As in Alvarez and Jermann (2000) individual i's participation constraint for any state $s_{t+1}$ tomorrow can be written as a constraint on the claims she can issue against $s_{t+1}$ income. This borrowing constraint is "not too tight" in the words of Alvarez and Jermann (2000) if it assures participation but does not constrain contracts otherwise

$$a_i(s_{t+1}) + Rb_{i,t+1} \geq A_i(s_{t+1}) = min\{\alpha(s_{t+1}) : V(z_i(s_{t+1}), \alpha(s_{t+1}), 0) \geq W(z_i(s_{t+1}))\} \tag{3}$$

Note that bonds are redundant in this setting, although including them facilitates somewhat the setup of the planner's problem in an open economy where aggregate bond holdings, denoted $B$, are potentially non-zero.
Importantly, the portfolio constraint (3) limits the issuance of assets that demand net repayments in high income periods, when the outside option of default is most attractive. On the one hand, this reduces transfers from high to low income individuals under insurance contracts. But on the other, it defines a maximum level of debt that individuals, and thus the country on aggregate, can sustain. The attractiveness of default during periods of high individual income, determined by the value of the outside option of financial autarky $W$, is thus the main determinant of the aggregate net asset position in stationary equilibrium.

### 2.3 The household's problem

Every period, households maximise their expected utility by choosing current consumption and assets subject to budget and borrowing constraints. As shown in Alvarez and Jermann (2000) this problem has a recursive representation as

$$\begin{aligned} V(z(s), a(s), b) &= max_{c,\{a(s')\},b'}\{u(c) + \beta E_s V(z', a(s'), b')\} \\ s.t.\ c + \sum_{s'} a(s')q(s') + b' &\leq Rb + a(s) + z(s) \\ a(s') + Rb' &\geq A(s') \\ A(s') &= min\{\alpha(s') : V(z(s'), \alpha(s'), 0) \geq W(z(s'))\} \end{aligned}$$

where $c, b', a'$ are policy functions of the state variables $(z(s), a(s), b)$.

### 2.4 Definition of competitive equilibrium

Following Alvarez and Jermann (2000), for given initial distributions of assets and income, a competitive equilibrium in this economy is a set of asset prices $q(s'), R$, a set of individual decision rules $c, b', a'(s')$ with associated value functions $V(z, a, b)$
such that

1. $V(z,a,b)$ is the households maximum value function associated to the household problem given $q(s'), R$

2. $V(z,a,b)$ is attained by $c, b', a'(s')$

3. Markets for state-contingent assets clear
$\int_{\mathbb{I}} a_i(s_t) = 0, \ \forall s_t, t$

4. The interest rate on bonds is equal to the world interest rate $R$.

The competitive equilibrium is called "stationary" if prices and aggregate bond holdings are constant, and the distribution of individual consumption and wealth holdings is stationary through time.

# 3 Aggregate savings and individual heterogeneity in stationary equilibrium: analytical results

In this section I show analytically how, unlike with unconstrained complete markets, individual participation constraints ensure the existence of a stationary equilibrium when world interest rates differ from the rate of time preference in an economy. And I show how, across stationary equilibria, a rise in income risk can leave consumption inequality unchanged, but decreases aggregate asset holdings. I show that this generally holds in the two income version of the model, and provide sufficient conditions for higher income risk to decrease assets in the more general case. To derive these results I exploit the constrained efficient nature of the economy that allows me to solve the associated planner's problem as in Marcet and Marimon (2009).

## 3.1 The planner's problem and first order conditions

Alvarez and Jermann (2000) show that a version of the first welfare theorem applies to the closed economy version of this environment. The assumption of a given interest rate changes the aggregate feasibility constraint but, together with an appropriate No-Ponzi condition, leaves this result intact. This allows me to focus on participation-constrained efficient allocations. In particular, I use the technique developed by Marcet and Marimon (2009), who show how the efficient competitive equilibrium allocation solves the following planners problem. For a given bounded measurable weighting function $\mu_{i,0} : \mathbb{I} \to R^+$ in a linear social welfare function $\Omega = \int_{\mathbb{I}} \mu_{i,0} E_0 \sum_0^\infty \beta^t u(c_{i,t})$ the problem of the planner is to distribute ressources optimally subject to individuals' participation constraints and the aggregate ressources of the economy

$$\mathbb{VV}(\Phi_{\mu_{i,0}}, B_0) = max_{\{c_{i,t}\}} \int_{\mathbb{I}} \mu_{i,0} \sum_{t=0}^{\infty} \beta^t u(c_{i,t}) \tag{4}$$

$$s.t. \int_{\mathbb{I}} c_{i,t} + B_{t+1} = \int_{\mathbb{I}} z_{i,t} + R_t B_t, \ \forall t$$

$$V_{i,t} \geq W(z_{i,t}), \ \forall t, i$$

$$B_t \geq -\frac{Y}{R-1}, \ \forall t$$

where the planner's maximum value $\mathbb{VV}$ is a function of $\Phi_{\mu_{i,0}}$, the initial distribution of planner weights induced by $\mu_{i,0}$, and aggregate bond holdings $B_0$. $V_{i,t}$ denotes the expected value of the consumption sequence the planner gives to agent i starting in period t, and the last line is a No-Ponzi condition on aggregate bonds $B$, which I assume to be 0 in period 0. Also, I assume that $\mu_{i,0}$ only takes a finite number of values.

Note that the problem in (4) is not recursive in the cross-sectional distribution of income. Intuitively, the planner optimally provides an increase in value $V_{i,t}$ to participation-constrained individual i by an increase in both current and future consumption. But this requires the planner to keep her consumption promise even if individual i receives a negative income shock tomorrow. The solution thus has potentially infinite history dependence. But Marcet and Marimon (2009) show how, based on the Lagrangian associated to the sequential planner's problem, this history-dependence can be encoded in a time varying value of individual welfare weights $\mu_{i,t}$. Particularly, the assumptions on $\Phi_{\mu_{i,0}}$, utility and transition probabilities ensure that the problem is sufficiently well-behaved to have a saddle-point representation that is recursive in a time-varying distribution of weights $\Phi_{\mu_{i,t}}$ and aggregate bond holdings[2]

$$\mathbb{VV}(\Phi_{\mu_i}, B) = \inf_{\gamma_i \geq 0} \max_{\{c_i\}} \int_{\mathbb{I}} [(\mu_i + \gamma_i) u(c_i) - \gamma_i W_i] + \beta E[\mathbb{VV}((\Phi_{\mu_i'}, B')] \tag{5}$$

$$s.t. \int_{\mathbb{I}} c_i + B' = \int_{\mathbb{I}} z_i + RB$$

$$\mu_i' = \mu_i + \gamma_i \tag{6}$$

$$B_t \geq -\frac{Y}{R-1} \ \forall t \ \Phi_{\mu_i}$$

$$\Phi_{\mu_{i,0}}, B_0 \ given$$

[2]To see this, note that that the initial weighting function $\mu_{i,0}$ only takes a finite number of values, and that for every $t < \infty$ the set of possible income histories $Z^t$ is finite and bounded. So the exogenous state space is the Euclidian Product of a countable number of compact sets, and thus, according to Tychonoff's theorem, compact. Also, given the No-Ponzi condition, aggregate bond holdings are bounded and thus lie in a convex compact set, implying that feasible consumption allocations are just a simplex, and thus a convex set, every period. With concave utility, the constraint set is therefore compact and convex, and non-empty since autarky is feasible and incentive-compatible. The Problem thus fulfills conditions A1 to A5 in Marcet and Marimon (2009), and therefore has a recursive saddle-point representation. For further detail, see the proof of uniqueness and existence in the Appendix.

where $\gamma_i$ corresponds to the multiplier on i's participation constraint in the sequential problem (4). Note that the weights of individuals in the social welfare function are now updated every period to meet participation constraints.[3] And when $\gamma_i$ is zero, so i is unconstrained, (6) ensures promise-keeping by the planner. Intuitively, by increasing multipliers the planner allocates a higher than expected consumption path to constrained individuals with positive income shocks, to keep them "happy" with the contract. The absolute weights of the remaining, unconstrained individuals are constant, but decline relative to those for individuals with positive income shocks. This leads to a gradual decline in consumption for these individuals until they either receive a positive income shock, or reach the level of constant consumption that, given prospects for future shocks, just meets the participation constraint corresponding to their income level. The first order conditions[4] for individual consumption imply

$$\frac{U'(c_i)}{U'(c_j)} = \frac{\mu_j + \gamma_j}{\mu_i + \gamma_i} \tag{7}$$

Thus, since $U'(c)$ is decreasing, individuals with a higher weight receive higher consumption. Also, from the first order condition for aggregate bond holdings, the interest rate is tied to the ratio of the multipliers $\lambda$, associated to the aggregate feasibility constraint in (5)

$$R = \frac{\lambda}{\beta E[\lambda']} = \frac{\beta\lambda}{\lambda'} = \frac{U'(c_i)(\mu_i)}{\beta U'(c_i')(\mu_i + \gamma_i)} \tag{8}$$

where the second equality exploits the absence of aggregate uncertainty and the law of large numbers,[5] and the third uses the intratemporal optimality conditions for consumption. Importantly, the interest rate determines the slope of marginal utility for those consumers who remain unconstrained ($\gamma_j = 0$)

$$U'(c_i) = \beta R U'(c_i') \tag{9}$$

Given monotonicity of $U'$, this provides a law of motion for the consumption of unconstrained agents. Particular, if $R = \frac{1}{\beta}$ insurance is necessarily perfect in this economy in the long run

[3] Again, despite the continuum of agents, the values of multipliers remain countable, since $\mu_i' = \mu_i + \gamma_i$ is a function of current income and the past value of $\mu_i$ only. So, given my assumption of a countable support of $\Phi_{\mu_{i,0}}$, the number of individual multipliers remains countable.

[4] Note that continuously differentiable utility and a convex constraint set imply that the value function is differentiable. Also, Inada conditions and concavity of the utility function imply that the first order conditions, together with participation constraints, are sufficient to characterise the optimum.

[5] Since the state space is finite every period, the assumption of independent shocks over a continuum of agents ensures that the law of large numbers applies. Formally, $\int_{\mathbb{I}} x(i,t) = \sum_{Z\times\{\mu_{i,t}\}} \int_{\mathbb{I}} I_{\mu,z} = \sum_{Z\times\{\mu_{i,t}\}} \amalg_{\mu_t,z}$ where $I_{\mu,z}$ is the indicator function of the set $\{i : \mu_i = \mu, z_i = z\}$ and $\amalg_{\mu_t,z} \in [0,1]$ is the mass of individuals with weight $\mu$ and income $z$ in period $t$. So we can replace integrals with summation over countable sets. Given the continuum of agents $i \in \mathbb{I}$, this ensures that the law of large numbers applies. So the joint distribution of income and weights $\mu$ tomorrow is known today. On the law of large numbers in economies with a continuum of agents and independent idiosyncratic risk, see Uhlig (1996).

as individual consumption never declines, but increases when participation constraints bind until finally all individuals are at the same maximum level. To focus on the interesting case where consumption heterogeneity persists in the long-run, I look at world interest rates lower than the rate of time preference $R < \frac{1}{\beta}$. In this case, asset prices are higher, and insurance thus more costly, leading to imperfect risk-sharing. Moreover, the higher the cost of insurance, and the lower $R$, the faster falls consumption of unconstrained agents as demonstrated by the law of motion (9). Accordingly, note that if $\frac{U'(z_1)}{\beta U'(z_N)} > 1$, (9) immediately yields a minimum interest rate $R_{min} = \frac{U'(z_1)}{\beta U'(z_N)} > 1$ below which all individuals simply consume their endowments. This is because, whenever $1 < R < R_{min}$, there are no participation-compatible unconstrained transitions in (9). So individual consumption is simply equal to individual income. In the following, I thus concentrate on cases with $R_{min} < R < \frac{1}{\beta}$.

### 3.2 Existence, uniqueness and stationarity of equilibrium

The planner's problem of the previous section has a unique solution (see the appendix for a proof). However, we do not know if the constrained optimal allocation is stationary in terms of the long-run behaviour of aggregate consumption and its distribution across individuals. For example, in a standard open economy with complete domestic markets that are not participation-constrained, $R < 1/\beta$ implies that consumption levels are forever declining. So no stationary solution exists. With participation constraints, however, we can rule out this type of non-stationarity, as the total value that the planner can distribute to individuals declines with the level of aggregate resources. A permanently downward-sloping path of aggregate consumption thus necessarily violates individual participation constraints at some point in the future.

To see how individual participation constraints imply the existence of a stationary equilibrium, note that for given individual weights $\mu_i$, individual contract values decline with decreasing aggregate consumption. This requires stronger increases in relative weights of participation-constrained individuals $\gamma_i$ as individual participation constraints become more binding. Since future consumption relaxes participation constraints not only in the same but in all previous periods, more binding participation constraints increase the planner's marginal valuation of future resources relative to today's. Thus, in an economy with participation constraints facing an interest rate $R < 1/\beta$, the aggregate consumption decline slows down over time, as participation constraints become ever more binding and the planner's marginal rate of intertemporal substitution increases, until the economy reaches its stationary state with constant aggregate consumption. In other words, individual consumption volatility effectively replaces the non-stationarity of aggregate consumption. Thus, individual participation constraints provide an additional way of "closing small open economies" (Schmidt-Grohé et al 2003).

To see this algebraically, take the case of CRRA preferences, $u = \frac{c^{1-\sigma}}{1-\sigma}$. There, we can use (8) to express the planner's marginal rate of intertemporal substitution as a product of aggregate consumption growth and a term that measures volatility in individual consumption

$$R = \frac{1}{\beta}[\frac{C'}{C}\frac{\int_{\mathbb{I}}(\mu_i^{1/\sigma})}{\int_{\mathbb{I}}(\mu_i+\gamma')^{1/\sigma}}]^{\sigma} \quad (10)$$

Thus, for a given interest rate, a rise in individual volatility, measured as a stronger per-period increase in the weighted average of individual planner weights $\frac{\int_{\mathbb{I}}(\mu_i^{1/\sigma})}{\int_{\mathbb{I}}(\mu_i+\gamma')^{1/\sigma}}$, slows down the aggregate consumption decline.[6] In the resulting unique stationary equilibrium, participation constraints pin down a constant level of consumption for all constrained agents, while unconstrained agents' consumption is determined by their consumption in the previous period via the law of motion (9), given the exogenous interest rate $R$. The formal proof of existence and uniqueness is by construction of the stationary equilibrium for a given interest rate. The following section describes the resulting stationary consumption distribution. Algebraic details are given in Broer (2009b).

### 3.3 Income risk and aggregate debt in stationary equilibrium

This section shows how an increase in the riskyness of incomes can reduce aggregate assets in the stationary equilibrium of this economy. The intuition for this result is straightforward. With interest rates below the rate of time preference ($R < \frac{1}{\beta}$), individuals have incentives to front-load consumption and accumulate liabilities over time. Participation constraints, however, limit their ability to do this, by restricting individual liabilities to levels where default is less attractive than continuation of contracts. By lowering the value of financial autarky, and thus default, higher income risk relaxes participation constraints, and increases the liabilities constrained individuals can accumulate.

The previous intuition holds for all agents that are constrained at any given level of current income $z_i$. To show that its implication also holds for aggregate assets in the economy, however, we need to take into account that a rise in risk changes, first, the distribution of current incomes, and second the savings behaviour of unconstrained individuals. In particular, an increase in income risk implies that, across stationary equilibria, a positive mass of individuals moves to higher income values, which may relax or tighten their incentive constraints. Also, in simple savings models unconstrained individuals will typically increase their precautionary savings when incomes become riskier. In most economies the net effect is analytically untractable. In the complete markets environment of this paper, however, even though some individuals increase their savings in response to higher income risk, we can exploit the analytical solution of the

[6] Note that this term is absent in small open economies with unconstrained complete markets, where thus, even without aggregate uncertainty, no stationary equilibrium exists for $R < \frac{1}{\beta}$.

consumption distribution in Broer (2009), who generalises results in Krueger and Perri (2005), to show that the net effect of relaxing constraints is to reduce aggregate assets. Working with the distribution of consumption, rather than assets directly, is convenient for two reasons. First, unconstrained individuals' consumption can be recursively defined as a function of consumption in the most recent constrained period using the law of motion (9). So a fall in constrained consumption decreases all unconstrained consumption paths that follow it and thus reduces aggregate consumption along those paths. Second, in stationary equilibrium aggregate assets $A$ simply rise linearly with aggregate consumption $C$, according to the stationary budget constraint of the economy $A = \frac{C-Y}{R}$.

#### 3.3.1 The case of two income levels

For concreteness, this section first considers a simple version of the economy, where the income process described above takes only two values $\{z_h, z_l\} = \{y_0 + \frac{1}{\nu}\epsilon, y_0 - \frac{1}{1-\nu}\epsilon\}$, $\epsilon \geq 0$, where $\nu = \frac{1-q}{2-q-p}$ is the stationary mass of high-income individuals for the transition matrix $F = [p, 1-p; 1-q, q]$. Monotonicity and absolute continuity require $0 < 1-q < p < 1$. This limited commitment economy with two income levels is similar to those considered in previous contributions (e.g. Kehoe and Levine (2001), Krueger and Perri (2005, 2006), Thomas and Worrall (2007)). More particularly, the environment presented here generalises the continuum economy in Krueger and Perri (2005) to the case of persistent incomes, assuming that this persistence is not too different in high and low income states:

$$p, q > 1/2 \tag{11}$$

$$\frac{\beta - 1}{\beta} < p - q < \frac{1-\beta}{\beta} \tag{12}$$

In this context, a "**marginal rise in income risk**" is simply a small change $d\epsilon > 0$. The specification of $Z$ ensures that this is a mean-preserving spread for all values of $p, q$, and thus leaves aggregate resources unchanged.

Broer (2009b) shows how one can use the law of motion (9) together with the participation constraint of high-income individuals to derive a closed-form of the consumption distribution in this case, assuming CRRA preferences. Figure 2 illustrates these results, for general preferences, by showing the consumption path of an individual that receives a sequence of income shocks.[7]

[7] Broer(2009) derives the closed form of the consumption distribution $\Phi_C : \mathbb{C} \subseteq R^+ \longrightarrow [0,1]$ in this simple case for CRRA preferences. The distribution has a discrete support $c_1, c_2, ..., c_m$ with $c_i = c_1(\beta R)^{\frac{i}{\sigma}}, 1 < i < m$, $c_m = y_0 - \frac{1}{1-\nu}\epsilon$ and $m = min\{x \in N : x > \frac{\sigma[ln(y_0 - \frac{1}{1-\nu}\epsilon) - ln(c_1)]}{ln(\beta R)}\}$. The upper bound is $c_1 = \{\frac{(1-\sigma)(1-\beta q(\beta R)^{\frac{1-\sigma}{\sigma}})}{1+\beta(1-p-q)(\beta R)^{\frac{1-\sigma}{\sigma}} - (1-p)\beta^m q^{m-1}(\beta R)^{\frac{m(1-\sigma)}{\sigma}}}[\frac{1-\beta(p+q)-\beta^2(1-p-q)}{1-\beta q} W_h - (1-p)\beta^m q^{m-2}(qW_l - \frac{(1-q)W_h}{1-\beta q})]\}^{\frac{1}{1-\sigma}}$ and

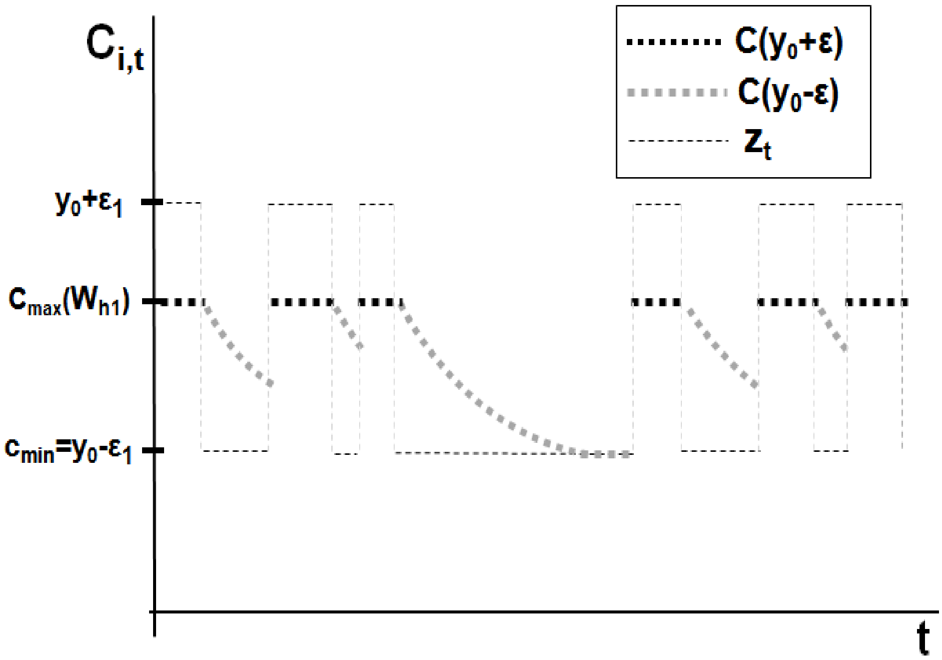


Figure 2: A path of individual consumption in the two income version of the model.

Consumption at high income is constant through time, at a level $c_{max}$ that depends on the value of autarky $W(z_h)$ via the participation constraint, but is always lower than high income ($c_{max} < z_h$) which individuals are happy to accept in return for some insurance in the future. After a low income shock, individuals move smoothly down in consumption along a sequence according to the law of motion (9), with a lower bound equal to $z_l$, which exactly solves her participation constraint at low income. So consumption in all unconstrained states moves up and down with the level of constrained consumption at high income, and thus with the value of financial autarky $W(z_h)$. Since all individuals experience independent realisations of the same income process, the cross-sectional distribution of consumption is simply the "row-sum" across the time series in figure 2.

What is the effect of a rise in risk $d\epsilon$ on the consumption path in figure 2, and thus on its cross-sectional distribution? A rise in $\epsilon$ increases the riskyness of future incomes, which acts to reduce autarky values $W(z)$. If this was the only effect, a rise in income risk would simply lead to a downward shift in the whole stationary distribution of consumption in line with the change in autarky value at high income, and thus trivially reduce aggregate consumption and assets. But, importantly, given persistence, $d\epsilon$ also changes the expected lifetime incomes of poor and rich individuals. Particularly, it increases current and expected ressources of the income-rich.

thus increases in $W_h$. The frequency distribution declines geometrically at rate $q$, and is given by $\Phi(c_1) = \frac{1-q}{2-q-p} = \nu$,$\Phi(c_i|_{1<i<m}) = \nu(1-p)q^{i-1}$,$\Phi(c_m) = (1-\nu)q^{m-1}$.

For them, the incentive-improving second-order effect of higher risk is thus counteracted by a first-order effect of increasing mean income. Figure 3 shows how, at moderate levels of risk (low $\epsilon$), the net effect is to raise the attractiveness of default for the income-rich by increasing the utility value of their income stream. Only as risk rises beyond some level $\epsilon^{\star}$, implying sufficiently low $z_l$, the default-deterring second-order effect of higher risk dominates. This is because the marginal utility loss from a further reduction in low income $U'(z_l)\frac{1}{1-\nu}d\epsilon$ increases without bound, due to Inada conditions.[8]

So for high levels of risk $\epsilon > \epsilon^{\star}$, it is immediate that a further rise in risk $d\epsilon > 0$, which reduces autarky values for both high and low income individuals and thus relaxes all participation constraints in the economy, decreases aggregate consumption and assets in stationary equilibrium. This holds similarly for the more general version of the economy with $N > 2$, as I will show in the next section. The challenge, however, is to show that negative relationship between aggregate consumption and assets, on the one hand, and income risk on the other also holds for $\epsilon < \epsilon^{\star}$, where the incentive-improving second-order effect of higher risk does not dominate the rise in mean incomes of rich individuals. This is done, in the following proposition, which shows that, for a given interest rate, aggregate consumption decreases as income risk rises in this limited commitment economy. The proof is based on a dual-argument, exploiting the fact that, for a given interest, the stationary consumption distribution illustrated in this section minimises the aggregate resources needed to provide participation-compatible levels of life-time utility to constrained individuals.[9]. Therefore, if we can show that in response to a rise in risk

[8]To see this algebraically, note that autarky values are

$$W_h = \frac{(1-\beta q)u(y_0 + \frac{1}{\nu}\epsilon) + \beta(1-p)u(y_0 - \frac{1}{1-\nu}\epsilon)}{1-\beta(q+p) - \beta^2(1-(q+p))} \tag{13}$$

$$W_l = \frac{\beta(1-q)u(y_0 + \frac{1}{\nu}\epsilon) + (1-\beta p)u(y_0 - \frac{1}{1-\nu}\epsilon)}{1-\beta(q+p) - \beta^2(1-(q+p))} \tag{14}$$

Given the assumptions on transition probabilities, $W_l$ is always declining in $\epsilon$, while the high income-autarky value $W_h$ is concave in $\epsilon$ with a maximum at some $\epsilon* > 0$. It increases for $\epsilon < \epsilon*$, decreases for $\epsilon > \epsilon*$ and crosses the perfect insurance value at $\bar{\epsilon} > \epsilon*$. To see this, take the first derivative of autarky values with respect to $\epsilon$

$$\frac{dW}{d\epsilon} = (I - \beta F)^{-1}[\frac{1}{\nu}u'(y_0 + \frac{1}{\nu}\epsilon), -\frac{1}{1-\nu}u'(y_0 - \frac{1}{1-\nu}\epsilon)] \tag{15}$$

The persistence assumptions assures that for $\epsilon = 0$ the rise in current utility dominates the fall in future expected utility. With strictly positive entries of $F$, however, Inada conditions on $u$ translate to $W_h$, so marginal utility goes to infinity as the low income realisation goes to zero: as $\epsilon \longrightarrow y_0$, $\frac{dW_l}{d\epsilon} \longrightarrow -\infty$. By the intermediate value theorem and continuity, there exists an $\epsilon*$ with $\frac{dW_h(\epsilon*)}{d\epsilon} = 0$, and $\bar{\epsilon} > \epsilon*$ with $W_h(\bar{\epsilon}) = 0$. Also, the concavity of the utility function translates to the concavity of autarky values as a function of $\epsilon$

$$\frac{dW^2}{d\epsilon^2} = (I - \beta F)^{-1}[(\frac{1}{\nu})^2 u''(y_0 + \frac{1}{\nu}\epsilon), (\frac{1}{1-\nu})^2 u''(y_0 - \frac{1}{1-\nu}\epsilon)] < 0 \tag{16}$$

[9]See Krueger and Perri (2005) who derive the consumption distribution in the two income case from an

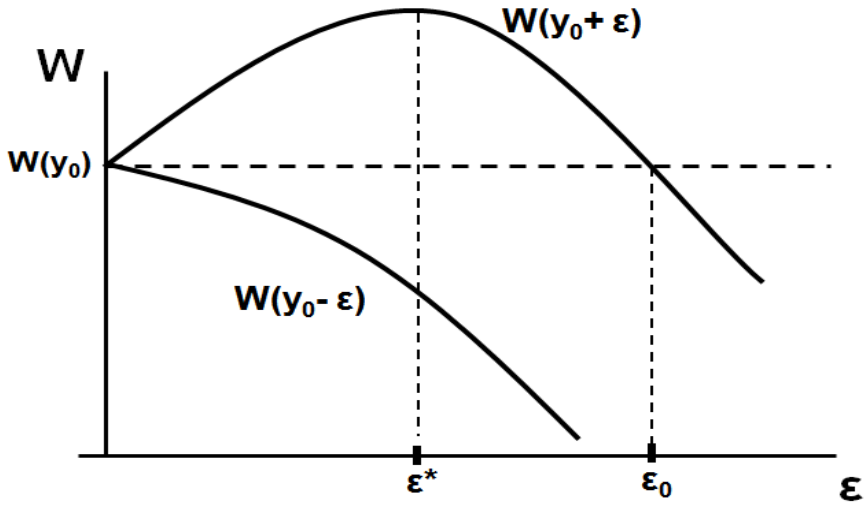


Figure 3: The figure depicts the values of autarky at high and low income in the two income version of the limited commitment continuum economy.

there exists some change in stationary consumption levels that is participation-compatible and reduces aggregate consumption, the change in aggregate equilibrium consumption is necessarily negative. So excess demand for consumption in stationary equilibrium is down-sloping in income risk in this limited commitment economy.

**Proposition 1** In the economy with two income values, a rise in income risk $d\epsilon > 0$ leads to a fall in stationary consumption, and assets.

**Proof**

I first construct, in response to a marginal rise in income risk $d\epsilon$, a resource-neutral incentive-compatible change in the stationary consumption allocation $\{dC\}$ for which participation-constraints of high-income individuals become slack. By reducing consumption of high-income individuals by a small amount we can thus derive a participation-compatible allocation associated with a decline in aggregate consumption.

To see this, consider $\widetilde{dC} : dc_i = dz_i$, i.e. a change in consumption equal to that in incomes for all individuals. Trivially, this is resource-feasible and meets participation constraints for low income individuals, as their constrained level of consumption moves 1-for-1 with income, according to figure 2. To see that $\widetilde{dC}$ leads to slack participation constraints at high income, denote as $\Delta dW$ the difference between the change in contract value implied by $\widetilde{dC}$ and that in autarky value implied by $dz$, at high income. This can be written as

expenditure minimisation problem using the technique developed in Atkeson and Lucas (1992, 1995)

$$\Delta dW = dV(z_h) - dW(z_h) \quad (17)$$

$$= [1 - \beta p - (1-p)(1-q)\sum_{i=2}^{m} q^{i-2}\beta^i]\{\frac{1}{\nu}[u'(c_1) - u'(z_h)] - \frac{1-p}{1-\nu}\sum_{i=1}^{m}\beta^i q^{i-1}[u'(c_i) - u'(z_l)]\}d\epsilon > 0 \quad (18)$$

where $c_i,\ i = 1, ..., m$ is consumption along the downward-sloping, unconstrained path in figure 2, with $m$ the maximum number of transitions before an individual becomes constrained at low income. Note that in $\Delta dW$, all states where individuals are constrained at low income cancel, as there, lifetime utility after the change $\widetilde{dC}$ is equal to that under autarky. Since marginal utility is decreasing, the marginal utility gain from a rise $\frac{1}{\nu}d\epsilon$ in consumption at $c_1 < z_h$ is necessarily higher than that at autarky consumption $z_h$. Equally, the marginal utility loss of a given fall in unconstrained consumption $c(z^s) > z_l$ is necessarily lower than that in autarky. So the implied change in the contract value is necessarily more positive than that in autarky values, or $\Delta dW > 0$ and participation constraints at $z_h$ are slack. Now reduce consumption in high income periods by a small number $\varsigma > 0$. For sufficiently small $\varsigma$ this does not violate participation constraints, but leads to a fall in aggregate consumption equal to $\nu\varsigma$. Since the optimal change in the stationary consumption allocation $\{dC^\star\}$ minimises participation-compatible expenditure, it implies a fall in aggregate consumption not smaller than $\nu\varsigma$. So aggregate consumption, and therefore the stationary level of assets, both decline in risk $\epsilon$. ■

### 3.3.2 The general case $N > 2$

It is trivial to show that a proof similar to that in proposition 1 applies in an economy with three income states ($N = 3$), where only the upper and lower values are affected by changes in risk. It does not, however, apply to a more general economy with $N > 3$. But we can exploit a generalised version of the Laffer-curve type relationship between autarky values and individual income risk in figure 3, to derive a sufficient condition for assets to decline with risk. For this, we need to show that even when a small group of individuals receives very large increases in current income as inequality rises, their autarky values eventually decrease. Inada conditions and the strictly positive transition probabilities in $F$ ensure that this is the case.

**Proposition 2** In the general economy with $N > 2$, consider a sequence of rises in income risk $\{dz\}$ indexed by $k$. There is an element $K$ of this sequence, such that $dz_k$ causes aggregate consumption and assets to fall for $k > K$.

**Proof**

Given the markov structure of income, the value of default as a function of $z^k$, the kth element in the sequence of income vectors implied by $\{dz\}$, can be written as

$$W(z^k) = \sum_{t=0}^{\infty} (\beta F)^k U(z^k) = (I - \beta F)^{-1} U(z^k) = \mathbb{F} U(z^k) \tag{19}$$

Due to the positive entries of $F$ both the mass of individuals at any given income value $\Phi_{Z_j}$, and their probability to transit to lowest income $Z_N$ $\mathbb{F}_{jN}$ are bounded below, by $min_j \Phi_{Z_j} > \zeta$ and $min_i \mathbb{F}_{iN} > \vartheta$ respectively, for some numbers $\vartheta, \zeta > 0$. The impact of a small change in income risk on the stationary autarky values is given by

$$dW(z^k) = \mathbb{F} U'(z^k) dz \tag{20}$$

$maxdW(z^k)$ can be bounded by considering a group of individuals with minimum mass $\zeta$ at 'middle' income j (the lowest income (highest marginal utility) value that experiences a rise in consumption when risk increases) who receive a maximum transfer of resources, bounded by $dz_{jk} < Y$. Neglecting all negative terms apart from that at lowest income we get

$$max_{n \in \{1,\ldots,N} dW(z_n^k) < sum_{i=j}^{N} \mathbb{F}_{nj} U'(z_j) \frac{Y}{\zeta} - \vartheta \beta U'(z_N^k) \vartheta \tag{21}$$

Since $z_N^k$ is strictly decreasing in $k$, the second term goes to minus infinity while the first is finite and independent of k. So from some $K$ onwards, all elements of $dW$ are necessarily negative and a rise in risk strictly relaxes all participation constraints in the economy.
The result than follows trivially from the fact that consumption of all unconstrained agents can be recursively defined from their last constrained consumption level using (9). By relaxing constraints on front-loading of consumption, a fall in all autarky values thus lowers stationary equilibrium consumption for all individuals and leads to a fall in aggregate stationary consumption and assets. ■

This section has shown that excess demand for consumption decreases in income risk in an economy with limited commitment to contracts, leading to a fall in stationary assets as risk increases. This is always the case in the popular specification with two income values. And it holds at sufficiently high levels of risk in the more general case.

### 3.4 The decoupling of income and consumption inequality in an open economy

So far, we have seen that in an economy with limited commitment to contracts, foreign asset holdings decrease after a rise in domestic income risk. But how does this affect consumption inequality? The following result shows that in an open economy facing a given world interest rate, the inequality of consumption can become completely independent from that of income.

**Corollary 3**

*For $N = 2$ and CRRA preferences, if $R$ is high enough for the mass at the lower bound of the distribution to be negligible ($\Phi(c_m) \approx 0$), the variance of log-consumption is*

$$Var_c = \Lambda[\frac{\log(\beta R)}{\sigma}]^2 \tag{22}$$

*where $\Lambda > 0$ is a function of transition probabilities only. So (log) consumption inequality is entirely determined by world interest rates $R$, where a higher $R$ lowers domestic consumption inequality. If there is a non-negligible mass at the truncation point, $\Phi(c_m) > 0$, this is an upper bound for the cross-sectional variance of individual consumption.*

For the simple algebra that leads to the result see Broer (2009b). The intuition is straightforward: Income risk affects the stationary distribution of consumption mainly via the participation constraint at high income that determines its upper bound, and thus the position of the distribution. Apart from the truncation at $z_l$, which becomes negligible at high interest rates, the shape of this distribution, however, depends entirely on the value of $R$, via the law of motion (**??**). With CRRA preferences, the relative values of consumption are thus only a function of the interest rate and parameters. So international interest rates determine consumption inequality, while income risk determines mean consumption, and thus asset holdings.

Corollary 3 results from the geometric consumption distribution with $N = 2$ when the participation constraint at $z_l$ is not binding. For the general case $N > 2$, the consumption distribution can be characterised by $N$ minimum participation-compatible consumption levels, associated to $N$ incomes and autarky values, that provide the upper bounds for geometric sub-distributions (see Broer 2009b for details). The shape of the sub-distributions is again independent of the upper bound, with variance that decreases in $R$. However, changes in income risk now change relative autarky values and thus do not move the subdistributions in parallel. So the shape of the overall consumption distribution is not independent of income risk. But it is easy to show that the width of the support of the distribution decreases with R.

## 3.5 Extensions of the simple model

The environment analysed in this section is based on relatively standard assumptions about the stochastic process of individual income and preferences. Nevertheless, there are two natural extensions to the analysis: First, the assumption that default leads to complete financial autarky seems restrictive. Particularly, permanent exclusion from insurance markets seems in line with the relatively long delay in eliminating a default episode from personal credit scores, which, for example, is 10 years for chapter 7 personal bankruptcy filings in the US. But it is hard to see how agents could be prevented from storing part of their post-default income as a buffer against negative future income shocks. The possibility to draw on savings in bad times, however, may significantly reduce the punishment of exclusion from insurance markets. How important this effect is depends crucially on the rate of return on savings after default $R^{aut}$. The assumption of financial autarky is equivalent to $R^{aut} = -1$. As the rate of return rises, self-insurance through savings becomes more powerful, improving the trade-off between the mean and volatility of consumption after default especially for the income-rich. This is crucial, as it might break the Laffer-curve type relationship between autarky values and income risk presented, for the two income case, in figure 3. So with saving after default at high interest rates, an increase in income risk may tighten participation-constraints of the income-rich independent of the level of risk. The exact level of $R^{aut}$ above which this is true depends on how the value function in the post-default income fluctuation problem varies with the rate of return and income risk, which has not been characterised analytically.[10] Unfortunately, therefore, the relationship between income risk, the attractiveness of default and, ultimately, aggregate debt is difficult to characterise analytically in an environment with savings. But the following section considers the importance of saving after default in a calibrated version of the economy.

A second natural extension of the environment would be to relax the assumption of positive entries in the income transition matrix $F$, which so far implied a strictly positive probability of moving from highest to lowest income in one period. Again, I leave this issue to the quantitative analysis in the following section, which calibrates the income process to empirical estimates for the US economy.

# 4 Individual risk and global imbalances: income uncertainty and the US net foreign asset position 1980-2003

The previous section showed that in an open, debt-constrained economy, rises in income risk can lower aggregate savings and asset positions. But importantly, in the general case with $N > 2$, this only holds for an initial level of income risk that is sufficiently high. The sign and importance of the effect of changes in income risk on asset positions thus depends on the particular economy under analysis. Also, the independence of stationary consumption inequality from income risk only holds for the special case with two income values, at a given exogenous interest rate. Thus, this section considers a version of the model that is calibrated to match some stylised features of the US economy. Particularly, I use the stochastic process for US individual incomes estimated by Krueger and Perri (2006), and compare debt holdings and consumption inequality in stationary

[10] Miao(2002) presents analytical comparative statics for this problem, but does not consider changes in income risk.

equilibria corresponding to the two endpoints of their sample, respectively 1980 and 2003. In a first step, I derive the demand function for foreign debt and the consumption inequality as a function of world interest rates in both periods. I then proceed to a general equilibrium analysis of a stylised two country economy, where the US trades bonds with a large developing country, calibrated to capture the evolution of individual income inequality in China. There, I assume domestic asset trade is limited to uncontingent assets, resulting in a rise of precautionary savings in response to an increase in individual income risk.

To analyse the effect of changes in income risk on global imbalances, a focus on the US economy seems natural, both for its large and persistent current account deficits over recent decades, and for its large increase in individual income inequality, which exceeded that observed in almost all other OECD countries during this period.[11] Although this section introduces a large emerging economy as a counterpart, the two country nature of the analysis necessarily neglects the role of other large countries for the equilibrium of the world economy. Particularly, it might seem that the analysis cannot account for the accumulation of surpluses by some advanced economies such as Germany and Japan over the last 25 years. While feasibility prevents a formal analysis of this issue in the context of the quantitative model of this paper, it is at least interesting to note that, according to proposition 2, the financial deepening effect of higher income risk is more likely to play a role in economies with high initial levels of consumption risk. That advanced economies with lower levels of inequality respond to moderate increases in income volatility by a rise in savings is thus, a priori, not evidence against the economic mechanism presented in this paper. Before turning to the results I briefly describe the calibration, and the algorithm I use to compute the stationary equilibria.

## 4.1 Calibration

I calibrate the income process following Krueger and Perri (2006), using their estimates for the years 1980 and 2003, the endpoints of their sample. The authors assume the log of post tax labour income plus transfers (LEA+) $log(z_t)$ to be the sum of a group specific component $\alpha_t$ and an idiosyncratic part $y_t$. The latter, in turn, is the sum of a persistent AR(1) process $m_t$, with persistence parameter $\rho$ and variance $\sigma_m^2$, plus a completely transitory component $\varepsilon_t$ which has mean zero and variance $\sigma_\varepsilon^2$.

[11]Comparative studies of income inequality measures have found that, apart from the United Kingdom, other OECD countries have experienced less important increases in income inequality since 1980 than the US (see e.g. Brandolini et al 2007).

The process for LEA+ is thus of the form

$$\begin{aligned} log(z_t) &= \alpha_t + y_t \\ y_t &= m_t + \varepsilon_t \\ m_t &= \rho m_{t-1} + \nu_t \\ \varepsilon &\sim N(0, \sigma_\varepsilon^2) \\ \nu_t &\sim N(0, \sigma_\nu^2) \end{aligned} \tag{23}$$

Using data from the Consumer Expenditure Survey (CEX), the authors first partial out the group-specific component $\alpha_t$ as a function of education and other variables, identifying the variance of the idiosyncratic part of income $y_t$, as well as (from the short panel dimension of the CEX) its first order autocorrelation. They then fix $\rho = 0.9989$, the value estimated by Storesletten et al (2004), which allows them to identify $\sigma_\nu^2$ and $\sigma_\varepsilon^2$.
The results show an increase in the variance of labour income of 18 percentage points between 1980 and 2003, the two periods I focus on. 11 percentage points are due to an increase in within-group inequality, out of which roughly two thirds are accounted for by an increase in the importance of persistent shocks, and one third by that of transitory shocks.
In my exercise I abstract from changes in the common wage rate and differences in the group specific component, which, in the present model as in that of Krueger and Perri, translate fully into consumption differences by construction.
As a baseline calibration, I choose a CRRA utility function with coefficient of relative risk aversion of 1 (log-preferences), and a discount factor of 0.96. I then look at the sensitivity of the results to changes in parameters. And I look at the case when agents who default are excluded from all financial transactions in the current period, but allowed to invest in non-contingent bonds in the future to smooth income shocks over time. This reduces the impact of higher income risk on the attractiveness of default.

## 4.2 Model Solution

To solve the model, I first approximate the persistent process for $m_t$ with a 7-state Markov chain using the standard Tauchen and Hussey (1991) method.[12] Following Krueger and Perri (2006) I choose a binary process for the transitory shock. The computational algorithm then follows Broer (2009), who describes the recursions to derive the stationary consumption distribution in more detail.

[12] Note that this method accords to my assumption of widening the support $Z$ to increase risk, but leaving the transition probabilities unchanged.

### 4.3 Partial equilibrium results

#### 4.3.1 Income risk and net foreign assets

Figure 4 shows the main quantitative result of this paper: a rise in income risk calibrated to the experience of the US economy between 1980 and 2003 strongly reduces the stationary demand for assets in an economy with limited commitment to contracts. For example, at an interest rate of 3.5 percent (which yields a zero foreign asset position in 1980), the observed rise in income risk leads to a fall in the stationary level of net foreign assets of more than 50 percent of annual GDP.

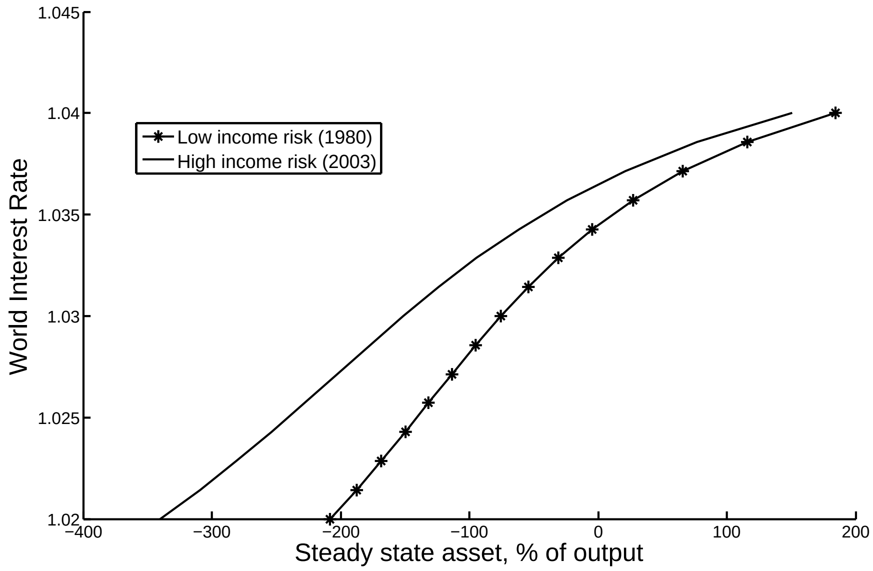


Figure 4: Asset demand function, baseline calibration.

So far, the results of this paper were derived under the arguably strong assumption that default be punished by complete exclusion from financial transactions. Figure 5 relaxes this assumption and considers a version of the economy where individuals can invest part of their income at the world interest rate from the period following default. This option to self-insure by saving makes default more attractive, as it allows agents to smooth consumption in low income periods even without access to insurance contracts. And importantly, postponing consumption by saving becomes less costly, and therefore self-insurance more powerful, at higher interest rates. Figure 5 shows that this can break the monotonicity of stationary foreign assets in the level of income risk. Particularly, for high interest rates, self-insurance through saving is so powerful that the rise in current income for the income-rich now dominates the second-order effect of higher income risk, leading to an increase in stationary assets as more attractive default tightens borrowing constraints. However, at a level of interest rates corresponding, for example, to the average real return on US treasury bills over the period of analysis, the observed rise in income risk implies a fall in stationary assets of 11 percent of GDP.

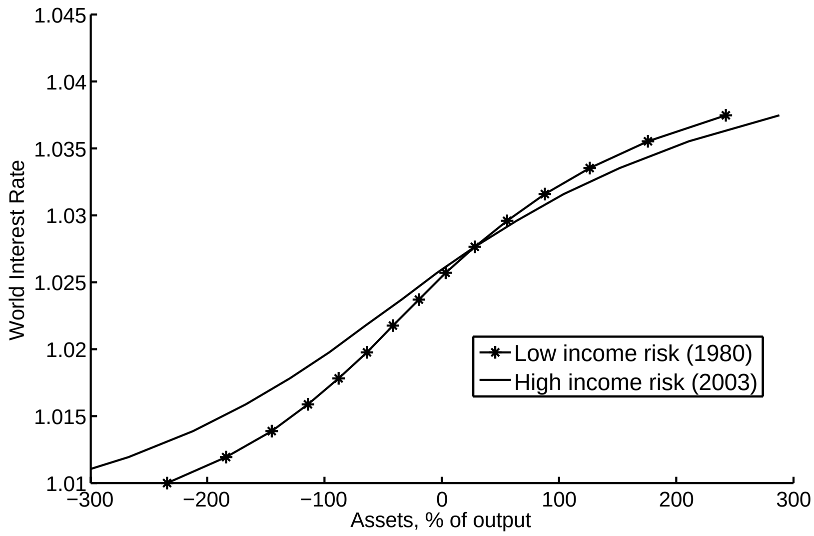


Figure 5: Asset demand function, log-preferences, saving at world interest rates permitted from the period following default.

Finally, figure 6 shows that with higher risk aversion ($\sigma = 2$), which strengthens the punishment of more volatile consumption after default, the observed rise in income risk decreases stationary assets for any real interest rate below 3.5 percent.

For a more general version of the economy, where agents can save some of their income even after default, the relationship between income risk and stationary assets thus depends on the level of world interest rates. This is why the next section endogenises the equilibrium interest rate in a general equilibrium analysis of a simple two country economy. Before, however, I analyse how consumption inequality in this economy is affected by changes in income risk and the world interest rates.

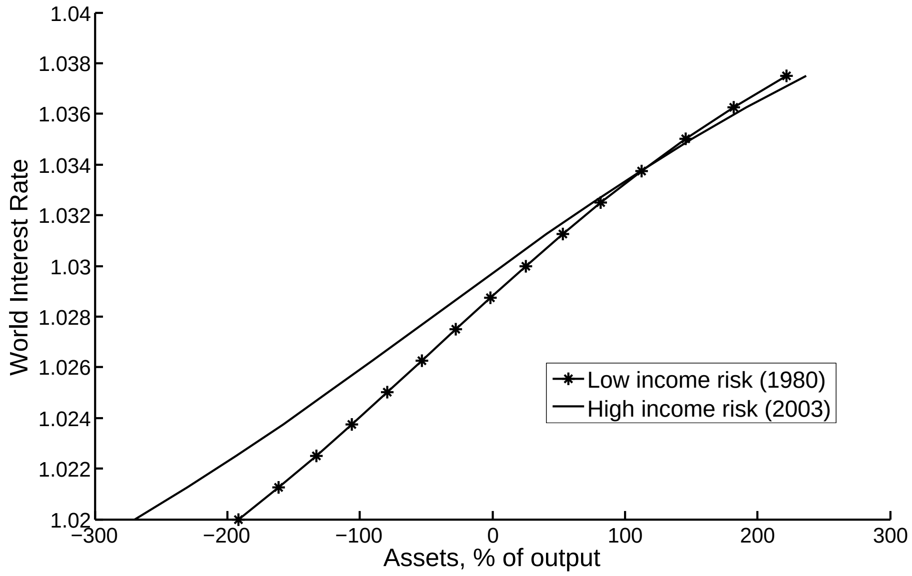


Figure 6: Asset demand function, higher risk aversion ($\sigma = 2$), saving at world interest rates permitted from the period following default.

#### 4.3.2 Income and consumption risk

In the theoretical analysis, I showed that income risk mainly determines the position of the consumption distribution, while its shape, via the law of motion (9), depended largely on the level of interest rates. Corollary 3 made this statement precise for the two income case. In a more general version of the economy, however, changes in income risk affect relative autarky values, and thus also the shape of the stationary distribution. Figure 7 shows, however, that even in the calibrated economy, interest rates remain the main driver of consumption inequality: the change in consumption volatility due to a change in income risk is an order of magnitude smaller than the changes caused by movements in the world interest rate. Thus, the limited commitment economy opens a new, interesting channel of transmission from the international economy to the level of domestic inequality in individual consumption levels.

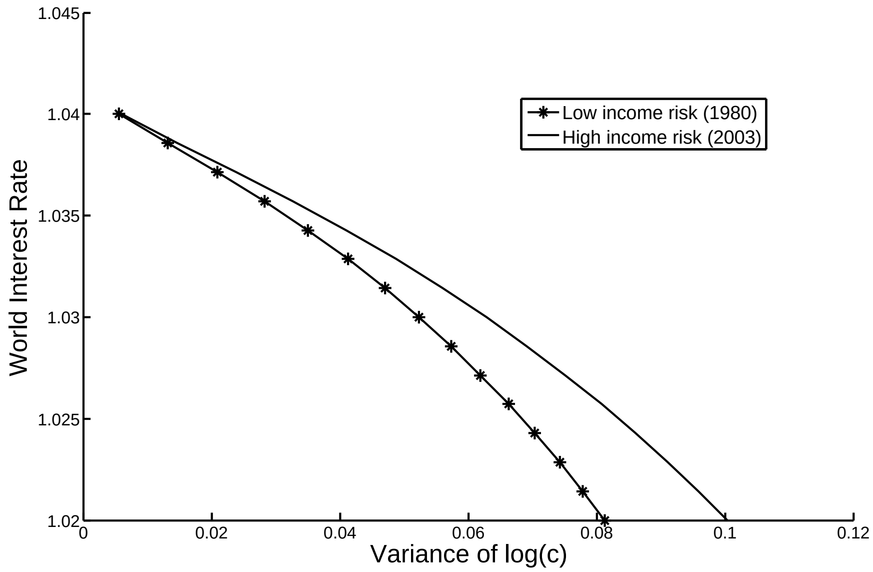


Figure 7: Variance of log consumption, baseline calibration.

### 4.4 Endogenous financial deepening meets the "savings glut": The effect of rising individual risks in developed and emerging economies

So far, the analysis was agnostic about the determinants of savings outside the US, taking as given a world interest rate. But of course, in a closed world economy, the fall in US savings caused by increased idiosyncratic risk affects the equilibrium interest rate. This section thus looks at the general equilibrium in a simple economy consisting of two countries that differ both in their domestic financial market structures and the evolution of idiosyncratic risk that their agents experience over time. Particularly, I present a stylised world economy consisting of China and the US. Both countries experience a rise in idiosyncratic income uncertainty in line with their historical experience, but differ in their ability to insure against this risk through domestic financial trade. Particularly, US financial markets are assumed to be complete but subject to participation constraints as before, allowing individuals to save at the world interest rate after they default on contracts. Chinese consumers, on the other hand, do not have access to complete domestic financial markets. Rather, I assume that individuals there can only engage in self-insurance through trade in bonds subject to a borrowing limit. As before, I abstract from aggregate risk. International asset trade is limited to non-contingent bonds, whose prices all agents take as given. A stationary equilibrium of the world economy is thus a process for individual consumption in both countries, an aggregate net asset position between the two countries and a market clearing interest rate.

### 4.4.1 Individual risk and equilibrium foreign asset positions

The analysis concentrates on the effect of changes in idiosyncratic risk on equilibrium net foreign asset positions over the last 25 years. The process of idiosyncratic risk in the US is unchanged from the previous section. Unfortunately, equivalent estimates of an income process with group-specific heterogeneity, as well as persistent and transitory within-group risk, is infeasible for China, where the necessary household panel survey is not available for the period of interest. We are thus left with estimates of cross-sectional income inequality. This is a problem, as we cannot identify the different components of individual income risk from cross-sectional data alone. But the calibrated model provides a mapping from a specific income process to the cross-sectional consumption inequality and a savings demand schedule. I thus calibrate the components of the income process to capture the Gini coefficients of consumption and income for Chinese urban regions reported in Perloff and Wu (2005) in 1985, plus a zero initial foreign asset position. Assuming that the income process in China has the same permanent-persistent-transitory structure as in the US, including the persistence parameter of 0.9989, this provides three targets for three parameters, namely the variances of the permanent, persistent and transitory component of the income process in (23).[13] The increase in idiosyncratic risk in China is then calibrated to capture the observed rise in both Gini coefficients until 2001. For this, I assume that the change in permanent income differences in China is entirely captured by the rise of Urban-Rural inequality. But I look at the sensitivity of the results to this assumption below. The results assume furthermore a relatively tight borrowing limit corresponding to average quarterly income. As country weights, I use relative GDP of both countries from the Penn World tables in 1980 and 2003.

The appendix reports the implied estimates of the income process in China. In line with the similar Gini coefficients for consumption and income, inequality in the 1980s is estimated to be mainly determined by permanent income differences: both the variance of persistent and transitory income shocks are small. But the observed rise in consumption and income inequality until the early 2000s, stronger for income than for consumption, is in line with a strong increase in both the variance of persistent and transitory shocks, by 7.5 and 6.6 percentage points respectively.

Figure 8 plots the resulting equilibria for the early 1980s and the early 2000s. Chinese assets are plotted with a negative sign, such that the intersections of the demand and supply schedules

[13] For the permanent part of income risk, I use a uniform distribution of log-income values with 5 support points, and calibrate the support width to capture the moments of the data. For the AR(1) component of the income process I choose a 5-state discretisation using the Tauchen and Hussey (1991) method.

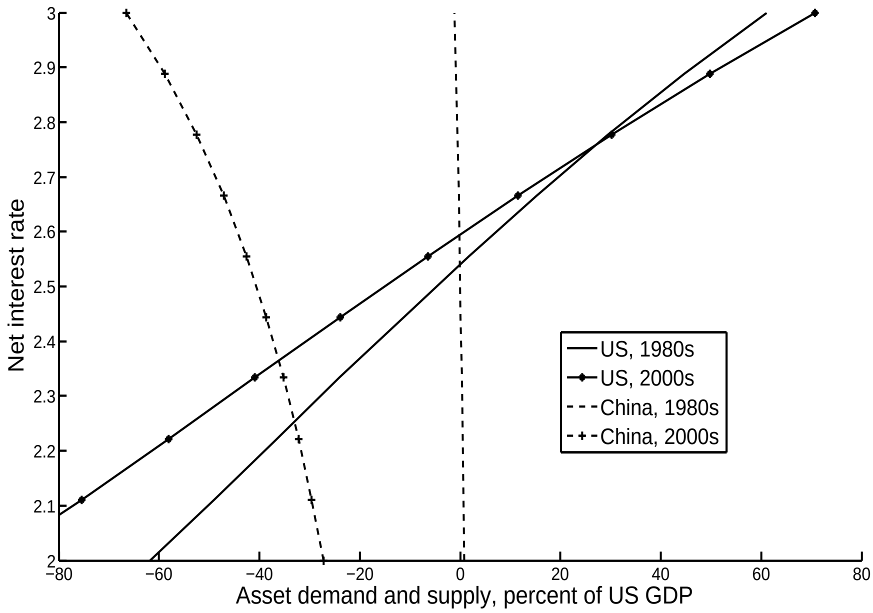


Figure 8: Asset demand and supply in a two country world economy.

give equilibrium asset positions and interest rates. The initial net interest rate of 2.5 percent is low relative to the discount factor of 0.96, as in many models of imperfect insurance. The increase in risk in the US results in the familiar fall in the savings demand schedule as a result of financial deepening. But in China, the strong rise in idiosyncratic risk after the early 1980s results in a strong rise in precautionary savings. This is exactly as we would expect in a self-insurance economy, where the financial deepening effect of higher income risk is absent, and the precautionary savings effect is relatively strong. The corresponding net effect is a fall in the US net foreign asset position to minus 36 percent of GDP, and a fall in the world interest rate of about 17 basis points.

### 4.4.2 Sensivity analysis

As it is impossible to distinguish the effect on cross-sectional inequality of increases in permanent income difference from those of the very persistent shocks in the model, Figure 8 was based on the assumption that increases in permanent income differences are entirely captured by the difference between urban and rural regions. Since precautionary savings are largely unaffected by changes in permanent inequality but rise with persistent shocks to income, this may overstate the equilibrium savings. Therefore, Figure 9 shows how the results change when I make the opposite assumption of unchanged persistent shocks (which requires some recalibration also of the variance for transitory shocks, to match both Gini coefficients). As expected, the rise in equilibrium US liabilities is lower, but at 23 percent of GDP still sizeable.

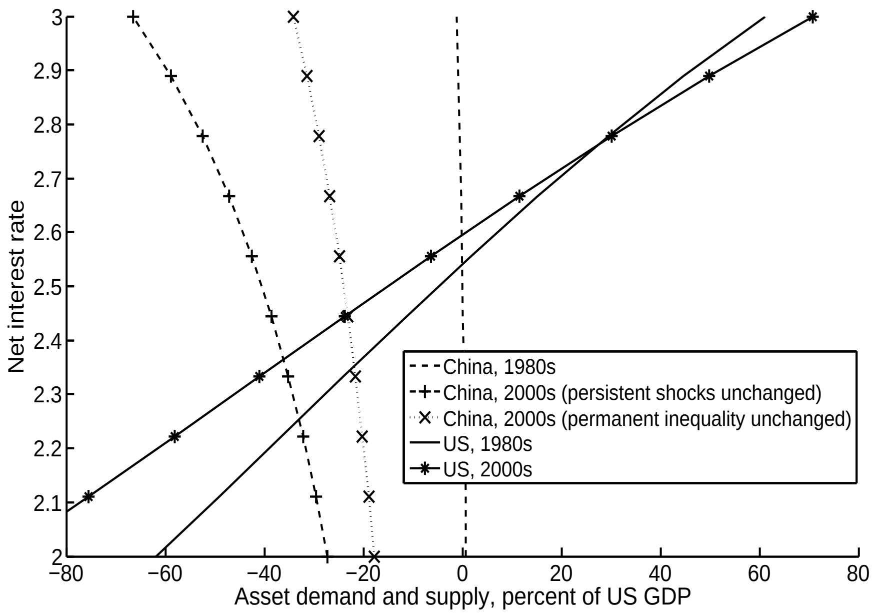


Figure 9: Asset demand and supply in a two country world economy, sensitivity.

## 5 Conclusion

This paper looked at the link between domestic income uncertainty, consumption inequality and net foreign asset positions in an economy where financial markets suffer from enforcement constraints. Domestic financial markets were assumed to be complete, but constrained by individuals' option to default on contracts, at the price of permanent exclusion from insurance markets. I showed that, contrary to economies with unconstrained complete markets, this economy has a well-defined stationary equilibrium for any given world interest rate. Higher income risk can indeed lower aggregate savings by making the punishment of default, financial autarky, less attractive, thus endogenously "deepening" financial markets. However, changes in income risk have only a small effect on consumption inequality, which depends mainly on the international interest rate. A calibration of the model to the US case showed that the changes in income risk observed between 1980 and 2003 might indeed explain an important part of the fall in the net foreign asset position. This holds not only at a constant world interest rate, but

also in the general equilibrium of a simple world economy where the US trades bonds with a country that has less sophisticated markets and experiences a strong increase in idiosyncratic risk similar to that seen in China. The "glut" in precautionary savings there and the endogenous financial deepening in the US, both caused by rising idiosyncratic risks, result in a significant deterioration of the US net foreign asset position, and a small fall in the world interest rate.
Future research should generalise this analysis in at least two directions: first, one should also take account of the change in aggregate macroeconomic risk, which declined over the period of analysis. And second, an adequate equilibrium of the world economy should not only take into account advanced countries with deficits and emerging surplus economies, but also countries like Germany or Japan, that experienced surpluses yet have relatively developed domestic financial markets. In this context, the model's prediction of an inverse U-shape relationship between net foreign asset positions and individual income risk is especially interesting.

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# 7 Appendix

## 7.1 Proof of existence and uniqueness

**Result:** *For every given world interest rate $R_{min} < R < \frac{1}{\beta}$, there exists a unique equilibrium allocation in the small country that is equal to the solution of the planner's problem for an appropriate weighting function $\mu$ in the social welfare function.*

**Proof**

I prove existence of a unique solution to the planner's problem by checking that the conditions for a simplified version of Proposition 3 in Marcet and Marimon (2009) hold in this economy. Given the finite space of individual endowments $Z$ we can apply a version of Tychonoff's theorem to see that the Euclidian product $Z^T$ is compact for countable T. So the exogenous vector of individual states lies in a compact (Borel) subset of the Euclidian Space $R^T$. And of course, the discrete transition function satisfies the Feller property (Assumption A1 in Marcet and Marimon (2009)). Second, given the No-Ponzi condition, for any given $B_t, R, Y$ the set of feasible consumption allocations $c_{i,t} : \int_{\mathbb{I}} c_i, t \leq \frac{Y}{R-1} + B_t, \forall t$ is just a simplex, so the choice vector lies in a compact and convex set (Assumption A2 in Marcet and Marimon (2009)). Third, note that our objective function is continuous, but unbounded. However, since aggregate ressources are bounded each period, so is $\int_{\mathbb{I}} U(c)$ (Assumption B1 in Marcet and Marimon (2009)). Finally, individual autarky is incentive compatible and resource feasible. So the constraint set is convex, compact, and non-empty.[14]

Given the continuous objective function, the original sequential problem (4) therefore has a solution. Also, Marcet and Marimon (2009) show that, given any initial weighting function $\mu$, these conditions suffice to show that an allocation $\{c_{i,t}\}, i \in \mathbb{I}, t \geq 0$ solves the original problem if and only if there is a sequence of multipliers $\gamma_{i,t},\ i \in \mathbb{I}, t \geq 0$ such that $\{c_{i,t}, \gamma_{i,t}\},\ i \in \mathbb{I}, t \geq 0$ solves the saddle-point functional equation (5).

Uniqueness of the equilibrium is assured by the strict concavity of the utility function $u$.■

[14] Strictly, we have to show that the constraint set has a non-empty interior, or that there is a real number $\varepsilon > 0$, such that $\int_{\mathbb{I}} c_i, t - Y \geq \varepsilon$ and $\int_{\mathbb{I}} [E[\sum_{t=0}^{\infty} (\mu_{i,t} + \gamma_{i,t}) U(c_{i,t}) - W(z_i)] > \varepsilon$.

In fact, without knowing the solution of the problem, the existence of $\varepsilon > 0$ is not trivial to prove. However, once we have the solution, the condition is easy to check. For now, I show the existence of $\varepsilon$ for the i.i.d. version of the special case, with $p = q = 1/2$ and $B_{t+1} = B_t = 0$. For this case it is easy to see that as long as the income uncertainty is big enough, or $\epsilon > \nu : \frac{U'(y_0+\nu)}{U'(y_0-\nu)} = \frac{2-\beta}{\beta}$, there are numbers $\xi, \widehat{\varepsilon} > 0$ such that a programme of the form $c(y_h) = y_h - \xi, c(y_l) = y_h + \xi - \widehat{\varepsilon}$ fulfills the conditions above. Intuitively, the expected discounted gain from higher consumption in future low-income states is big enough to allow a ressource-feasible reallocation of current consumption from high to low income agents. Thus the interior of the constraint set is strictly non-empty (Assumption B2 in Marcet and Marimon (2009)). But, as we will see, this history independent sharing rule is not optimal.

# 8 Tables and figures

**Table 1:**

**Income risk and savings in a simple world economy - variances of income components for China**

| | permanent | persistent | transitory | Gini income | Gini consumption |
|---|---|---|---|---|---|
| 1985 | 0.08 | 0.038 | 0.034 | 0.19 | 0.17 |
| 2001 | 0.08 | 0.13 | 0.10 | 0.27 | 0.21 |

The table reports the variances of components of an income process for Chinese urban regions that has the same structure as that reported in the text for the US: in the absence of information on group-specific attributes, (between-group) permanent income differences are modelled as a log-uniform distribution with 5 support points, while within-group income risk is the sum of a an AR(1) process with persistence parameter 0.9989 (discretized as a 7 state Markov process), plus a purely transitory binary shock (see the text for details). The parameters are chosen to target the Gini coefficients for consumption and income from Perlach and Wu (2005) for urban regions, and a zero net foreign asset position in 1980.