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Abstract

We develop a model that incorporates task-based production into a matching model with intrafirm wage bargaining. Unlike in existing task-based models, the representative firm derives the optimal task allocation as a function of capital and labor, rather than relative factor prices. Embedding this mechanism in a model with strategic employment choice, we show how the properties of task-level technology affect the extent of overhiring.

Keywords: task approach; search and matching; Stole-Zwiebel bargaining; overhiring; wage bargaining; elasticity of complementarity

JEL Classification: J23, D24, E23

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1 Introduction

This note makes two contributions to the literature. First, we develop a modeling framework that integrates the task-based approach with labor and capital into a matching environment, where wages are determined through individual bargaining after the firm’s employment decision. In task-based production models, popularized by Acemoglu and Autor (2011) and related literature on technical change, firms are typically modeled as price takers in the labor market, and the optimal task allocation is pinned down by market-determined relative factor prices.¹ In contrast, our approach requires the firm to determine the optimal assignment of tasks as a function of relative factor inputs.

Second, we include in this approach strategic employment choice in the spirit of Stole and Zwiebel (1996a, 1996b), where firms select employment to influence wage bargaining outcomes. Following the Stole-Zwiebel logic, firms have an incentive to “overhire” to dilute worker bargaining power and reduce wage costs. As discussed by Cahuc and Wasmer (2001), the overhiring effect arises under decreasing returns to labor, which we obtain here by treating capital as fixed. We show how an endogenous elasticity of complementarity resulting from our task-based framework affects the extent of overhiring, and examine the role of the task-level technology in this relationship.

To highlight the mechanisms, we limit the analysis to partial equilibrium. However, the approach can be integrated into general-equilibrium models with Stole-Zwiebel bargaining, including those with capital that is optimally chosen but predetermined at the bargaining stage (Cahuc et al., 2008), with decreasing marginal revenue product via monopolistic competition (Ebell and Haefke, 2009; Beugnot and Tiball, 2010), or with any concave employment-based production function (Acemoglu & Hawkins, 2014).

2 Firms’ behavior

2.1 Optimal task allocation

A unit mass of identical firms operates in discrete time. At the end of period t , the representative firm produces the final good $\tilde{Y}_t$ by combining the services of a continuum

¹Most models in this area assume perfect competition in the labor market. Recently, Marczak et al. (2025) extended a task-based framework to include search and matching frictions, and labor unions’ wage setting.

of tasks $y_t(i)$, $i \in [0, 1]$, via a Cobb-Douglas technology:

$$\tilde{Y}_t = \exp \left[\int_0^1 \ln y_t(i) di \right].$$

The firm assigns capital $K_t > 0$ and workers $L_t > 0$ to the different tasks according to the task-specific production function

$$y_t(i) = \alpha_K(i) k_t(i) + \alpha_L(i) l_t(i),$$

where $k_t(i)$ and $l_t(i)$ denote the capital and labor input assigned to task i in period t , respectively, and

$$K_t = \int_0^1 k_t(i) di \quad \text{and} \quad L_t = \int_0^1 l_t(i) di.$$

The continuously differentiable functions $\alpha_K(i) > 0$ and $\alpha_L(i) > 0$ describe the task-related productivities of capital and labor. We assume that task complexity increases with i , and workers' comparative advantage increases in more complex tasks. Formally, the relative task productivity schedule $\bar{\alpha}(i)$ is strictly increasing:

$$\bar{\alpha}(i) \equiv \frac{\alpha_L(i)}{\alpha_K(i)} \quad \text{with} \quad \bar{\alpha}'(i) > 0.$$

At Stage 1, the firm solves for the optimal allocation of any given K_t and L_t across tasks to maximize output. If the productivity profiles $\alpha_K(i)$ and $\alpha_L(i)$ are time-invariant, optimization needs to be performed only once; otherwise, this problem is solved in each period. Since there is no time dependency in this stage in our case, we drop time subscripts. The firm solves:

$$\begin{aligned} \max_{\{k(i), l(i)\}} \mathcal{L} &= \exp \left[\int_0^1 \ln (\alpha_K(i) k(i) + \alpha_L(i) l(i)) di \right] \\ &+ \mu \left[K - \int_0^1 k(i) di \right] + \lambda \left[L - \int_0^1 l(i) di \right] \\ \text{s.t.} \quad &k(i) \geq 0, \quad l(i) \geq 0 \end{aligned}$$

The first-order conditions yield the binding resource constraints and

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial k(i)} &= \frac{\tilde{Y}}{y(i)} \alpha_K(i) - \mu \leq 0, & k(i) &\geq 0, & \frac{\partial \mathcal{L}}{\partial k(i)} k(i) &= 0, \\ \frac{\partial \mathcal{L}}{\partial l(i)} &= \frac{\tilde{Y}}{y(i)} \alpha_L(i) - \lambda \leq 0, & l(i) &\geq 0, & \frac{\partial \mathcal{L}}{\partial l(i)} l(i) &= 0.\end{aligned}$$

The resulting task allocation follows a knife-edge pattern: each factor fully specializes in its comparative-advantage region. The threshold I that partitions the range of tasks into these regions is uniquely determined by the relative shadow prices of capital and labor and relative productivity at the margin:

$$\bar{\alpha}(I) = \frac{\lambda}{\mu}, \quad I \in (0, 1). \quad (1)$$

The following specialization pattern is obtained:

- For $i < I$ (capital specialization): $y(i) = \alpha_K(i) k(i)$

$$\frac{\tilde{Y}}{k(i)} = \mu \Rightarrow k(i) = k = \frac{K}{I} \quad \text{and} \quad \frac{\partial \tilde{Y}}{\partial l(i)} < \lambda \Rightarrow l(i) = 0.$$

- For $i > I$ (labor specialization): $y(i) = \alpha_L(i) l(i)$

$$\frac{\tilde{Y}}{l(i)} = \lambda \Rightarrow l(i) = l = \frac{L}{1-I} \quad \text{and} \quad \frac{\partial \tilde{Y}}{\partial k(i)} < \mu \Rightarrow k(i) = 0.$$

- Interior solutions ($k(i) > 0, l(i) > 0$) are of measure zero due to $\bar{\alpha}'(i) > 0$.

The optimal task allocation yields the following indirect production function:

$$Y = B(I) \left(\frac{K}{I} \right)^I \left(\frac{L}{1-I} \right)^{1-I}, \quad (2)$$

where $B(I) \equiv \exp \left[\int_0^I \ln \alpha_K(i) di + \int_I^1 \ln \alpha_L(i) di \right]$ captures task productivity heterogeneity. Y resembles a standard Cobb-Douglas function, with a crucial difference: the exponents are endogenous and depend on L and K . The production function in (2) reduces to a standard Cobb-Douglas form only when I is exogenously fixed. The shadow price ratio equals the marginal rate of technical substitution between capital and labor ($\text{MRTS}_{K,L}$).

With eq. (2):

$$\frac{\lambda}{\mu} = \frac{\partial Y / \partial L}{\partial Y / \partial K} \equiv \text{MRTS}_{K,L} = \frac{1 - I}{I} \frac{K}{L}, \quad (3)$$

which combined with (1) gives:

$$\bar{\alpha}(I) = \frac{1 - I}{I} \frac{K}{L} \quad \Rightarrow \quad I = I\left(\frac{K}{L}\right) \quad \text{with } I'(\cdot) > 0. \quad (4)$$

2.2 Employment decision

After the firm decides on the task allocation for any level of K_t and L_t , the next stage of the firm's optimization decision entails setting an optimal level of production factors. The firm's employment decision precedes wage bargaining and is subject to search and matching frictions. With fixed capital ($K_t = K > 0$), the production technology exhibits decreasing returns to labor, making employment a strategic variable in wage negotiations.² Following the intra-firm bargaining framework (Stole & Zwiebel, 1996a, 1996b; Cahuc & Wasmer, 2001), firms tend to overhire to dilute individual workers' bargaining power.

Timing is as follows. At the beginning of period t , the employed workforce is L_{t-1} and the number of unemployed is U_t . The firm posts V_t vacancies at cost s per vacancy. Matches are realized via a constant returns to scale matching function $M(\bar{V}_t, U_t)$, where $\bar{V}_t$ is the total measure of vacancies posted by all firms. The job filling rate is $m(\theta_t) = M(\bar{V}_t, U_t) / \bar{V}_t$, with tightness $\theta_t = \bar{V}_t / U_t$, both taken as given by the single firm. Hence, employment evolves as:

$$L_t = (1 - q)L_{t-1} + m(\theta_t)V_t, \quad (5)$$

where q denotes the exogenous separation rate. After recruitment, the firm and workers bargain individually over wages w_t given the employment level, so $w_t = w(L_t)$. Finally, production takes place as described above.

In *Stage 2*, the firm solves the Bellman equation:

$$\Pi(L_t) = \max_{V_t} \left\{ Y_t - w(L_t)L_t - r_t K - sV_t + \frac{1}{1 + r_t} \Pi(L_{t+1}) \right\}$$

subject to the indirect production function (2) and the law of motion (5). In the above

²Cahuc et al. (2008) consider the optimal choice of capital jointly with employment in each period prior to wage bargaining. With only one type of labor, as in our framework, endogenizing capital leads to underinvestment but does not qualitatively affect the employment decision. We therefore hold capital fixed to focus on the novel mechanism.

equation, r_t denotes the interest rate. The first-order condition is

$$(1 - I_t) \frac{Y_t}{L_t} - w_t - \frac{dw_t}{dL_t} L_t + \frac{1 - q}{1 + r_t} \frac{d\Pi_{t+1}}{dL_{t+1}} = \frac{s}{m(\theta_t)}, \quad (6)$$

with envelope condition

$$\frac{d\Pi_t}{dL_t} = (1 - I_t) \frac{Y_t}{L_t} - w_t - \frac{dw_t}{dL_t} L_t + \frac{1 - q}{1 + r_t} \frac{d\Pi_{t+1}}{dL_{t+1}}. \quad (7)$$

The firm takes into account that hiring an additional worker allows a wage reduction for all employed workers, reflected in the term $(dw_t/dL_t)L_t$. Combining (6) and (7) yields the job creation condition:

$$\frac{s}{m(\theta_t)} = (1 - I_t) \frac{Y_t}{L_t} - w_t - \frac{dw_t}{dL_t} L_t + \frac{1 - q}{1 + r_t} \frac{s}{m(\theta_{t+1})}. \quad (8)$$

Equation (8) states that the expected hiring cost per worker equals the net value of the marginal worker, including the wage effect on incumbent workers and continuation value.

3 Individual Wage Bargaining

Given the timing, the present discounted utility of an employed worker in period t is

$$\Psi_{E,t} = w_t + \frac{1}{1 + r_t} [q \Psi_{U,t+1} + (1 - q) \Psi_{E,t+1}].$$

The corresponding utility for an unemployed worker is

$$\Psi_{U,t} = z_t + \frac{1}{1 + r_t} [p_{t+1} \Psi_{E,t+1} + (1 - p_{t+1}) \Psi_{U,t+1}],$$

with z_t denoting unemployment benefits and $p_{t+1} = \theta_{t+1}m(\theta_{t+1})$ the exit rate from unemployment. The worker's surplus is $R_t = \Psi_{E,t} - \Psi_{U,t}$, whereas the firm's surplus from one additional worker is $J_t = d\Pi_t/dL_t$, given by (7). Denoting the bargaining power of the worker with $\phi \in [0, 1]$, in *Stage 3* the wage is determined by Nash bargaining

$$w_t = \arg \max_{w_t} \left\{ R_t^\phi J_t^{1-\phi} \right\},$$

which results in the sharing rule $(1 - \phi)R_t = \phi J_t$. Substituting definitions and rearranging yields the following differential equation in w_t :

$$w(L_t) = (1 - \phi) \left[z_t - \frac{1 - q - p_{t+1}}{1 + r_t} R_{t+1} \right] + \phi \left[(1 - I_t) \frac{Y_t}{L_t} - \frac{dw_t}{dL_t} L_t + \frac{1 - q}{1 + r_t} \frac{s}{m(\theta_{t+1})} \right].$$

The solution is derived in Appendix A.1. After taking into account the sharing rule, we obtain

$$w(L_t) = (1 - \phi)z_t + \phi \frac{1}{1 + r_t} s \theta_{t+1} + \int_0^1 x^{\frac{1-\phi}{\phi}} (1 - I_t(L_t x)) \frac{Y_t(L_t x)}{L_t x} dx. \quad (9)$$

The bargained wage is a weighted sum of z_t and the average vacancy costs in the economy, plus an additional term represented by the integral expression. The latter captures all inframarginal products of labor, with their respective weights being higher for those closer to the margin. Under Stole-Zwiebel bargaining with identical workers, all employees are treated as marginal and receive the same wage.

4 Hiring Externality in the Task-based Matching Environment

Using (9) to calculate $(dw_t/dL_t) L_t$, and assuming a steady state, the job creation condition (8) can be written as

$$\underbrace{(1 - I) \frac{Y}{L}}_{\frac{dY}{dL}} = \underbrace{w + \frac{r + q}{1 + r} \frac{s}{m(\theta)}}_{\text{wage and hiring costs}} + \underbrace{L \int_0^1 x^{\frac{1}{\phi}} \frac{d^2 Y(Lx)}{dx^2} dx}_{\text{wage adjustment}}$$

In standard matching models with constant returns to scale, the wage adjustment term vanishes, so the MPL equals wage and hiring cost per worker. Here, integrating Stole-Zwiebel bargaining with downward-sloping MPL generates a negative wage adjustment term, capturing a hiring externality—by expanding employment, the firm reduces the wage bargained for all incumbents, incentivizing overhiring beyond the point where marginal revenue equals the cost of a single worker.

The task-based approach introduces new insights into overhiring. The endogenous assignment of tasks becomes a novel determinant of the wage adjustment term. Specifically,

as shown in Appendix A.2,

$$\frac{d^2 Y}{dL^2} = -c(I)I(1-I)\frac{Y}{L^2} = -c(I)\frac{I}{L}\frac{dY}{dL} < 0, \quad (10)$$

where $c(I)$ denotes the elasticity of complementarity. It is defined as

$$c(I) \equiv \frac{d \ln \text{MRTS}_{K,L}}{d \ln(K/L)} = 1 - \frac{1}{(1-I)} \frac{d \ln I}{d \ln(K/L)}, \quad (11)$$

which is obtained by differentiating the log version of (3). Next, implicit differentiation of the logged first-order condition (4) yields $d \ln I / d \ln(K/L)$, which after substituting into (11) gives

$$c(I) = \frac{(1-I) \varepsilon_{\bar{\alpha},i}(I)}{1 + (1-I) \varepsilon_{\bar{\alpha},i}(I)}, \quad \text{where} \quad \varepsilon_{\bar{\alpha},i}(I) \equiv \left. \frac{d \ln \bar{\alpha}(i)}{d \ln i} \right|_{i=I} > 0, \quad 0 < c(I) < 1. \quad (12)$$

$\varepsilon_{\bar{\alpha},i}(I)$ is the elasticity of the relative task productivity schedule with respect to a one-percent change in the task index i . Note that $c(I) = 1/\sigma(I)$, where $\sigma(I) > 1$ is the elasticity of substitution.

For a standard Cobb-Douglas production function with fixed task allocation, we have $\sigma(I) = \sigma = 1$ and $c = 1$. In contrast, the general case with optimal choice of I is associated with $c(I) < 1$ and, therefore, smaller decline in MPL according to (10). This reduces the incentive for overhiring. The smaller decline in MPL is caused by task reallocation—more tasks are assigned to workers as employment increases.

Moreover, the magnitude of the hiring externality now depends on the curvature of $\bar{\alpha}(i)$. To illustrate this, consider an isoelastic function $\bar{\alpha}(i)$, such that $\varepsilon_{\bar{\alpha},i}(I) = \varepsilon$. This assumption simplifies the exposition without loss of generality, as an isoelastic function $\bar{\alpha}(i)$ can exhibit different shapes depending on whether $\varepsilon \gtrless 1$. The sensitivity of $c(I)$ to ε , holding I fixed, is given by

$$\left. \frac{\partial c(I)}{\partial \varepsilon} \right|_{I=\text{const}} = \frac{1-I}{[1 + (1-I)\varepsilon]^2} > 0.$$

An increase in ε means that a one-percent increase in the task index i leads to a larger percentage increase in the productivity of workers relative to capital, $\bar{\alpha}(i)$. This can be interpreted as the two factors becoming more dissimilar at each task i , and thus more complementary. When the firm increases L , it reduces I as more tasks are allocated to

workers. However, with a higher ε , a smaller reduction in I is needed to achieve the same decrease in $\bar{\alpha}(I)$. According to (11), a smaller value of $d \ln I / d \ln(K/L)$ is associated with a larger $c(I)$. As a result, an increase in ε has a more pronounced negative effect on $d^2 Y / dL^2$ through its impact on $c(I)$, and thus leads to stronger overhiring.³

5 Conclusions

By integrating task-based production into a matching model with individual wage bargaining, this note shows that firms not only choose the total amount of factor inputs but also endogenously determine their allocation across tasks based on the input levels. Incorporating strategic employment choice, we find that: (i) the firm's ability to adjust the task allocation dampens overhiring incentives compared to the case with fixed task allocation; (ii) the elasticity of complementarity, which affects the extent of overhiring, is itself endogenously determined, making overhiring dependent on task allocation; and (iii) the task-dependent technology, specifically the elasticity of the relative task productivity schedule, influences the elasticity of complementarity and thus the degree of overhiring. The proposed model offers a promising framework for addressing questions widely explored in the task-based literature, such as the implications of automation or artificial intelligence, thereby opening new avenues for future research.

A Appendix

A.1 Solution to the Wage Differential Equation

Ignoring for the moment the term

$$A_t \equiv (1 - \phi) \left[z_t - \frac{1 - q - p_{t+1}}{1 + r_t} R_{t+1} \right] + \phi \frac{1 - q}{1 + r_t} \frac{s}{m(\theta_{t+1})},$$

that does not depend on L_t , the wage equation reduces to

$$\frac{dw_t}{dL_t} + \frac{1}{\phi L_t} w(L_t) = \frac{1}{L_t} (1 - I_t) Y_t.$$

³Beyond the direct effect of ε on the change in the MPL via $c(I)$, the total change in MPL in (10) also includes indirect effects through changes in I on $c(I)$, L , and Y . As these are second-order effects, interpreting the total effect would offer limited additional insight while considerably complicating the analysis due to interdependencies among these variables in the job creation and wage equations.

This is a linear ODE in $w(\cdot)$ with integrating factor $L_t^{1/\phi}$. Multiplying both sides by $L_t^{1/\phi}$ and integrating yields the general solution:

$$w(L_t) = L_t^{-1/\phi} \left[\int_0^{L_t} x^{(1-\phi)/\phi} (1 - I_t(x)) \frac{Y_t(x)}{x} dx + D \right].$$

We assume that $\lim_{L_t \rightarrow 0} L_t w(L_t) = 0$, which implies $D = 0$. Rescaling and adding A_t gives the final solution:

$$w(L_t) = A_t + \int_0^1 x^{(1-\phi)/\phi} (1 - I_t(L_t x)) \frac{Y_t(L_t x)}{L_t x} dx.$$

A.2 Second derivative of the indirect production function

Recalling that the MPL is $dY/dL = (1 - I)Y/L$, the second derivative of Y is

$$\frac{d^2 Y}{dL^2} = -I(1 - I) \frac{Y}{L^2} - \frac{Y}{L} \frac{\partial I}{\partial L}.$$

To compute $\partial I / \partial L$, consider the log first-order condition (4),

$$0 = \ln \bar{\alpha}(I) + \ln L - \ln K + \ln I - \ln(1 - I).$$

Implicit differentiation yields

$$\frac{\partial I}{\partial L} = \frac{I(1 - I)}{L[1 + (1 - I)\varepsilon_{\bar{\alpha},i}(I)]}, \quad \varepsilon_{\bar{\alpha},i}(I) \equiv \left. \frac{d \ln \bar{\alpha}(i)}{d \ln i} \right|_{i=I}.$$

Therefore,

$$\frac{d^2 Y}{dL^2} = -I(1 - I) \frac{Y}{L^2} \left(1 + \frac{1}{1 + (1 - I)\varepsilon_{\bar{\alpha},i}(I)} \right).$$

Using the definition of $c(I)$ in (12), this yields the expression in (10).

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