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Equivalisation is a crucial component of calculating statistics on income inequality and poverty, with these statistics sensitive to how incomes are equivalised. Ireland is among a number of countries to equivalise incomes using a country-specific equivalence scale, based (loosely) on the relativities of social welfare payments in the mid-1980s. We use three methods and data from three-decades of the Irish Household Budget Survey to investigate whether there is an empirical basis to support the use of the existing national scale over alternatives. We find that while estimates from the Engel method yield equivalence scales for additional adults that are not dissimilar to those used for the national equivalence scale, those from the Almost Ideal Demand System are much higher. Conversely, estimates from all three methods yield equivalence scales for children that are much smaller than those used for either the national or modified OECD equivalence scales. While the downward trend in income inequality (as measured by the Gini coefficient) does not appear to be sensitive to the choice of scale, rates of income poverty – particularly child poverty – are.

[1] Household Budget Survey data were collected by the Central Statistics Office and provided by the Irish Social Science Data Archive (ISSDA). Funding for this paper through the Economic and Social Research Institute's Tax, Welfare and Pensions Research Programme (supported by the Departments of Public Expenditure and Reform, Employment Affairs and Social Protection, Health, Children and Youth Affairs and Finance) is gratefully acknowledged, as are the helpful comments of the programme steering committee, participants at the Economic and Social Research Institute seminar series and the Irish Economic Association annual conference.

1. Introduction

Equivalisation is a crucial, if often overlooked, component of calculating statistics on income inequality and poverty. This process adjusts ("equivalises") the income of households of differing sizes and composition, with common practice being to divide the income of a household by a set of "equivalence scales": a function of the number of adults and children in the household. As well as reflecting (infrequently discussed) implicit value judgements (Deaton and Muellbauer, 1986; Atkinson et al., 1995), research has shown that the choice of these scales can make meaningful differences to measures of income inequality and poverty (e.g. Regan and Kakoulidou 2022; Mysikova and Zelinsky, 2019; Aaberge and Melby, 1998; Jenkins and Cowell, 1994).

Despite this, different – largely ad-hoc – equivalence scales have been adopted by international organisations with little explicit justification. For example, while Eurostat use what is known as the "modified OECD scale" in the construction of their statistics on income distributions, the OECD, somewhat counterintuitively, use the square-root of household size for their income distribution statistics database. In addition, research has suggested that country-specific equivalence scales may more accurately reflect the economic well-being of each country (Mysíková et al, 2022), not least as the cost and distribution of spending on necessities varies across countries depending on the household size (Goedemé et al, 2019).

Ireland is among a number of countries to have adopted such a country-specific equivalence scale – the "national scale" – which is used by its government (e.g. Government of Ireland, 1997; 2020), national statistical organisation (CSO, 2022) and researchers (e.g. Doorley et al., 2022,).[2] However, the rationale for this national scale – one of three used by (Callan and Nolan, 1987) in early work on the distribution of income in Ireland – is ad-hoc, based (loosely) on the relativities of social welfare payments in the mid-1980s.[3] Figure A.1. in Appendix shows that although these relativities have remained largely constant over the horizon for adult dependents, those for child dependents have increased by around a fifth while the national scale has remained unchanged. Moreover, the fact that the relativities of social welfare payments for adult dependents have remained largely constant reflects policy decisions by successive Governments to keep these payments aligned for reasons of simplicity and precedent rather than as the result of some empirical assessment of the appropriate relativities of payments or of the appropriate national scale.

This paper investigates whether there is an empirical basis to support the use of the existing national scale over alternatives. It does so using three different methods to derive equivalence scales from the patterns of expenditure observed in Household Budget Surveys covering the period 1987-2015. The first – the Engel (1895) method – is based on the assumption that households of differing composition have the same standard of living if they spend the *same share* of their total expenditure on some necessary good (e.g. food). The second – the Rothbarth (1943) method – assumes such households have the same standard of living if they spend the *same amount* on some set of "adult" goods (e.g. adult clothing, alcohol etc).[4]

[2] The national statistical organisations of almost all EU countries use the modified OECD scale, with other exceptions being Italy, Denmark, Romania, Slovakia and Sweden.

[3] This is highlighted by the fact that different institutions use different scales to estimate income poverty and inequality statistics for Ireland: the national statistical authority uses the national equivalence scale, EUROSTAT uses the modified OECD scale and the OECD uses the square-root scale.

[4] Such goods must be consumed only by adults: as a result of this assumption, the Rothbarth method can only be used to derive an equivalence scale for children and not additional adults in a household.

However, among the shortcomings that these methods have is that they are sensitive to the precise choice of necessary or adult good, with the appropriate choice of good not evident *a priori* (Deaton et al., 1989; Lewbel and Pendakur, 2008). To address this concern, we also adopt a third method which involves estimating the parameters of Deaton and Muellbauer's (1980a) Almost Ideal Demand System (AIDS) and using these estimates to derive equivalence scales on the basis of households' expenditure patterns across all goods. We then examine how measures of income inequality and poverty using all the estimated scales vary, comparing them to the estimates of Roantree et al. (2021) – who use the modified OECD scale – over the last three decades.[5]

We find that the while estimates from the Engel method yield equivalence scales for additional adults that are not dissimilar to those used for the national equivalence scale, those from the Almost Ideal Demand System are much higher. Conversely, estimates from all three methods yield equivalence scales for children that are much smaller than those used for either the national or modified OECD equivalence scales. While the downward trend in income inequality (as measured by the Gini coefficient) does not appear to be sensitive to the choice of scale, rates of income poverty – particularly child poverty – are. Although most of the scales yield an overall income poverty rate of c.15% in 2015 (the latest year of Household Budget Survey data collected), the income poverty rate for children in that year varies between 8.8% using the scales derived from the AIDS model to 17.7% using the modified OECD equivalence scale. The change in child poverty over time also varies substantially across the scales, illustrating the importance of the choice of equivalence scale in the measurement of poverty.

Our contribution to the literature is twofold. Firstly, we build on earlier work that sought to derive equivalence scales from patterns of household expenditure (e.g. Conniffe and Keogh, 1988; Conniffe, 1992; Blundell and Lewbel, 1991; Banks and Johnson, 1994; Bargain et al., 2010; Garvey et al., 2011). Such work has typically focused on children, whereas we derive scales for both adults and children. In addition, by exploiting the near three-decade horizon covered by our data we show – like (Daley *et al*., 2020), though using more than just the Engel method – that the equivalence scales implied by expenditure patterns are not constant over time, at least for children. We also contribute to the large literature that has shown measures of income inequality and poverty are sensitive to the choice of scale (e.g. Regan and Kakoulidou 2022; Mysikova and Zelinsky, 2019; Aaberge and Melby, 2005; Figini, 1998; Jenkins and Cowell, 1994; Banks et al., 1994; Coulter et al., 1992).

The rest of the paper proceeds as follows. Section 2 presents our empirical approach, data, and estimates of equivalence scales using the Engel, Rothbart and AIDS methods. Section 3 explores how these estimates translate into measures of income inequality and poverty while Section 4 concludes.

2. Estimating Equivalence Scales from household expenditure data

Three broad classes of equivalence scale have been adopted in research on the distribution of income (Atkinson et al, 1995). "Expert" scales – such as the modified OECD and Irish national equivalence scales – are based on the judgement of policy makers and academics. One criticism of these scales is that they are not necessarily grounded in economic theory or empirical data and could be considered

[5] (Roantree et al., 2021) construct a harmonised series of the Gini coefficient and income poverty rates using data from the 1987 ESRI Survey of Income Distribution, Poverty and Usage of State Services, the 1994-1999 waves of the Living in Ireland Survey and the 2004-present Survey of Income and Living Conditions (SILC).

ad-hoc in nature. Subjective survey scales are instead based on respondents self-assessed levels of income adequacy (e.g., Kapetyn and van Praag, 1978), while consumption or expenditure scales – which are the focus of this paper – are based on patterns of household expenditure. This section outlines our empirical approach to estimating such scales, the data we use and the estimates.

2.1. Empirical approach

The Engel methodology begins with Engel's law: the proportion of a household's expenditure that goes on food (the "food share") is a *decreasing* function of income / total expenditure. Accordingly, the Engel curve for food (which traces out the food share as a function of income or total expenditure) should slope downwards as one moves from lower to higher levels of income / total expenditure.[6] If household welfare is increasing in income / total expenditure, it follows that there is a monotonically decreasing relationship between the food share and household welfare; and, in particular, that two households have the same level of welfare if their food shares are equal.

The Engel curve can be parametrically specified as a function of total expenditure. In our analysis, we focus on the widely used Working-Leser form (equation (1)) - estimated by OLS -, which models the food share as a linear function of the log of income (Deaton, 1981; Garvey et al, 2011).

Denote by w_i the expenditure share of the *i*th good:

$$w_i = \beta_0 + \beta_1 lnY + \beta_2 n_c + \beta_3 n_a + \varepsilon_i \quad (1)$$

where Y is total household expenditure, and n_c and n_a are the number of children and adults in the household respectively.. In our analysis, we consider three sets of household expenditure: food, clothing and housing costs, as a well the combined expenditure on these three goods. Although the Engel's law relates to food, clothing and housing costs can also be considered as necessary goods, with research often using them in conjunction with food to estimate equivalence scales (Garvey et. al., 2011).

The parameters of equation 1 are estimated by OLS. Then, let n_a^0 be the number of adults in the reference household, n_c^0 the number of children in the reference household, and $n^0 = n_a^0 + n_c^0$ the size of the reference household (a common choice being, e.g., $n^0 = n_a^0 = 2,\ n_c^0 = 0$). For a "comparison household" type with number of adults n_a^h, number of children n_c^h, and total size n^h, the scale which converts a level of income for the reference household to an equivalent level of income for the comparison household is (Deaton and Muellbauer, 1986):

$$\frac{n^h}{n^0} \cdot exp\left[\frac{\beta_3}{\beta_1}\left(n_a^h - n_a^0\right) + \frac{\beta_2}{\beta_1}\left(n_c^h - n_c^0\right)\right] \quad (2)$$

This formula yields constant equivalence scales which do not depend on the level of total expenditure. Given the non-linear dependence of the scale on parameters, we follow Phipps and Garner (1991) in using the delta method (i.e., a Taylor series expansion around the point estimates) to compute standard errors for scales.

Modern approaches to Engel curve estimation have found nonlinear Engel curves for a number of commodities, such as alcohol and clothing. This has prompted the use of non-parametric estimation methods which make smoothness or differentiability requirements of the Engel curve but do not require the specification of a particular functional form (Engel and Kneip, 1996; Banks, Blundell and Lewbell; 1997). We supplement our parametric estimation with estimates using kernel regressions.

[6] For more details regarding the Engel model see (Deaton and Muellbauer, 1980b, ch.8).

Following Blundell et al (1997), we use the Epanechnikov kernel for the continuous variable lnY and the Liracine kernel for the discrete variables n_a and n_c. The Engel method has been subject to the criticism that it tends to overestimate the true equivalence scales for children (Nicholson, 1976; Deaton and Muellbauer, 1986). The Rothbarth method aims to overcome this objection.[7] Like the Engel method, it posits a single good the consumption of which is argued to correspond to levels of household welfare. However, the model does not rest on an empirical regularity like Engel's Law. Instead, it is stipulated *a priori* that some "adult-only" good is consumed only by parents and not by children. Further, parents' preferences for this good are assumed to be invariant under changes to the number of children: adding children to the household has only an income effect on parents' consumption of this good, not a substitution effect (the so-called "demographic separability" of parental and child preferences described by Deaton *et. al.*, 1989).

Under these assumptions, the Rothbarth model takes the absolute amount of expenditure on the "adult-only" good as an indicator of parental welfare, rather than its budget share (Tsakoglou, 1991; Banks and Johnson, 1994). On the Rothbarth model, different households spending the same amount on this "adult-only" good are at the same level of welfare. The assumptions of the model serve only to identify equivalence scales for children: the model says nothing about the effect on preferences for or consumption of the adult good when an additional adult enters the household. Accordingly, the Rothbarth method furnishes estimates of child equivalence scales only.

Formally, we choose a level of (log) total expenditure Y at which to evaluate the scales, Y^0, and set the variables n_a^0, n_c^0 to their reference values. We then solve for the equivalent level of income Y^h for a comparison household with characteristics n_a^h, n_c^h as:

$$Y^h = exp\left(\frac{Y^0 - \beta_0 - \beta_2 n_c^h - \beta_3 n_a^h}{\beta_1}\right) \quad (3)$$

The equivalence scale itself is calculated as is Y^h/Y^0 and is reported at the sample median / mean level of expenditure (Tsakloglou, 1991; Balisacan, 1991; White and Masset, 2002; Bargain and Donni, 2011).

Though the Rothbarth method arguably avoids the tendency of the Engel method to overstate child equivalence scales, it is subject to quite strong objections of its own. In particular, the identification assumption – that some goods are consumed only by parents and that parents' preferences for those goods are unchanged with the arrival of children to the household – is a strong and plausibly unrealistic assumption. The Rothbarth model also makes no allowance for difference in *inter-household* preferences for the adult good, which is particularly relevant for goods such as alcohol, tobacco, and gambling. Perhaps not unrelatedly, it has been observed that child scales estimated by the Rothbarth method are very sensitive to the choice of outcome good, something we will also find (Lancaster and Ray, 1997).

The Engel and Rothbarth methods are subject to idiosyncratic problems outlined above: further, all such "single equation" methods are subject to the objections of allowing no role for *price variation* in goods to affect equivalence scales (e.g., the assumption of the models that the addition of children to a household has a pure income effect) and of having no rigorous grounding microeconomic theory (links to utility or welfare are established via empirical regularities or *a priori* stipulations). For these reasons, we also estimate equivalence scales using a demand-system methodology. To formalise

[7] For formal treatments of the Rothbarth method, see (Gronau, 1988, 1991).

approach, we allow the household's *expenditure function* $E(\boldsymbol{p}, u)$ to depend on a vector of demographic variables $\boldsymbol{z}$. Then, the "true" equivalence scale S) is written:[8]

$$S(\boldsymbol{p}, u, \boldsymbol{z}_0, \boldsymbol{z}_h) = \frac{E_h(\boldsymbol{p}, u, \boldsymbol{z}_h)}{E_0(\boldsymbol{p}, u, \boldsymbol{z}_0)}$$

Here, $\boldsymbol{z}_0$ is a vector of demographic characteristics for the reference household and $\boldsymbol{z}_h$ for another household.

On this approach, an equivalence scale can be obtained by choosing functional forms for the reference household's expenditure function E_0 and the other household's expenditure function E_h. Equivalently, using Shephard's Lemma, one can specify the household's demand function or budget share function for each commodity, along with demographic variables. [9] Following the "generalised cost scaling" approach of Ray (1982, 1983),[10] we incorporate demographic variables into the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980a). In our commodity classification, we follow the specification of Savage(2016), who uses the same data as we do to explore the impacts of hypothetical indirect tax reforms.

The budget share equation for the *i*th good of the standard AIDS is:

$$w_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \beta_i \ln\left(\frac{y}{P}\right) \quad (4)$$

Where p_j is the price of the *j*th good, y is total expenditure, Greek letters are parameters to be estimated, and P is a translog price index:

$$\ln P = \alpha_0 + \sum_{i=1}^{n} \alpha_i \ln p_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \ln p_i \ln p_j \quad (5)$$

The parameter α_0 is specified by the researcher, with the common choice being to set it to slightly less than the largest value of log total expenditure observed in the data (Deaton and Muellbauer, 1980b; Banks et al., 1997).[11]

Following the specification of Ray(1983), demographic variables are incorporated into the AIDS as follows:

$$w_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \left(\beta_i + \boldsymbol{\eta}_i^T \boldsymbol{z}\right) \ln\left(\frac{y}{\bar{m}_0(\boldsymbol{z}) P}\right) \quad (6)$$

where $\boldsymbol{z}$ is a vector of demographic variables and $\bar{m}_0(\boldsymbol{z}) = 1 + \boldsymbol{\rho}^T \boldsymbol{z}$; the ρ and η vectors are parameters to be estimated In our model, demographic variables include the number of children in

[8] Incidentally, the definition of the "true" equivalence scale as a ratio of expenditure functions is almost identical to the definition of the "true" or Könus price index from index number theory.

[9] Shephard's Lemma asserts that the partial derivative of $E(\boldsymbol{p}, u)$ with respect to p_i is the (Hicksian or compensated) demand for good i; equivalently, that the partial derivative of $\ln E(\boldsymbol{p}, u)$ with respect to p_i is the budget share for good i.

[10] (Pollak and Wales, 1981) outline five other methods for incorporating demographic variables into demand system analysis.

[11] In keeping with the literature, we also impose some other parameter restrictions on the basis of theory e.g. symmetry of the Hicks-Slutsky matrix and linear homogeneity of the demand functions. For details on how these restrictions are imposed see (Savage,2016, pg.375) or (Poi, 2012).

the household and the number of adults in the household.[12] Following Ray, the parameters are interpreted as follows: the parameters ρ_1, ρ_2 are the equivalence scales for adults and children, and the $\boldsymbol{\eta}$ parameters are estimates of the sensitivity of the scales to the prices of our six commodities. We report only the equivalence scales below, leaving an investigation of the price-dependency of scales for future research.

Data on household expenditures and demographics comes from Household Budget Surveys from 1987 to 2015.[13] The price series for each commodity were taken from the subindices of the Consumer Price Index published by the Central Statistics Office.[14]

2.2 Data and Descriptive Statistics

We use data from all waves of the Irish Household Budget Survey (HBS) available from 1987 to 2015.[15] The survey follows a nationally representative sample of 6,000 to 7,000 households for two weeks and asks them to record their expenditures. It also collects rich demographic information such as age, sex and household size.

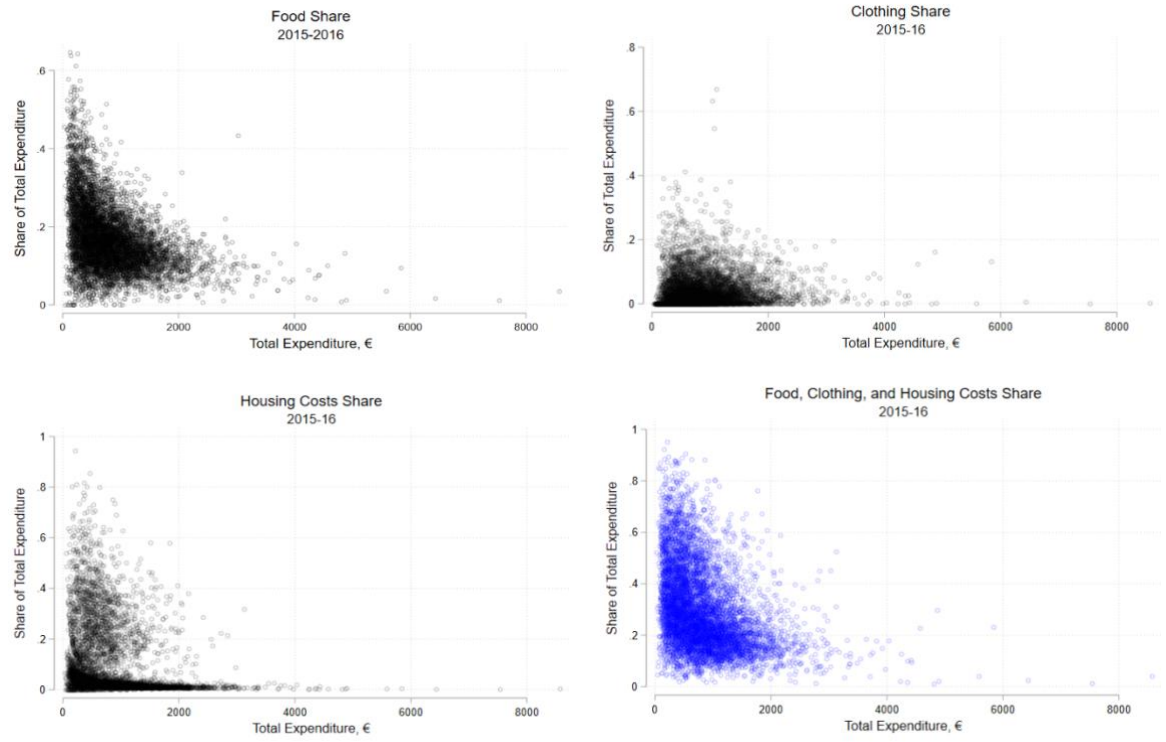
To derive equivalence scales using the Engel method, we use budget shares for food, clothing and housing costs, as well as the combination of these three. To derive equivalence sales using the Rothbarth method, we must identify "adult" goods which are assumed to only be consumed by adults. Closely following the existing literature in this area, we use budget shares for adult clothing, alcohol and tobacco and gambling as well as the combination of these three. We also use the available data on household size and composition (number of adults and children) in the HBS. Following the EUROSTAT definition, all individuals younger than 14 years of age are counted as children whereas older individuals are regarded as adults.

Figure 1 shows the distribution of the share of expenditure on food, clothing, housing costs and the combination of these three across total expenditure distributions for 2015.[16] Figure 2 shows the distribution of the share of expenditure on alcohol and tobacco, adult clothing and gambling across total expenditure distributions for 2015. Visually we observe that Engel's law holds.

The share of food expenditure declines with total expenditure. The same pattern is observable for clothing and housing costs although these relationships are a bit flatter. Figure 2 illustrates similar patterns for most of the "adult" goods. Expenditure shares on alcohol and tobacco decline with income while the relationship between gambling budget shares and income is less clear. Crucially, however, underlying the Rothbarth method, there is no assumption required of a relationship between the budget share and income.

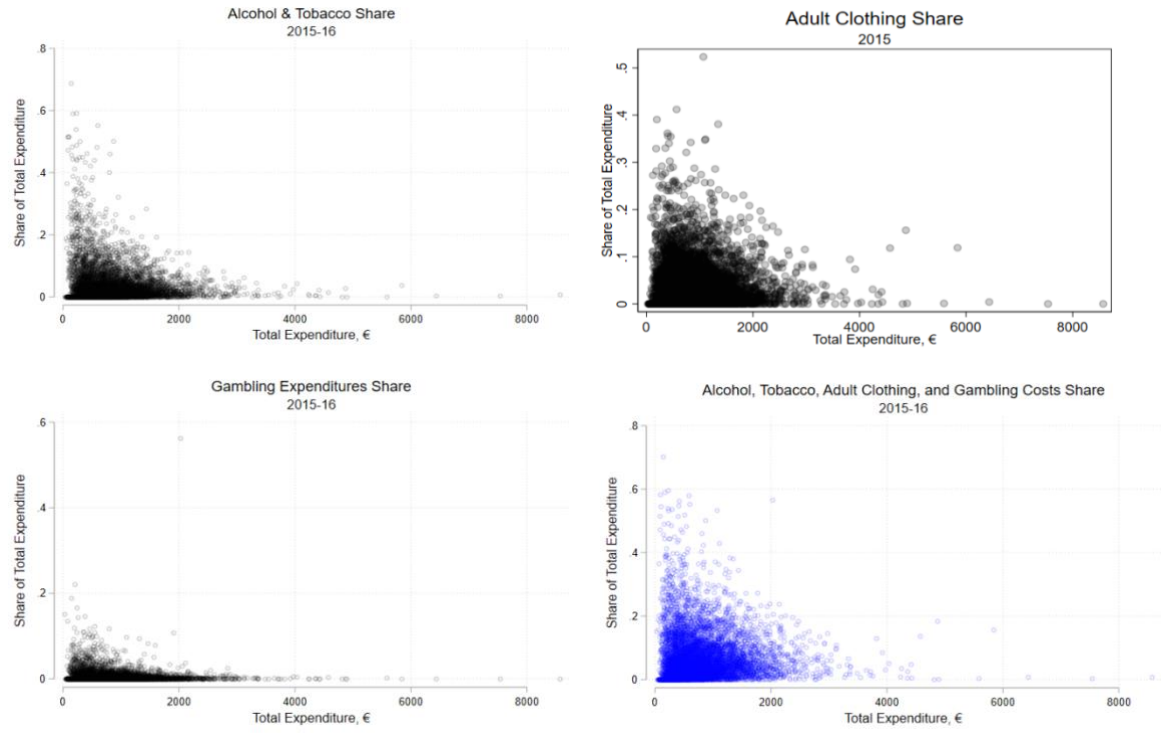
[12] As in (Michelini, 2001, pg.386), we subtract one from the adult variable to obtain the appropriate reference household.
[13] (Savage,2016) considers only the years 1987 to 2009.
[14] Available at https://data.cso.ie/table/CPM03
[15] The waves we include are 1987, 1994, 1999, 2004, 2009 and 2015.
[16] In the Appendix (Figures A.2 to A.11) we also provide the same graphs for the rest of the HBS waves.

Figure 1: Expenditure on Engel goods as share of total expenditure, HBS2015/16



Note: Own calculations using HBS 2015-16. Total expenditure is monthly.

Figure 2: Expenditure on Rothbarth goods as share of total expenditure, HBS2015/16



Note: Own calculations using HBS 2015-16. Total expenditure is monthly.

2.3. Equivalence scale estimates

This section presents estimates of equivalence scales using the Engel, Rothbarth and AIDS method. In all cases the first adult is normalised to 1. We present results for the additional adult and for children, distinguishing between younger and older children who are usually counted as adults. For each specification and each wave of HBS data, detailed point estimates and standard errors (computed using the delta method) are presented in the Appendix (Tables A.2 – A.6 and Figures A.16 – A.19).

2.3.1. The additional adult scale

Figure 3 presents our estimates of the additional adult scale using the Engel method for both the food share of expenditure and the combined share of food, clothing and housing expenditure, as well as the AIDS method. For the Engel method, we estimate both a Working-Leser and non-parametric form. The estimated scale changes over time and depends on the method or the choice of expenditure share we use. Using the AIDS method yields the highest scale, particular in recent years. Among the scales estimated using the Engel method, using the food share of expenditure leads to the highest scales. This is in line with the economies of scale theory: food consumption is subject to very poor economies of scale whereas other goods, such as housing and clothing, are subject to higher economies of scale. In 2015, we estimate that the adult scale is close to the national scale when we use the combined goods share of expenditure but higher than the national scale using the food share of expenditure or the AIDS method.

2.3.2. The child scale

For the estimation of the child scale, we use all three methods (the Engel, the Rothbarth and the AIDS method). Results using the Rothbarth method are presented separately to those which use the Engel or AIDS method to facilitate comparison with estimates of the adult scale.

2.3.2.1. The Engel and the AIDS method

Figure 4 plots the evolution of our estimated child scale using the Engel and AIDS method. All estimations in all time periods are below the national scale for children.

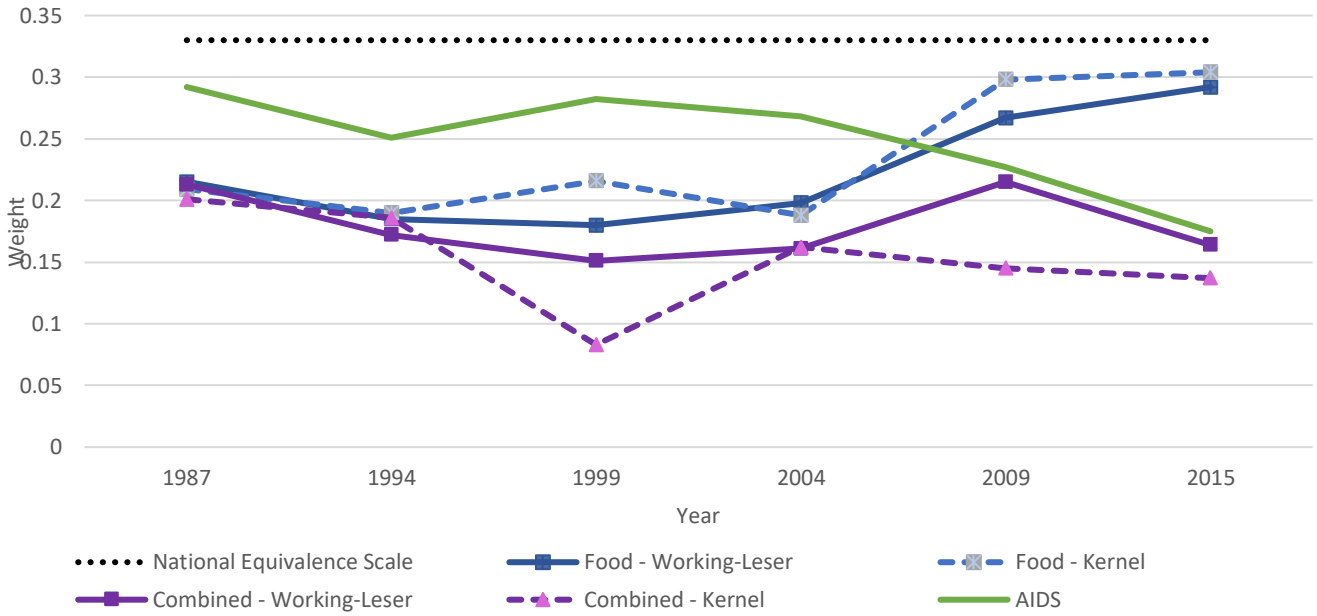
Although each estimation using the Engel method produces similar results in 1987 (with the child scale estimated close to 0.2), they deviate over time depending on the expenditure basket used. Using the combined goods basket, the estimated scale decreases over time - reaching a low of 0.08 – 0.15, depending on the functional form, in 1999 - and ends up at 0.14-0.16 in 2015. Using the food share, the estimated child scale is stable at 0.18-0.21 util 2004, when it starts to increase. By 2015, the child scale estimated using the Engel method and food shares is close to the national scale, at 0.3.

Between 1987 and 2003, the AIDS method yields a higher child scale than the Engel method, of 0.27-0.3. After 2004, the estimated scale decreases, reaching 0.18 in 2015.

Figure 3: Additional adult scale, Engel and AIDS method

Note: Own calculations using HBS. All derived additional adult scales are estimated using the Engel methodology, with the exception of the *AIDS* scale in which the demand system method is used. An analytic description of the methods can be found in Section 2.1. The *Food – Working-Leser* and the *Combined-Working-Leser* scales are derived using the Working-Leser function, and the share of expenditure spent on food and food, housing costs and clothing respectively. The *Food – Kernel* and the *Combined-Working-Leser* scales are derived using non-parametric regressions, and the share of expenditure spent on food and food, housing costs and clothing respectively. The additional adult scale in the *National Scale* is 0.66.

Figure 4: Child Scales, Engel and AIDS Methods

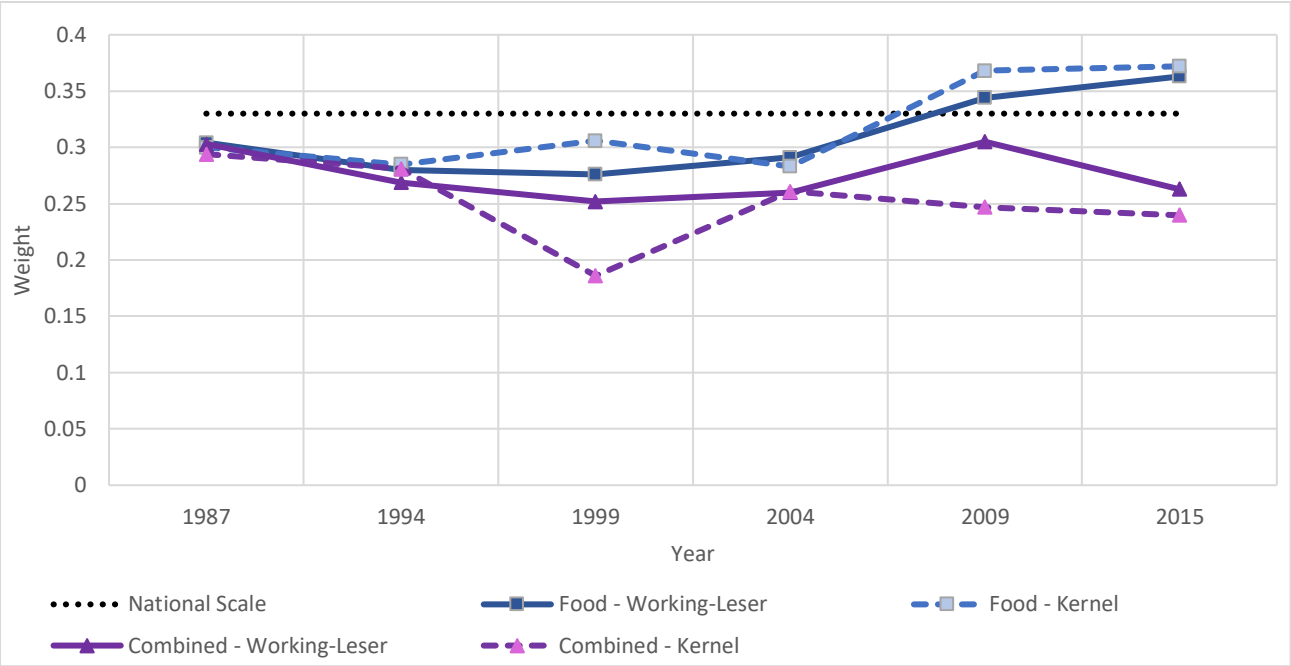


Note: Own calculations using HBS. The child scales presented are the weighted average of the scales for the first, second and third child. All derived scales are estimated using the Engel method, except for the *AIDS* estimate in which the demand system method is used. An analytic description of the methodology can be found in Section 2.1. The *Food – Working-Leser* and the *Combined-Working-Leser* scales are derived using the Working-Leser function, and the share of expenditure spent on food and food, housing costs and clothing respectively. The *Food – Kernel* and the *Combined-Working-Leser* scales are derived using non-parametric regressions, and the share of expenditure on food and food, housing costs and clothing respectively. The child scale in the *National Scale* is 0.33.

Figures 5 and 6 shows further estimates for the child scale using the Engel method, splitting results for the first child (Figure 5) from the second or third (or more) children (Figure 6). The number of children in a household plays a significant role in our estimates of equivalence scales. Estimated scales for the first child are higher than the estimated overall child scale depicted in Figure 5 and closer to the National Scale over time. The combined goods basket produces the lowest estimated scales for first children, which fall between 1994 and 2004 and have been reasonably stable since then. Using food shares, the estimated scale increases through time and, in 2015, is estimated to be higher than the National scale, at 0.36.

Figure 6 shows that the estimated scales for the second child are lower than those of the first, with the scale even lower for the third child.[17] This is unsurprising as additional children are subject to economies of scale (especially when we estimate equivalence scales using the combined goods approach). Using the food basket, the scale for the second child is 0.29 in 2015, whereas for the third child is 0.22 (using the combined goods basket the scales are 0.16 and 0.08 respectively).

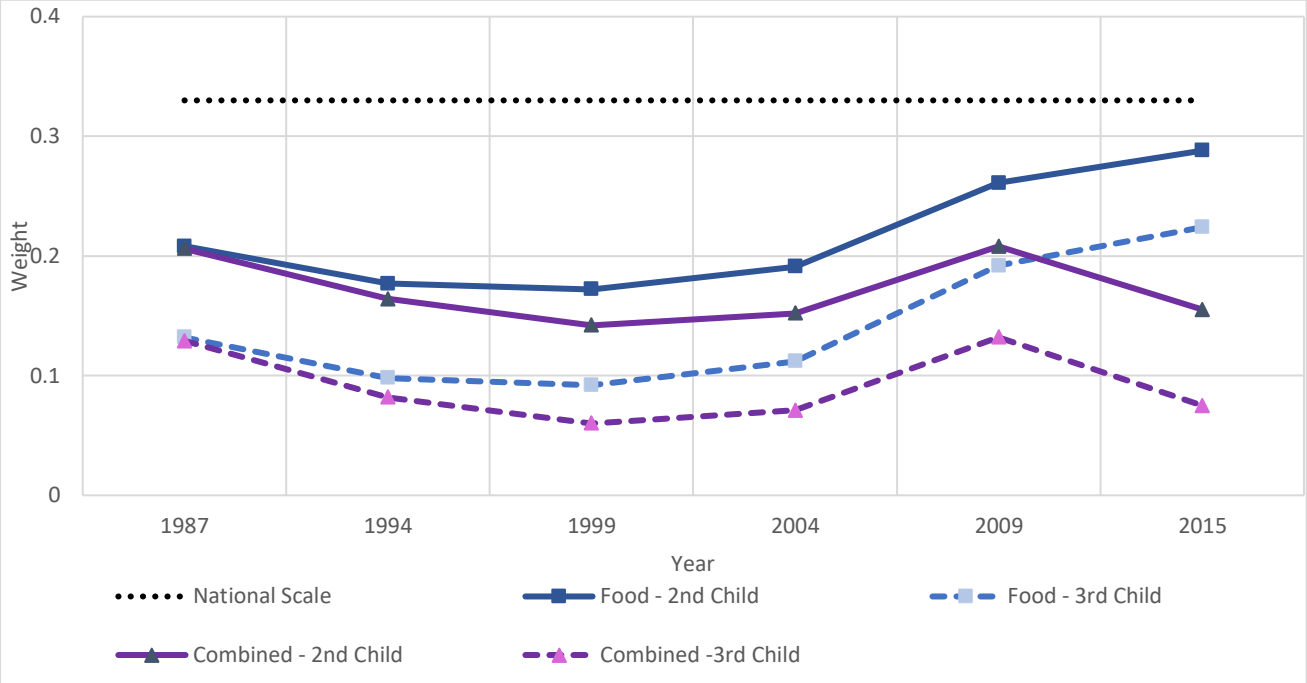
Figure 5: First Child scale – Engel method



Note: Own calculations using HBS. All derived scales are estimated using the Engel method. An analytic description of the methodology can be found in Section 2.1. The *Food – Working-Leser* and the *Combined-Working-Leser* scales are derived using the Working-Leser function, and the share of expenditure spent on food and food, housing costs and clothing respectively. The *Food – Kernel* and the *Combined-Working-Leser* scales are derived using non -parametric regressions, and the share of expenditure spent on food and food, housing costs and clothing respectively. The child scale in the *National Scale* is 0.33.

[17] In Figure 6 we present results only for the Working-Leser form. In the Appendix Table A.2. we also present results using the Kernel method.

Figure 6: Second and Third Child scales – Engel method



Note: Own calculations using HBS. All derived scales are estimated using the Engel method and the Working-Leser function. An analytic description of the methodology can be found in Section 2.1. The *Food – 2nd child* and the *Food-3rd child* scales are derived using the share of expenditure spent on food. The *Combined – 2nd child* and the *Combined-3rd child* scales are derived using the share of expenditure spent on food, housing costs and clothing. The child scale in the *National Scale* is 0.33

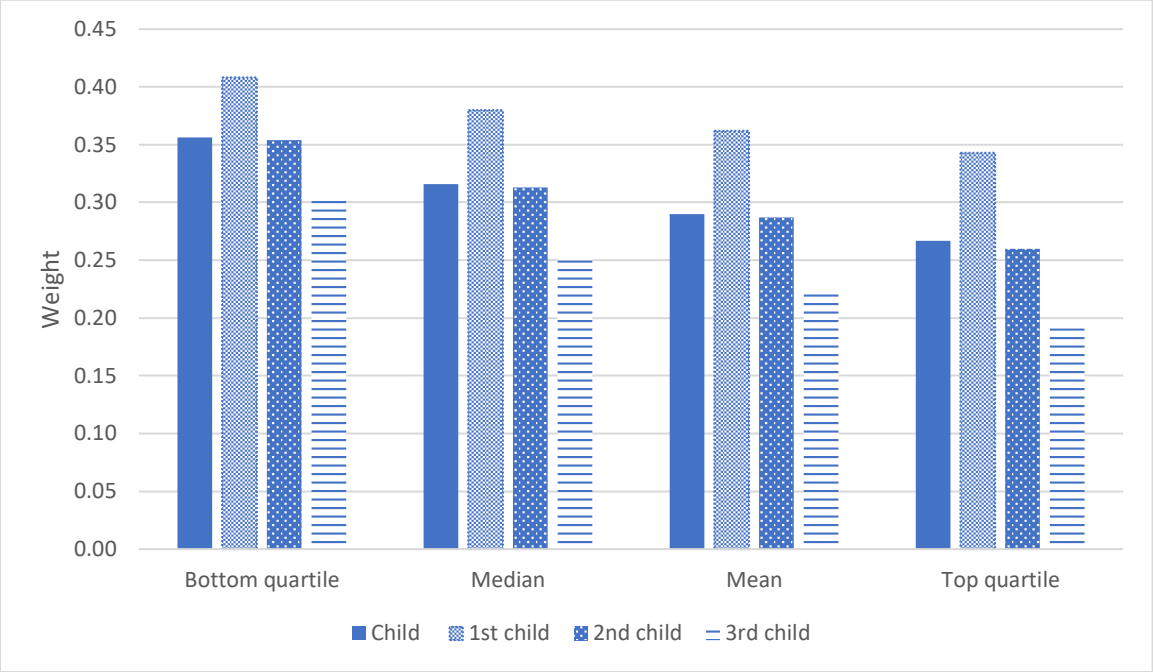
We next present results for given levels of expenditure (using the Working-Leser form and the food share of expenditure). We split households into quartiles based on their total expenditure and present results for the bottom and the top quartile, as well as the median and mean. In Figure 7, we present results for 2015 for the child scale, as well as the first, second and third child separately, but in the Appendix in Table A.3. we also present the estimated scales for previous years.

We find that the higher a household's total expenditure, the lower the child scale. For example, the total child scale is 0.36 in the bottom quartile, but 0.27 at the top quantile. This pattern holds for first, second and third children.

There are two possible reasons for this. Firstly, as Figure A.13 in the Appendix demonstrates, the share of expenditure on food is higher for the lowest quartiles, which have inherently poorer economies of scale than goods. Secondly, households in the bottom of the distribution are usually smaller in size and thus subject to smaller economies of scale (Figure A.14 in the Appendix).[18]

[18] In Figure A.12. in the Appendix we present the average number of children by quartile. The mean number of children by quartile is close in all quartiles.

Figure 7: Equivalence Scales for children



Note: Own calculations using HBS. All derived scales are estimated using the Engel method, the Working-Leser function and the share of expenditure spent on food. An analytic description of the methods can be found in Section 2.1.

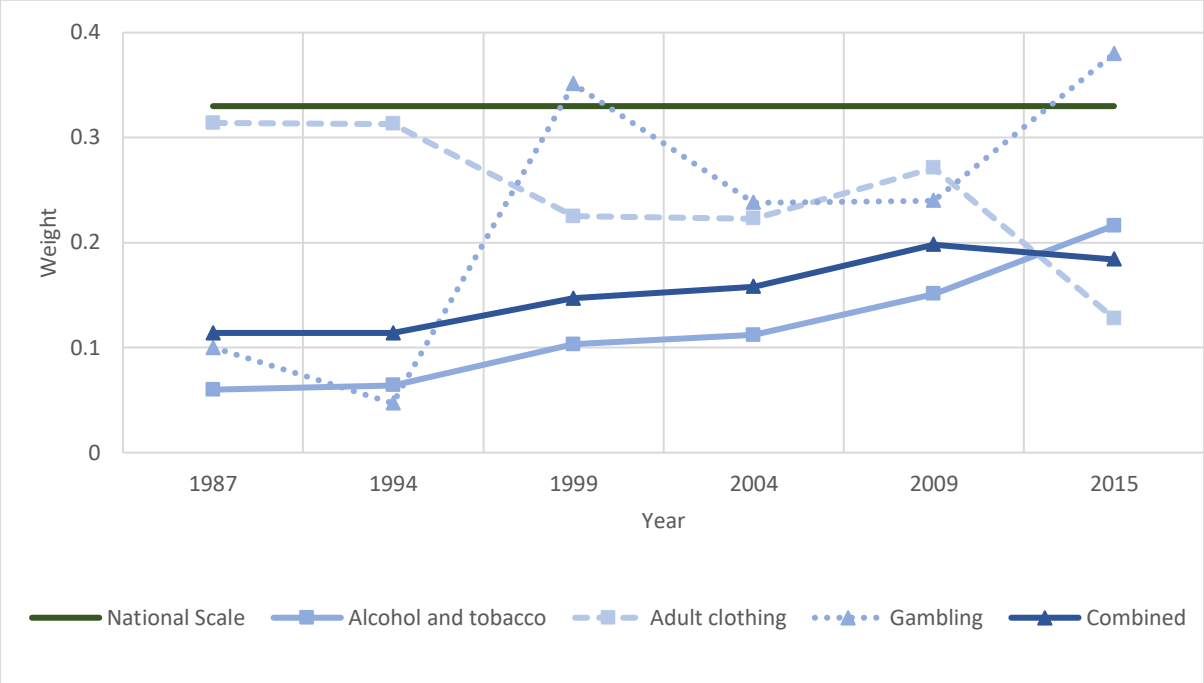
2.3.2.2. The Rothbarth method

We next present estimates of the child scale using the Rothbarth method which relies on the assumption that a household with children has a lower expenditure share on so-called "adult" goods compared with a household without children. We present result using the Working-Leser form, evaluated at median expenditure.[19] For this analysis we use four different adult goods baskets: alcohol and tobacco, adult clothing, gambling and a combined basket of all of these. In Figures 8 and 9, we present estimates of the child scale at the median expenditure.

Figure 8 presents estimates for the child scale over time using the Rothbarth method. Although our results depend on the adult good chosen, almost all scales are lower than the national scale, in line with our previous estimations using Engel's method. The child scale over time estimated using the alcohol and tobacco share of expenditure increases from 0.06 in 1987 to 0.22 in 2015. Using the adult clothing share of expenditure, the scale decreases over time, from 0.31 in 1987 to 0.13 in 2015. Using the gambling expenditure share leads to the largest fluctuations in the scale, with a low of 0.05 in 1994 and a high of 0.38 in 2015. The combined goods scale broadly follow the pattern for alcohol and tobacco, increasing from 0.11 in 1987 to 0.18 in 2015.

[19] We do not estimate the Rothbarth model non-parametrically as results are too sensitive to the income level chosen to evaluate them at (Bargain and Donni, 2010).

Figure 8: Child scale – Rothbarth method

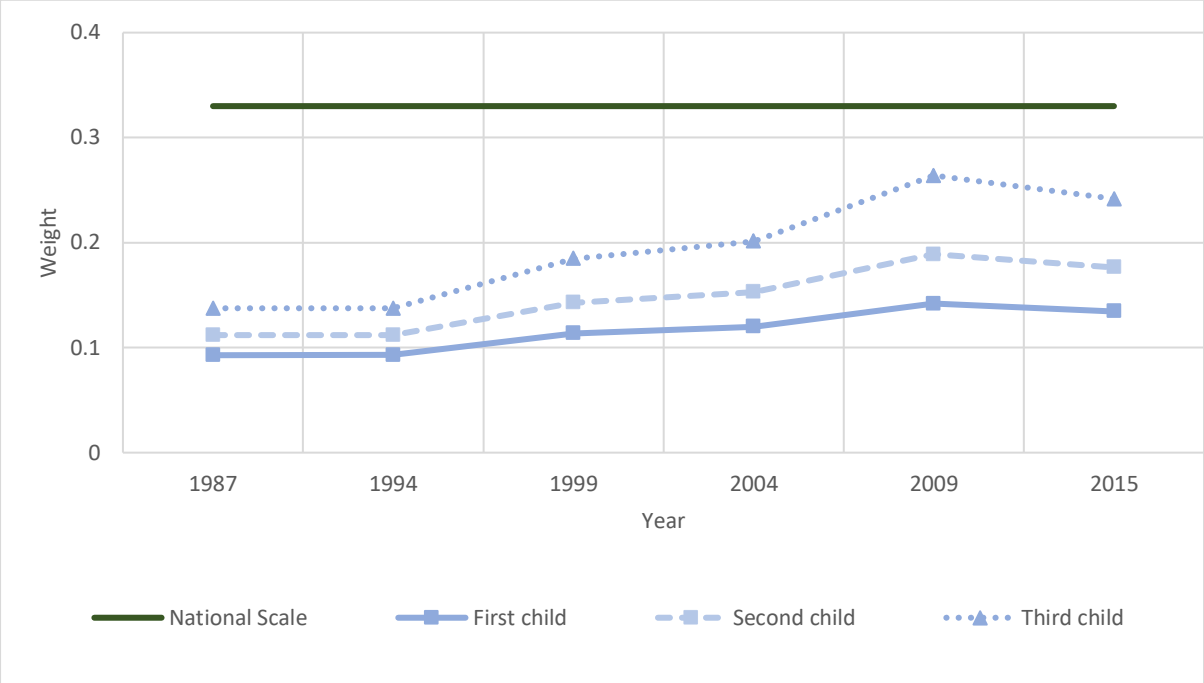


Note: Own calculations using HBS. All derived scales are estimated using the Rothbarth method at the median share of total expenditure. An analytic description of the methodology can be found in Section 2.1. The names of the scales indicate which share of expenditure is used. The *Combined* scale uses the share of expenditure spent on alcohol and tobacco, adult clothing and gambling. The child scale in the *National Scale* is 0.33

The Rothbarth method yields lower and more variable estimates than the Engel method. This is reasonable, as adult goods provide more flexibility for households to reduce their share of expenditure compared to the goods used in the Engel method. However, in both cases, the scales lie below the national scale.

In Figure 9 we also present results for the first, second and third child using the combined adult goods share of expenditure. Contrary to results using the Engel method, the child scale increases with subsequent children. This suggests that, with extra children, a family needs to decrease consumption of adult goods by proportionally more to keep their welfare at the same level. The child scale increases for all children up until 2009 and decreases marginally in 2015 (the child scale in 2015 is 0.13 for the first child, 0.18 for the second child and 0.24 for the third child).

Figure 9: Subsequent child scale – Rothbarth method



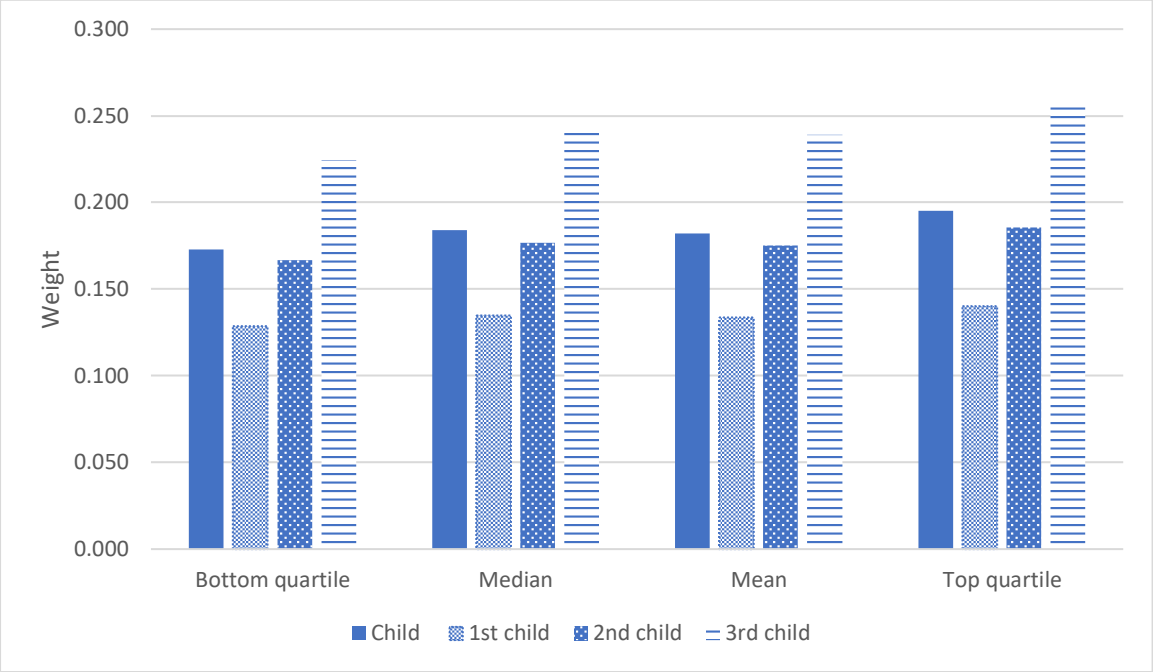
Note: Own calculations using HBS. All derived scales are estimated using the Rothbarth method with the share of expenditure spent on alcohol and tobacco, adult clothing and gambling at the median share of total expenditure. An analytic description of the methodology can be found in Section 2.1. The child scale in the *National Scale* is 0.33

We next estimate the child scale for different levels of expenditure using the Rothbarth method and the total adult goods share of expenditure (Figure 10).[20] The estimated scale for the first and second child is reasonably consistent at all levels of expenditure. The scale for the third child, however, is higher for the top expenditure quartile (0.22 for the bottom quartile vs. 0.26 for the top). The Rothbarth method relies on the assumption that the consumption of adult goods decreases with more children. However, households at the upper end of the expenditure distribution may have more flexibility in retaining the same level of expenditure on adult goods when they have more children.[21] Independently of the household's position in the expenditure distribution, economies of scale decrease (and thus the child scale increases) with subsequent children.

[20] In Figure 10 we present results for 2015, the rest of the years are in Table A.4 in the Appendix.

[21] Figure A.12 in the Appendix, presents the share of expenditure on alcohol and tobacco, adult clothing and gambling by quartile.

Figure 10: Equivalence Scales for children by share of expenditure, 2015, Rothbarth Method



Note: Own calculations using HBS. All derived scales are estimated using the Rothbarth method with the share of expenditure spent on alcohol and tobacco, adult clothing and gambling. The Rothbarth method estimates scales in a specific level of total expenditure, so the *Bottom quartile* scales refer to estimations made at the 25 percentile and *Top quartile* refer to estimations made at the 75%. An analytic description of the methodology can be found in Section 2.1.

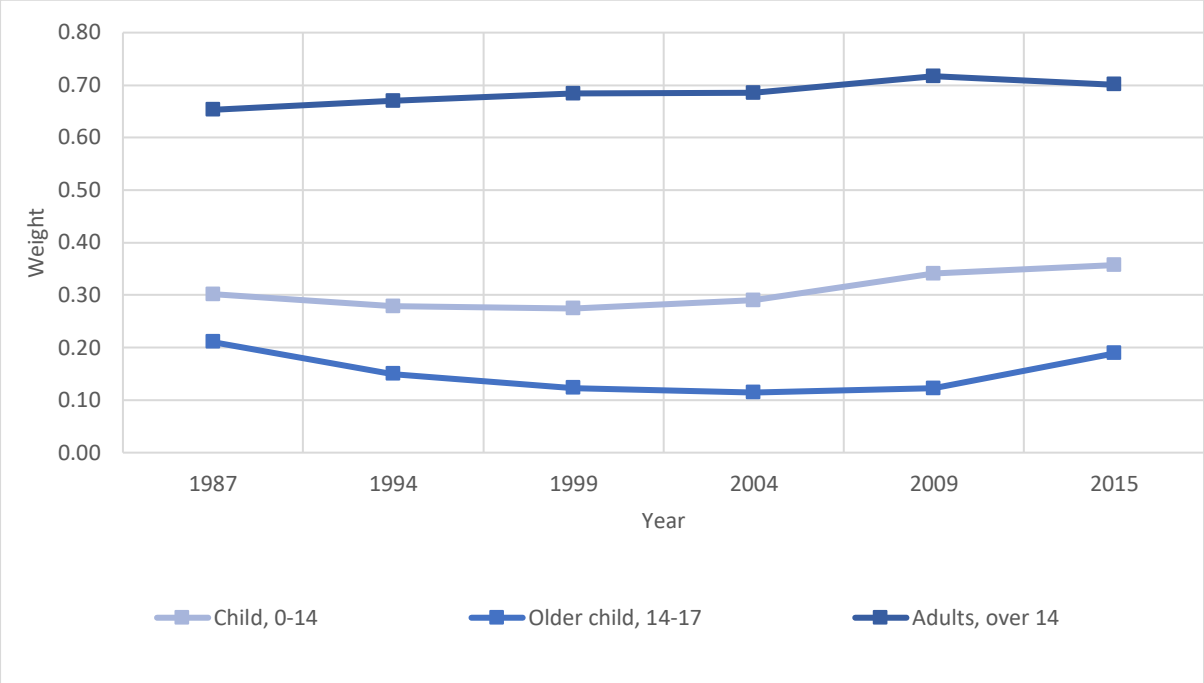
2.3.3. Scales for individuals ages 14 to 17

Official statistics in Ireland and internationally treat older children, ages 14 to 17, as adults to produce welfare statistics. Figure 11 shows our estimate of the scale for individuals aged 14 to 17 using the Working-Leser form for the Engel method and the food share of expenditure.[22] The estimated scale is lower than the estimated adult scale, but also lower than the estimated child scale. Specifically, we estimate the older child scale is 0.21 in 1987 while the child scale is 0.30 and the adult scale is 0.65. The estimated older child scale varies over the sampling period, between 0.11 and 0.21 and is consistently lower than the estimated child and adult scales.

Two observations emerge from these estimates. First, they suggest that there may be greater economies of scale with older children. Second, the equivalence scale used for welfare statistics in Ireland may be mis-categorising older children as adults, with implications for headline rates of income poverty and inequality.

[22] In Table A.6 in the Appendix, we present results using the food share of expenditure and the Kernel method.

Figure 11: Scales based on age - Engel Method



Note: Own calculations using HBS. All derived scales are estimated using the Engel method with the share of expenditure spent on food and the Working-Leser function. An analytic description of the methodology can be found in Section 2.1.

3. Inequality and poverty rates

One of the main uses of equivalence scales is to produce income inequality and poverty statistics. Following existing research (Callan et al.,1989; Figini,1998; Regan and Kakoulidou, 2022), we use the range of equivalence scales derived in previous sections to estimate for the period 1987-2015 the Gini coefficient and the At-Risk-Of-Poverty (AROP) rate for the whole population, as well as for children and those over 65 years of age.[23]

As the Rothbarth method produces scales only for children, we use the National scale's adult scale for these equivalence scales. We present also the same statistics using the National Scale (used by the Irish CSO), the modified OECD scale (used by EUROSTAT) and the square root scale (used by OECD). The estimated scales for adults and children in each equivalence scale are presented in Table 1 using data from 1987 ESRI Survey of Income Distribution, Poverty and Usage of State Services, the 1994/1999 waves of the Living in Ireland Survey and the 2004/2009/2015 wavs of Survey of Income and Living Conditions (SILC) harmonised by Roantree et al. (2021). We focus on Engel estimates using the Working-Leser form and the food or combined expenditure share; Rothbarth estimates using the combined adult goods expenditure share and the AIDS method. [24] The square root scale (not shown in the table) simply takes the value of the square root of the household size.

[23] The AROP rate population are those individuals that are below 60% of the median equivalised income.
[24] In Tables A.7-A.10 in the Appendix we present also results using the Kernel method.

Table 1: Equivalence scales

	National Scale		Modified OECD		Food (WL)	
	Adult	Child	Adult	Child	Adult	Child
1987	0.66	0.33	0.5	0.3	0.653	0.215
1994	0.66	0.33	0.5	0.3	0.670	0.185
1999	0.66	0.33	0.5	0.3	0.684	0.180
2004	0.66	0.33	0.5	0.3	0.685	0.198
2009	0.66	0.33	0.5	0.3	0.717	0.267
2015	0.66	0.33	0.5	0.3	0.701	0.292

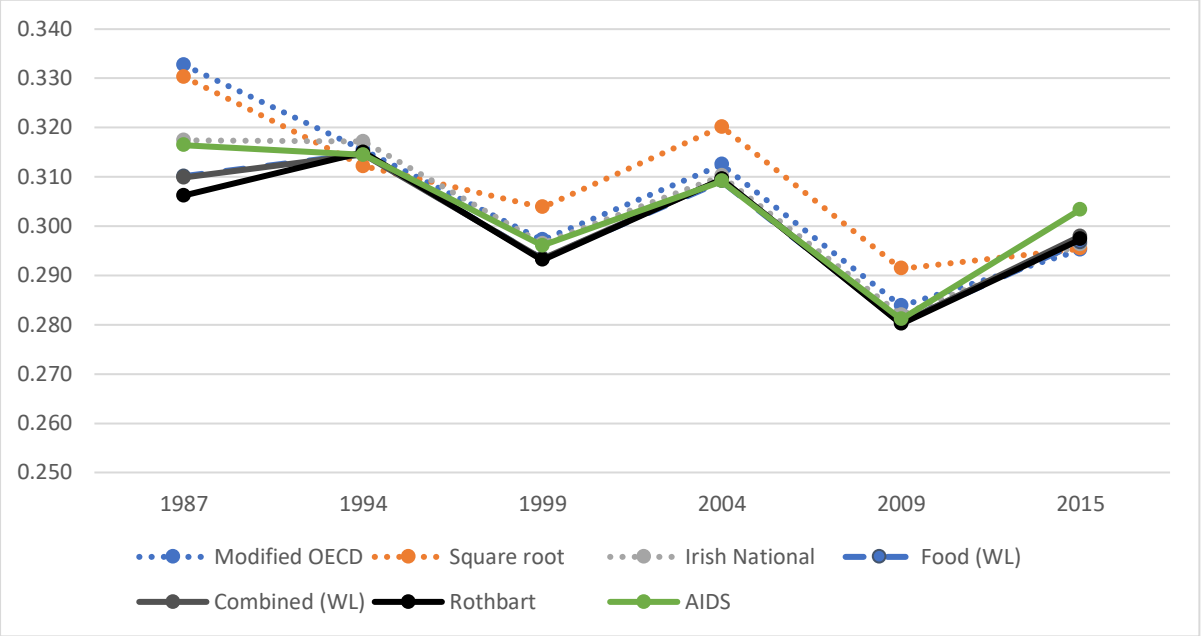
	Combined (WL)		Rothbarth		AIDS	
	Adult	Child	Adult	Child	Adult	Child
1987	0.663	0.213	0.66	0.114	0.597	0.292
1994	0.664	0.172	0.66	0.114	0.596	0.251
1999	0.683	0.151	0.66	0.147	0.857	0.282
2004	0.668	0.161	0.66	0.158	0.846	0.268
2009	0.630	0.215	0.66	0.198	0.864	0.227
2015	0.664	0.164	0.66	0.184	0.931	0.175

Note: Own calculations using HBS. An analytic description of the methodology can be found in Section 2.1 The *Food (WL)* scale has the derived scales using the Engel method, the Working-Leser form and the share of expenditure spent on food. The *Combined (WL)* scale has the derived scales using the Engel method, the Working-Leser form and the share of expenditure spend on food, housing costs and clothing. The *Rothbarth* scale has the derived child scales using the Rothbarth method, the Working-Leser form and the share of expenditure spend on alcohol and tobacco, adult clothing and gambling. As we do not have a derived scale for the additional adult from the Rothbarth method, we use the National scale (0.66), The *AIDS* scale has the derived scales using the demand-system approach. The *Adult* scale refers to the scale for the additional adults.

Figure 12 plots the estimated Gini coefficient over time using the range of equivalence scales. For most years, the Gini coefficient is similar when we apply any of our derived equivalence scales compared to the Irish national equivalence scales. It is slightly lower than the Gini coefficient estimated using either the OECD modified scale or the square root scale. The year 2015 is an exception and we see that our estimated scales lead to marginally higher Gini coefficients (0.297 to 0.303 compared to the range of the expert scales of 0.295 to 0.296).

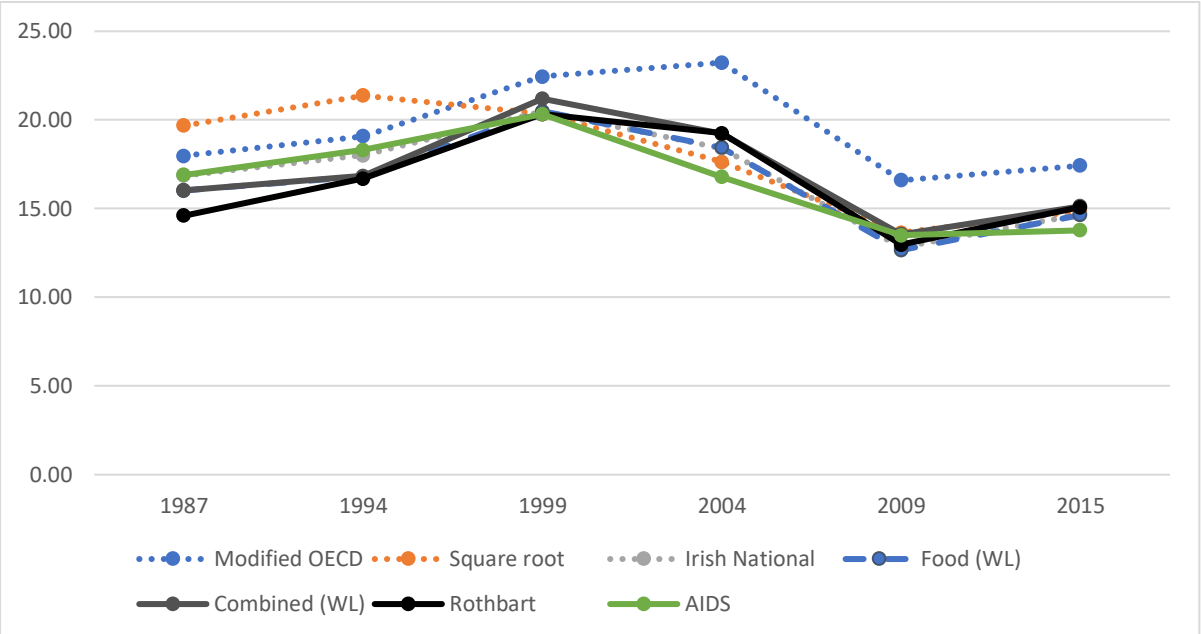
Figure 13 plots the AROP rate over time using the expert and estimated equivalence scales. In all the years examined, the AROP rate is higher when estimated using the square root scale and the Modified OECD scale. The Irish National Scale and the estimated scales using food shares, combined food, housing costs and clothing shares; adult good shares and the AIDS method yield similar results in most years.

Figure 1: Gini coefficient



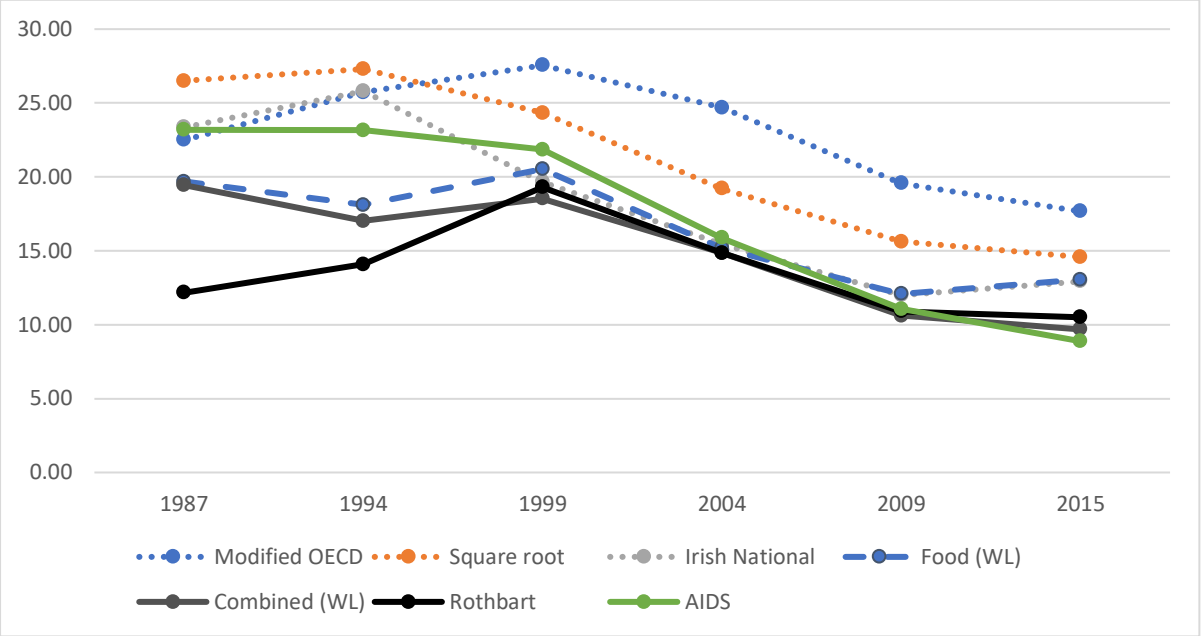
Note: Own calculations using HBS. The scales can be found in Table 1.

Figure 13: AROP rate



Note: Own calculations using HBS. The scales can be found in Table 1.

Figure 14: Child poverty



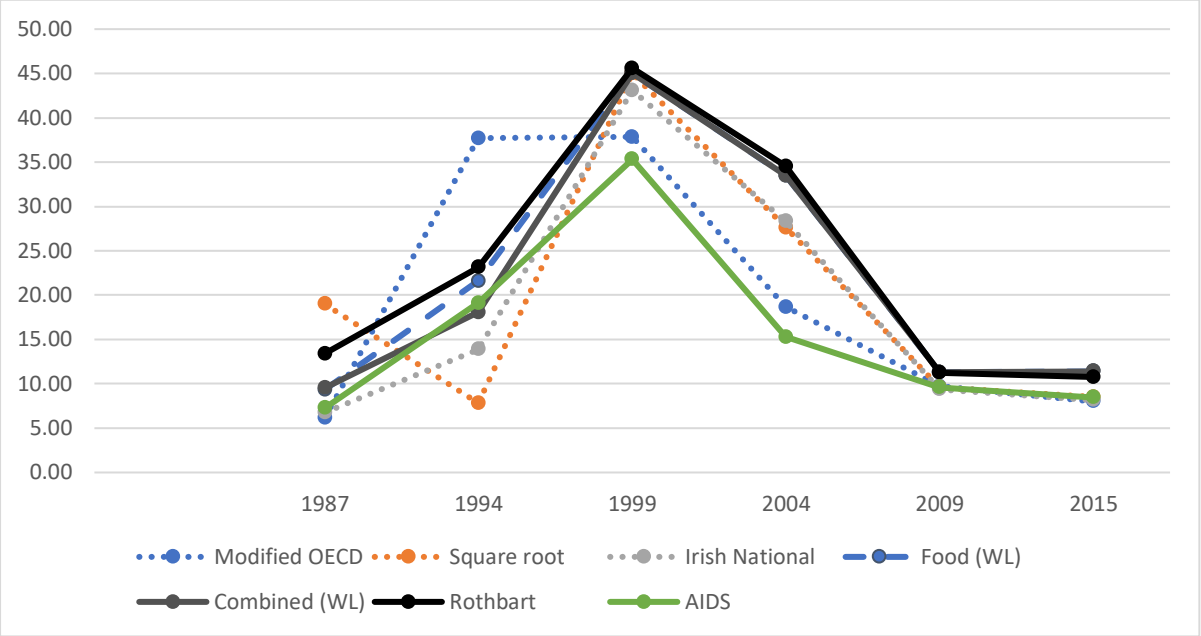
Note: Own calculations using HBS. The scales can be found in Table 1.

Figure 14 plots child poverty rates over time using each scale, where children are defined as aged < 14 years of age. Child poverty is considerably higher when using the expert equivalence scales (especially the square root scale) compared with our estimated scales. For example, in 2015, the child poverty rate is 17.7 per cent when using the square root scale, 14.6 per cent when using the Irish National scale and at 8.8 per cent when using the AIDS model. This is likely to be due to the lower value for child scales in the derived scales compared to the expert scales.

Perhaps more concerning is the fact that the pattern of changes in child poverty over time is different depending on the scale used. For example, using the national scale, we observe a decrease in child poverty between 1994 and 1999. However, using any of the derived scales, there is either an increase in child poverty or a much flatter pattern of change. This suggests that population changes over time, such as expenditure patterns or household size, may need to be explicitly incorporated into equivalence scales in order to accurately measure welfare changes for particular groups, such as children.

Elderly poverty varies considerably with the scale chosen (Figure 15). Even among the expert scales, there is a large discrepancy over time in the estimates of elderly poverty, with the square root scale usually yielding the highest estimates and the national scale yielding the lowest estimate. Each of the Engel and Rothbarth derived scales yields reasonably similar estimates of elderly poverty over time which follow most closely the modified OECD scale. The AIDS method yields a lower estimate of elderly poverty over time.

Figure 2: Elderly poverty



Note: Own calculations using HBS. The scales can be found in Table 1.

4. Conclusion

This paper has examined whether there is an empirical basis to support the use of the existing national scale in Ireland over alternatives. We did so using three different methods to derive equivalence scales from the patterns of expenditure observed in Household Budget Surveys covering the period 1987-2015.

To estimate the appropriate scales for a second (or subsequent) adult in a household, we use the Engel and AIDS methods. Engel estimates yield adult scales which are reasonably close to the national scale of 0.66 and, indeed, closer to the national scale than to the widely used modified OECD scale, which assigns a scale of 0.5 to additional adults. However, AIDS estimates suggest that adult scales might be considerably higher than this, especially in recent years for which we obtain estimates of close to 0.9.

We use the Engel, Rothbarth and AIDS methods to derive appropriate child scales. All specifications suggest that the child scale is below that inherent to both the national equivalence scale of 0.33 and the OECD modified scale of 0.3. Using food shares, the Engel method yields the highest child scales (of around 0.3 in 2015) while scales estimated using other methods yield scales of 0.15 to 0.18 in 2015.

We also derive a scale for those aged 14-17 who, in most equivalence scales, are considered as adults and not children. We estimate a scale of 0.2 for these individuals in 2015, which is below the derived child scale using the same method, but also significantly below the scale assigned to adults in either the national (0.66) or modified OECD (0.5) expert scales.

We test the sensitivity of estimates of income inequality and poverty over time to the use of three expert and a number of our derived equivalence scales. Broadly speaking, income inequality estimated by the Gini coefficient is not particularly sensitive to the equivalence scale used. We observe minor

differences in the level of income inequality but patterns over time are almost identical, regardless of the scale chosen.

Poverty rates, on the other hand, are more sensitive to the choice of equivalence scale. In particular, we observe large differences in the level and trend of child poverty, depending on the scale chosen. Estimated child poverty is systematically lower when measured using our derived scales compared to the expert scales. This suggests that the use of ad hoc expert scales may overstate the degree of child poverty but, more importantly, may under or overstate the change in child poverty as they are fixed over time while expenditure patterns and household size are not.

Our results have implications for those engaged in the estimation of welfare statistics using a national scale or an alternative expert scale, particularly those that have not been recently re-evaluated. At a minimum, it appears sensible to test the sensitivity of results – particularly poverty indices – to the use of alternative equivalence scales, including country and time specific scales.

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Appendix

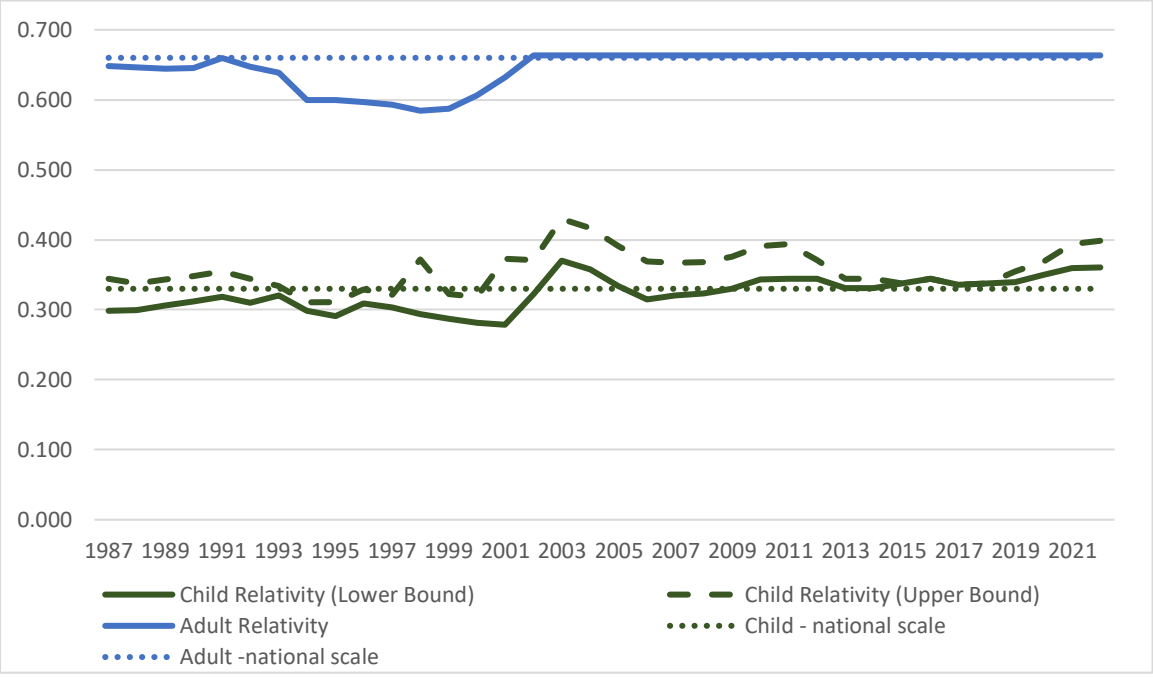
A. Evolution of relativities over time

The existing Irish national equivalence scale is one of three scales used by Callan et al (1989). It was derived using the existing parameters of the then Unemployment Assistance (now Jobseekers' Allowance). Recipients of this welfare payment receive an extra amount if there is a dependent adult (a spouse) or a child in the household. Callan et al (1989) used the ratio of the increase for a qualified adult (IQA) to the core payment as the adult equivalence and the ratio of the increase for a qualified child (IQC) plus the universal Children's Allowance payment (now Child Benefit) to the core payment as the child equivalence.

Figure A.1 shows how these relativities have changed over the last 40 years, specifically the evolution of the main rate as well as increases for child and adult dependants for the Jobseeker Benefit, the Jobseeker Allowance and the State Pension over the last forty years (from 1982 to 2022). Table A.1 presents these rates in nominal terms over time.

The implied relativity for adults has changed marginally over time, mainly because the IQAs always increase proportionally with the main rate (something that didn't happen for the IQCs). In the period between 1990 and 2000, the implied adult relativity fell, reaching a low of 0.58 in 1998. However, it has remained close to 0.66, the parameter used in the national scale, for the last two decades. For children, the implied relativity was close to 0.33 until the early 2000's, when it increased slightly. The current implied child relativity is between 0.36 and 0.4, depending on the number of children in a family.

Figure A.3: Adult and child scales and relativities, 1987-2022



Note: Own calculations. The adult relativity is the ratio of the IQA for Jobseekers Assistance to the core rate. The child relativity is the ratio of the IQC for Jobseekers Assistance (plus Child Benefit) to the core rate. In years when Child Benefit or

IQC varies by the number of children in a family, we choose the lowest and highest rate per child to construct the lower and upper bounds.

B. Tables

Table A.1. Welfare payments, nominal rates 1987-2022

Year	Unemployment Benefit / Jobseeker's Benefit (Nominal)	UB / JB Qualified Adult Increase	UB / JB Qualified Child Increase (First Child)	Child Benefit (First Child)
1987	42.3	27.4	9.37	3.76
1988	43.6	28.2	10.6	3.76
1989	45	29	10.9	3.76
1990	48	31	11.4	3.95
1991	50	33	12	3.95
1992	53	34.3	12.5	3.95
1993	55.6	35.5	12.8	5
1994	61	36.6	13.2	5
1995	62.5	37.5	13.2	5
1996	64.5	38.5	13.2	6.75
1997	67.5	40	13.2	7.25
1998	70.5	41.2	13.2	7.5
1999	93.33	54.85	16.76	10
2000	98.43	59.68	16.76	10.96
2001	108.56	68.57	16.76	13.49
2002	118.80	78.80	16.8	21.45
2003	124.80	82.80	16.8	29.4
2004	134.80	89.40	16.8	31.4
2005	148.80	98.70	16.8	32.9
2006	165.80	110.00	16.8	35.4
2007	185.80	123.30	22	37.5
2008	197.80	131.30	24	40
2009	204.30	135.60	26	41.5

2010	196.00	130.10	29.8	37.5
2011	188.00	124.80	29.8	35
2012	188.00	124.80	29.8	35
2013	188.00	124.80	29.8	32.5
2014	188.00	124.80	29.8	32.5
2015	188.00	124.80	29.8	33.75
2016	188.00	124.80	29.8	35
2017	193.00	128.10	29.8	35
2018	198.00	131.40	31.8	35
2019	203.00	134.70	34	35
2020	203.00	134.70	36	35
2021	203.00	134.70	38	35
2022	208.00	138.00	40	35

Notes: Cells in grey are given in Irish pounds. All other cells are given in euros. Cells in green are given for children aged over 12.

Table A.2. Second and third child scale estimates using the Engels - Kernel method

	Food share of expenditure		Combined goods share of expenditure	
	2nd child	3rd child	2nd child	3rd child
1987	0.201 (0.013)	0.125 (0.021)	0.194 (0.018)	0.116 (0.029)
1994	0.183 (0.011)	0.104 (0.017)	0.177 (0.051)	0.099 (0.082)
1999	0.209 (0.038)	0.133 (0.061)	0.064 (0.034)	0 (0.050)
2004	0.180 (0.016)	0.101 (0.025)	0.152 (0.044)	0.072 (0.069)
2009	0.294 (0.075)	0.233 (0.129)	0.135 (0.029)	0.054 (0.045)
2015	0.301 (0.022)	0.239 (0.038)	0.126 (0.043)	0.045 (0.066)

Own calculations using HBS . All derived scales are estimated using the Engel method and Kernel regressions. An analytic description of the methodology can be found in Section 2.1. The *Food – 2nd child* and the *Food-3rd child* scales are derived using the share of expenditure spent on food. The *Combined – 2nd child* and the *Combined-3rd child* scales are derived using the share of expenditure spent on food, housing costs and clothing.

Table A.3 Child scale by total expenditure, Engel method, 1987-2015

First quartile			
	1st child	2nd child	3rd child
1987	0.28	0.17	0.09
	(0.028)	(0.063)	(0.09)
1994	0.19	0.073	0
	(0.035)	(0.074)	(0.110)
1999	0.21	0.09	0.01
	(0.040)	(0.088)	(0.132)
2004	0.30	0.20	0.13
	(0.041)	(0.094)	(0.154)
2009	0.42	0.37	0.33
	(0.056)	(0.142)	(0.252)
2015	0.47	0.45	0.43
	(0.098)	(0.256)	(0.470)
Second Quartile			
	1st child	2nd child	3rd child
1987	0.32	0.23	0.16
	(0.015)	(0.037)	(0.060)
1994	0.31	0.21	0.13
	(0.017)	(0.040)	(0.065)
1999	0.26	0.15	0.07
	(0.022)	(0.048)	(0.075)
2004	0.28	0.17	0.09
	(0.026)	(0.059)	(0.094)
2009	0.36	0.29	0.23
	(0.030)	(0.072)	(0.122)
2015	0.41	0.35	0.30
	(0.029)	(0.073)	(0.128)
Third Quartile			
	1st child	2nd child	3rd child
1987	0.25	0.14	0.05
	(0.028)	(0.062)	(0.097)
1994	0.29	0.18	0.10
	(0.021)	(0.048)	(0.077)
1999	0.30	0.21	0.13
	(0.019)	(0.044)	(0.072)
2004	0.29	0.19	0.11
	(0.026)	(0.059)	(0.094)
2009	0.34	0.26	0.19
	(0.026)	(0.062)	(0.105)
2015	0.35	0.27	0.21
	(0.031)	(0.074)	(0.125)
Fourth Quartile			
	1st child	2nd child	3rd child
1987	0.31	0.21	0.14
	(0.014)	(0.033)	(0.053)
1994	0.30	0.20	0.12
	(0.015)	(0.034)	(0.056)
1999	0.31	0.22	0.14
	(0.016)	(0.038)	(0.063)
2004	0.35	0.26	0.20
	(0.016)	(0.039)	(0.066)

2009	0.37	0.30	0.24
	(0.019)	(0.047)	(0.080)
2015	0.35	0.27	0.21
	(0.022)	(0.052)	(0.089)

Note: Own calculations using HBS. All derived scales are estimated using the Engel method, the Working-Leser function and the share of expenditure spent on food. An analytic description of the methods can be found in Section 2.1. Standard errors are computed using the delta method.

Table A.4 Child scale by total expenditure, Rothbarth method, 1987-2015

First quartile	1st child	2nd child	3rd child	Child
1987	0.09 *(0.017)*	0.11 *(0.012)*	0.13 *(0.012)*	0.11 *(0.008)*
1994	0.09 *(0.019)*	0.10 *(0.014)*	0.13 *(0.014)*	0.11 *(0.009)*
1999	0.11 *(0.021)*	0.14 *(0.017)*	0.18 *(0.019)*	0.14 *(0.011)*
2004	0.11 *(0.021)*	0.14 *(0.017)*	0.18 *(0.019)*	0.15 *(0.011)*
2009	0.14 *(0.023)*	0.18 *(0.020)*	0.24 *(0.025)*	0.19 *(0.013)*
2015	0.13 *(0.027)*	0.17 *(0.024)*	0.22 *(0.029)*	0.17 *(0.015)*
Median				
	1st child	2nd child	3rd child	Child
1987	0.09 *(0.012)*	0.11 *(0.008)*	0.14 *(0.009)*	0.11 (0.006)
1994	0.09 *(0.013)*	0.11 *(0.010)*	0.14 *(0.012)*	0.11 (0.007)
1999	0.11 *(0.013)*	0.14 *(0.012)*	0.18 *(0.015)*	0.15 (0.008)
2004	0.12 *(0.012)*	0.15 *(0.014)*	0.20 *(0.017)*	0.16 (0.008)
2009	0.14 *(0.015)*	0.19 *(0.016)*	0.26 *(0.023)*	0.20 *(0.011)*
2015	0.13 *(0.017)*	0.18 *(0.018)*	0.24 *(0.026)*	0.18 (0.012)
Mean				
	1st child	2nd child	3rd child	Child
1987	0.09 *(0.012)*	0.11 *(0.008)*	0.14 *(0.009)*	0.11 *(0.006)*
1994	0.09 *(0.012)*	0.11 *(0.010)*	0.14 *(0.012)*	0.11 *(0.007)*
1999	0.11 *(0.013)*	0.14 *(0.012)*	0.18 *(0.015)*	0.15 *(0.008)*
2004	0.12 *(0.014)*	0.15 *(0.013)*	0.20 *(0.017)*	0.16 *(0.009)*
2009	0.14 *(0.015)*	0.19 *(0.016)*	0.26 *(0.023)*	0.20 *(0.011)*
2015	0.13 *(0.017)*	0.18 *(0.018)*	0.24 *(0.026)*	0.18 *(0.012)*
Top quartile				
	1st child	2nd child	3rd child	Child
1987	0.10 *(0.014)*	0.12 *(0.009)*	0.15 *(0.010)*	0.12 (0.006)
1994	0.10 *(0.015)*	0.12 *(0.011)*	0.15 *(0.013)*	0.12 (0.008)
1999	0.12 *(0.015)*	0.15 *(0.012)*	0.19 *(0.015)*	0.15 (0.008)
2004	0.13 *(0.015)*	0.16 *(0.013)*	0.22 *(0.018)*	0.17 (0.009)
2009	0.15 *(0.017)*	0.20 *(0.017)*	0.29 *(0.024)*	0.21 (0.011)

2015	0.14	0.19	0.26	0.20
	(0.019)	*(0.019)*	*(0.027)*	(0.013)

Note: Own calculations using HBS. All derived scales are estimated using the Rothbarth method with the share of expenditure spent on alcohol and tobacco, adult clothing and gambling. An analytic description of the methodology can be found in Section 2.1. The child scale in the *National Scale* is 0.33

Table A.5 Equivalence scales using the Kernel method

	Food (Kernel)		Combined (Kernel)	
	Adult	Child	Adult	Child
1987	0.658	0.299	0.663	0.294
	(0.003)	(0.006)	(0.003)	(0.007)
1994	0.607	0.289	0.654	0.280
	(0.003)	(0.004)	(0.006)	(0.022)
1999	0.748	0.306	0.703	0.186
	(0.006)	(0.016)	(0.012)	(0.016)
2004	0.684	0.283	0.661	0.261
	(0.008)	(0.007)	(0.010)	(0.020)
2009	0.775	0.368	0.625	0.245
	(0.023)	(0.031)	(0.017)	(0.013)
2015	0.712	0.372	0.649	0.240
	(0.011)	(0.009)	(0.006)	(0.019)

Note: Own calculations using HBS. The child scales presented are the weighted average of the scales for the first, second and third child. All derived scales are estimated using the Engel method and non-parametric regressions: an analytic description of the methodology can be found in Section 2.1. The *Food – Kernel* and the *Combined-Working-Leser* scales are derived using the share of expenditure on food and food, housing costs and clothing respectively.

Table A.6 Estimates for young and older children, Kernel method with the food share of expenditure

	Young children (0 -5 years old)	Middle children (6 – 13 years old)	Older children (14 – 17 years old)
1987	0.545	0.259	0.392
	(0.003)	(0.007)	(0.016)
1994	0.541	0.278	0.377
	(0.003)	(0.014)	(0.024)
1999	0.557	0.232	0.235
	(0.008)	(0.005)	(0.005)
2004	0.547	0.253	0.351
	(0.002)	(0.008)	(0.013)
2009	0.532	0.279	0.470
	(0.004)	(0.030)	(0.038)
2015	0.523	0.332	0.422
	(0.005)	(0.013)	(0.009)

Note: Own calculations using HBS. All derived scales are estimated using non-parametric regressions with the share of expenditure spent on food. An analytic description of the methodology can be found in Section 2.1.

Table A.7 Gini coefficient estimates

	Modified OECD	Square root	Irish National	Food (WL)	Food (Kernel)	Combined (WL)	Combined (Kernel)	Rothbarth
1987	0.333	0.330	0.317	0.310	0.310	0.310	0.309	0.306

1994	0.315	0.312	0.317	0.315	0.314	0.315	0.314	0.315
1999	0.297	0.304	0.296	0.293	0.294	0.293	0.294	0.293
2004	0.313	0.320	0.310	0.309	0.309	0.309	0.309	0.309
2009	0.284	0.291	0.282	0.281	0.281	0.280	0.281	0.280
2015	0.295	0.295	0.296	0.297	0.297	0.298	0.299	0.297

Note: Own calculations using HBS. The scales can be found in Table 1.

Table A.8 AROP rate estimates

	Modified OECD	Square root	Irish National	Food (WL)	Food (Kernel)	Combined (WL)	Combined (Kernel)	Rothbarth
1987	17.98	19.69	16.86	16.02	15.99	16.03	15.96	14.62
1994	19.09	21.37	18.01	16.80	17.85	16.85	17.23	16.70
1999	21.36	22.44	20.34	20.48	20.50	20.43	21.20	20.33
2004	20.45	23.24	17.64	18.41	18.44	19.18	19.21	19.26
2009	14.64	16.60	13.65	12.82	12.66	13.25	13.55	12.97
2015	15.80	17.43	14.86	14.66	14.64	15.12	15.13	15.09

Note: Own calculations using HBS. The scales can be found in Table 1.

Table A.9 Child poverty estimates

	Modified OECD	Square root	Irish National	Food (WL)	Food (Kernel)	Combined (WL)	Combined (Kernel)	Rothbarth
1987	22.50	26.49	23.37	19.69	19.33	19.45	18.80	12.18
1994	25.72	27.30	25.80	18.10	19.18	17.03	18.25	14.09
1999	25.83	27.55	24.32	19.67	20.53	19.50	18.52	19.30
2004	20.84	24.70	19.20	15.47	15.18	14.86	14.86	14.86
2009	16.86	19.56	15.62	12.00	12.08	12.06	10.62	10.90
2015	15.24	17.67	14.58	12.94	13.06	10.02	9.68	10.50

Note: Own calculations using HBS. The scales can be found in Table 1.

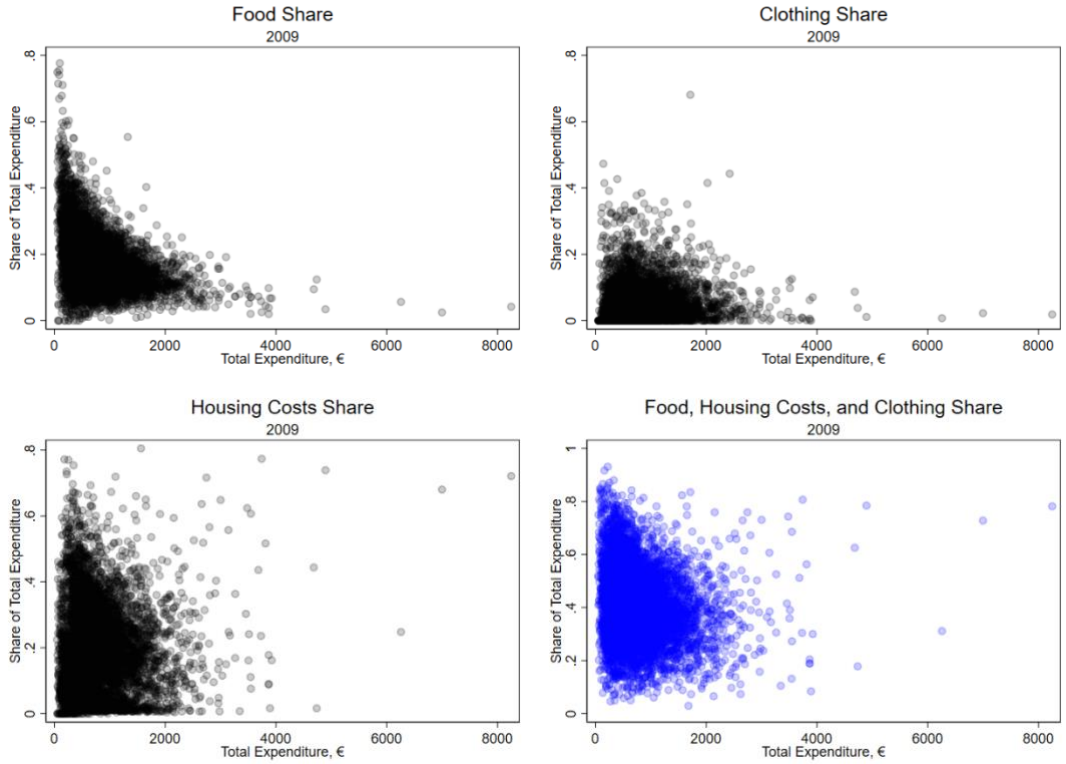
Table A.10 Elderly poverty estimates

	Modified OECD	Square root	Irish National	Food (WL)	Food (Kernel)	Combined (WL)	Combined (Kernel)	Rothbarth
1987	6.16	18.99	6.77	9.35	9.54	9.54	9.73	13.40
1994	22.31	37.70	7.82	13.89	21.61	18.07	18.07	23.13
1999	44.32	52.17	37.85	45.01	43.09	45.01	48.37	45.60
2004	36.96	50.13	18.68	27.60	28.33	33.48	34.18	34.52
2009	15.45	28.50	9.70	9.61	9.39	11.28	13.06	11.25
2015	12.41	22.66	8.01	8.52	8.18	11.42	11.84	10.77

Note: Own calculations using HBS. The scales can be found in Table 1.

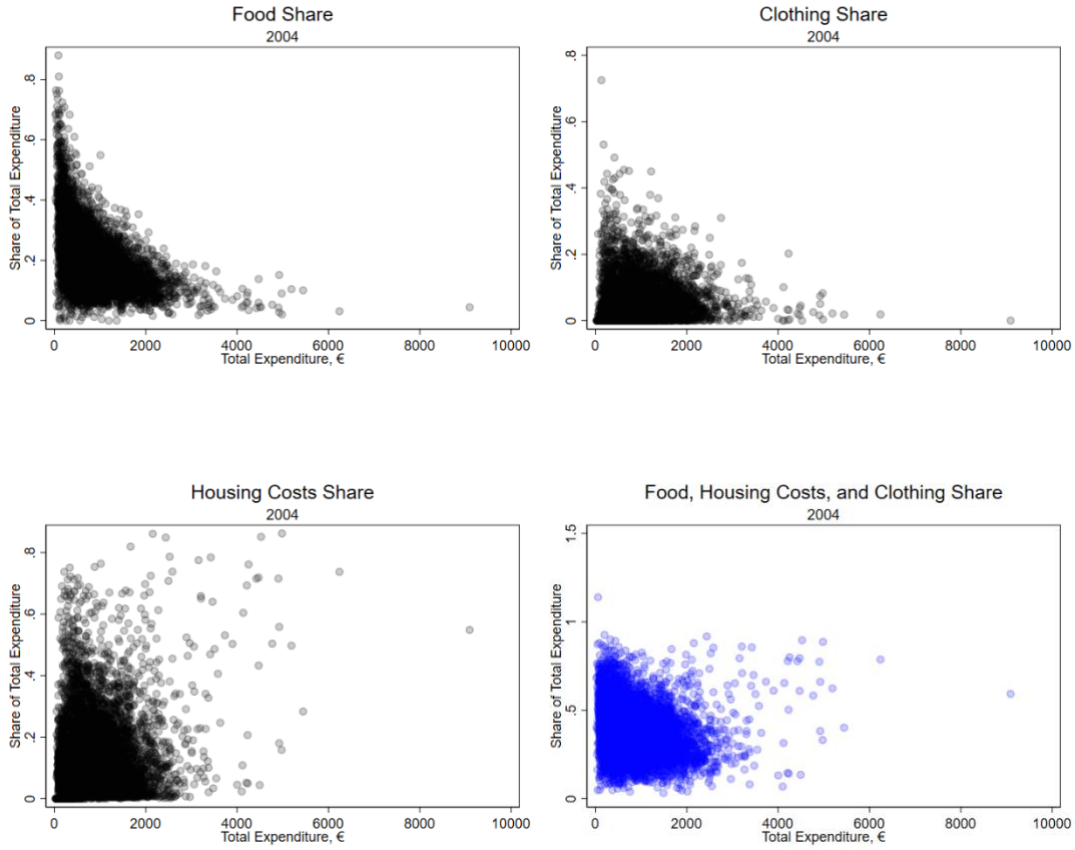
C. Figures

Figure A.2: Expenditure in goods used in Engel's method as share of total expenditure, HBS 2009



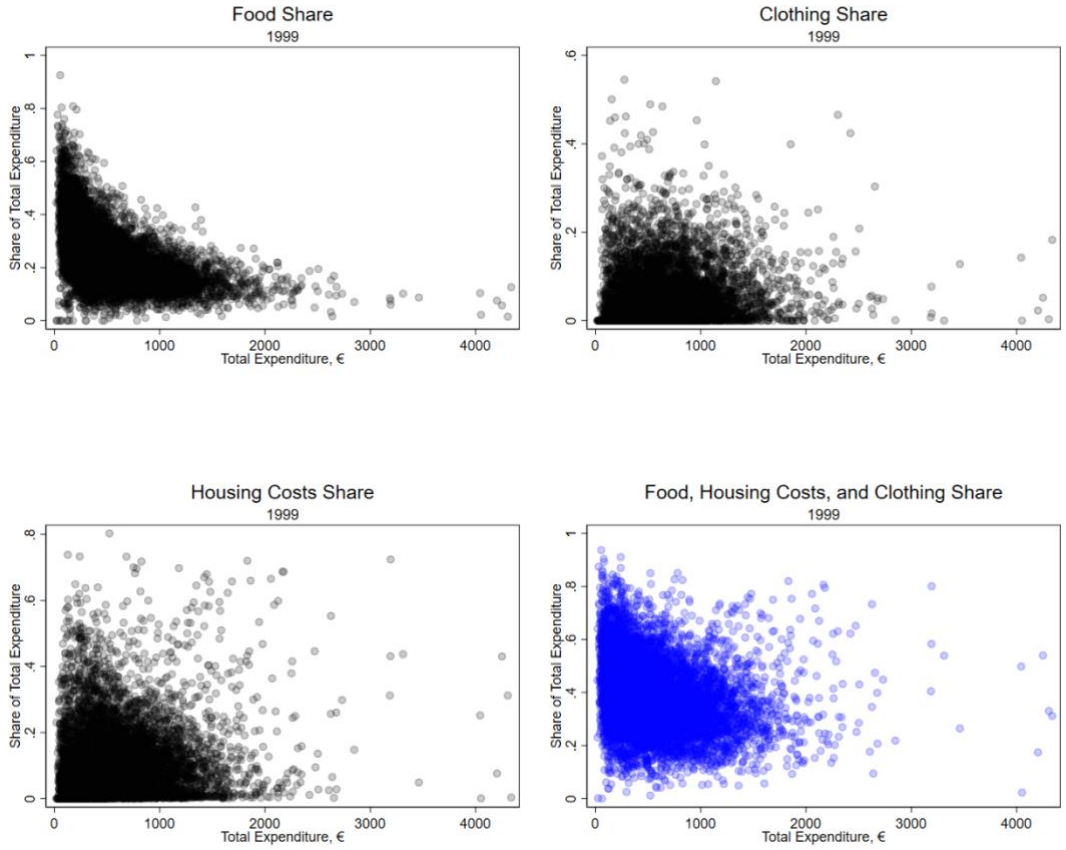
Note: Own calculations using HBS 2009-10. Total expenditure is monthly.

Figure A.3: Expenditure in goods used in Engel's method as share of total expenditure, HBS 2004



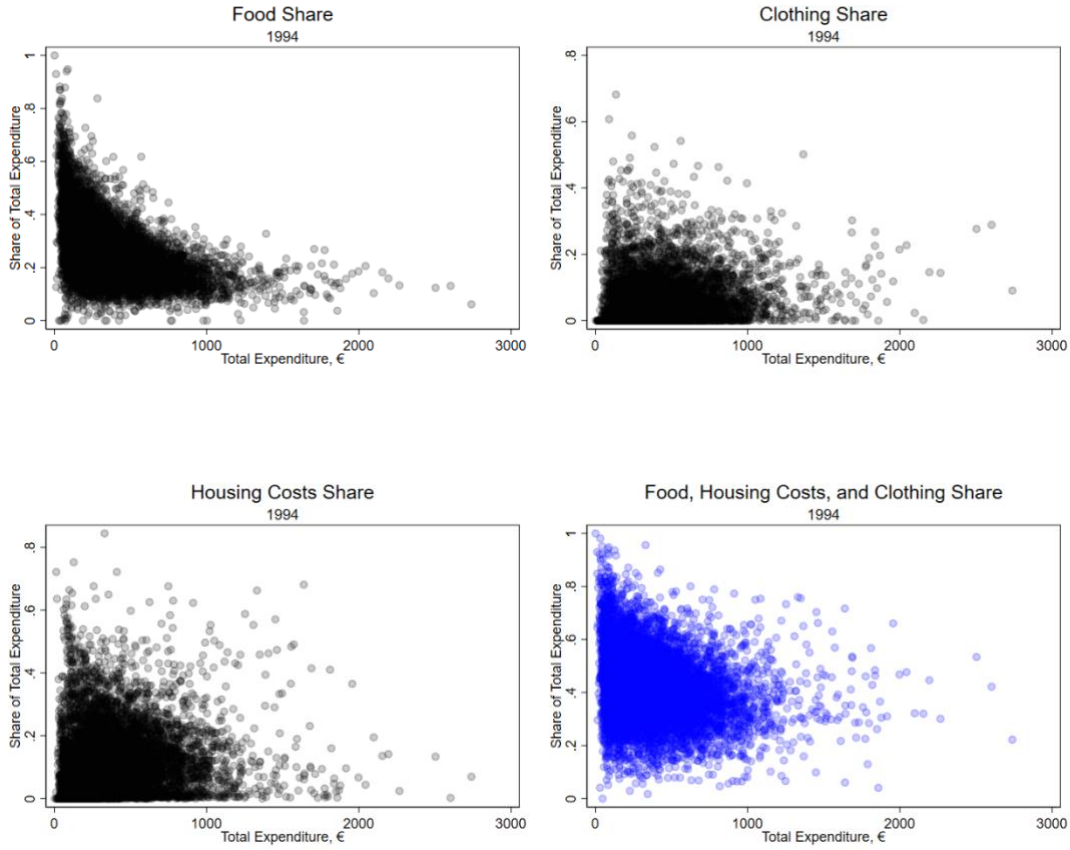
Note: Own calculations using HBS 2004-2005. Total expenditure is monthly.

Figure A.4: Expenditure in goods used in Engel's method as share of total expenditure, HBS 1999



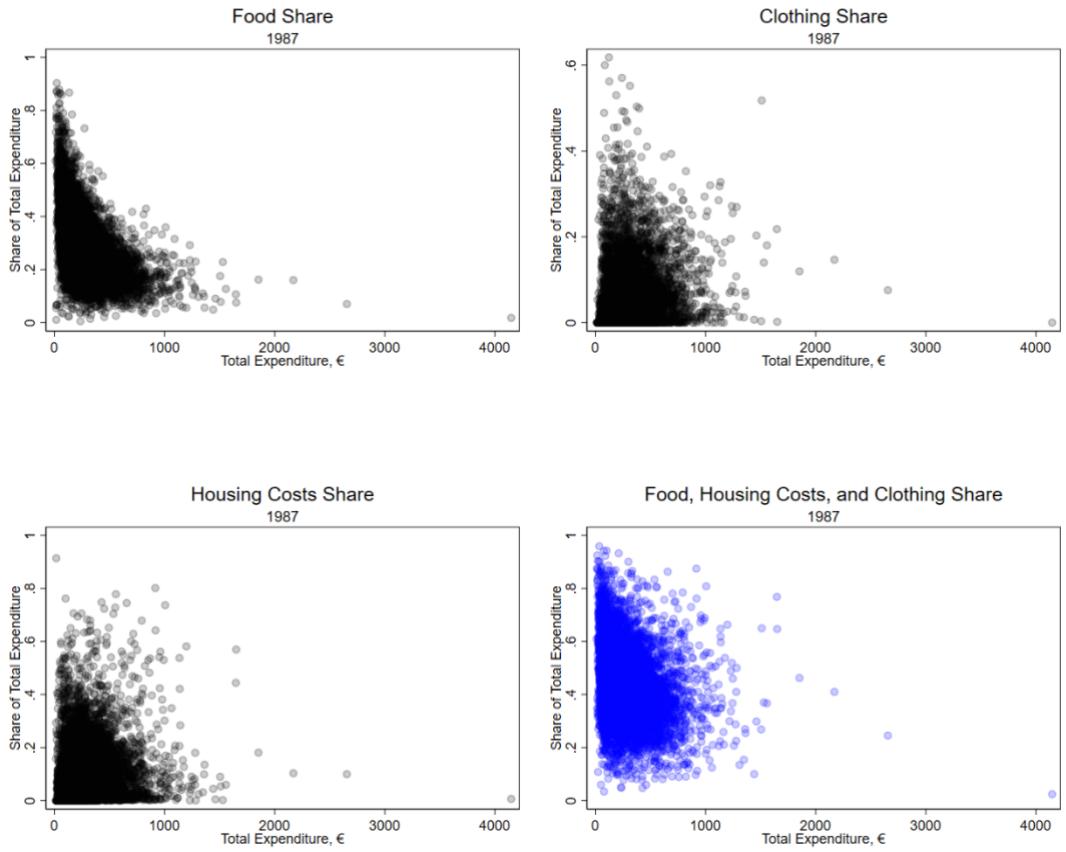
Note: Own calculations using HBS 1999-2000. Total expenditure is monthly.

Figure A.5: Expenditure in goods used in Engel's method as share of total expenditure, HBS 1994



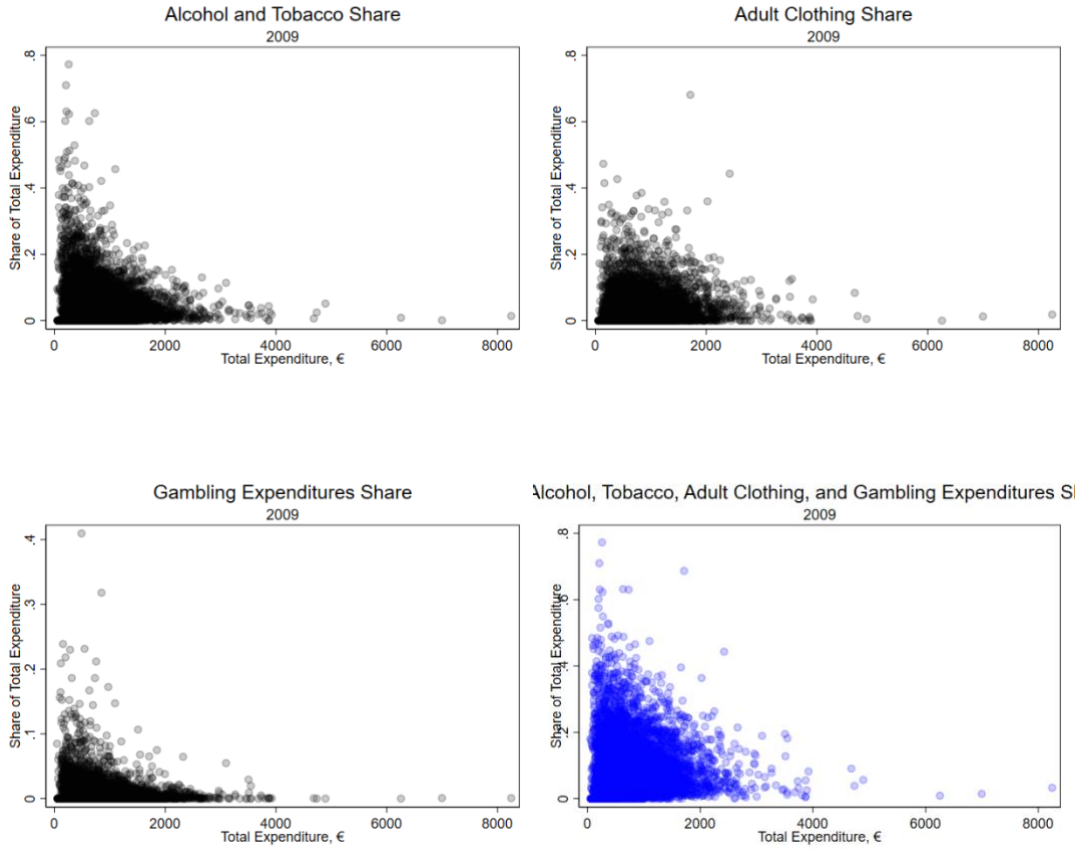
Note: Own calculations using HBS 1994-95. Total expenditure is monthly.

Figure A.6: Expenditure in goods used in Engel's method as share of total expenditure, HBS 1987



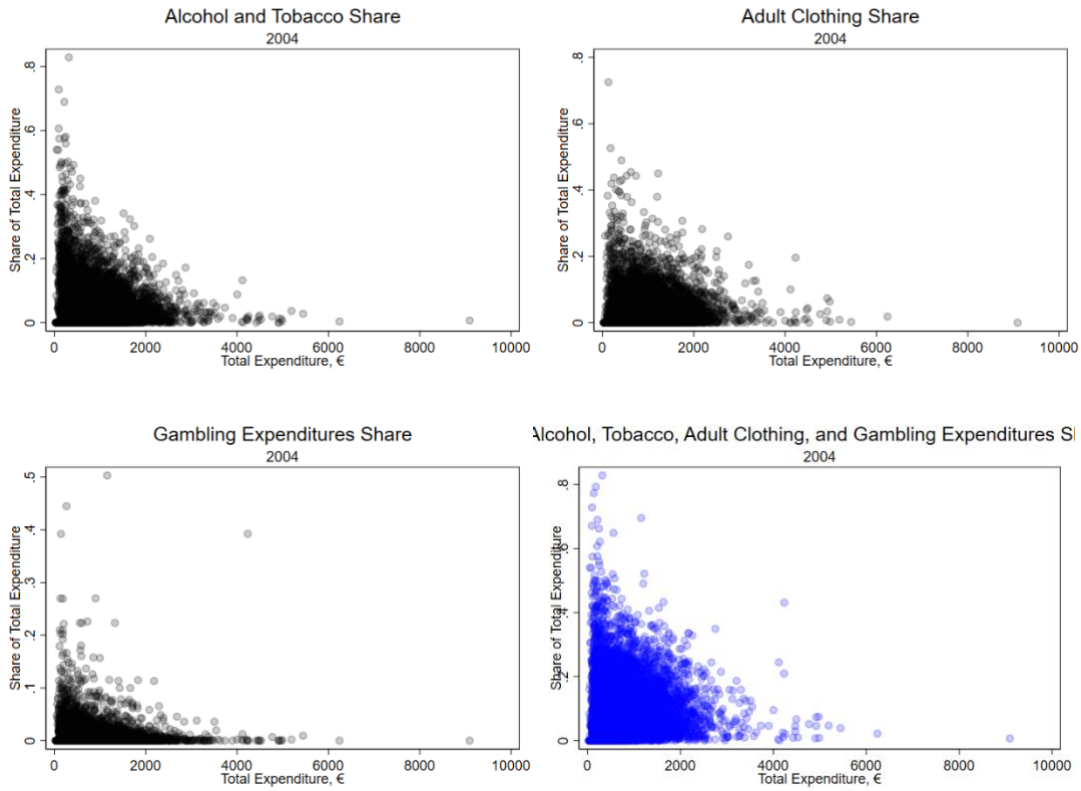
Note: Own calculations using HBS 1987-88. Total expenditure is monthly.

Figure A.7: Expenditure on goods used in Rothbarth method as share of total expenditure, HBS 2009



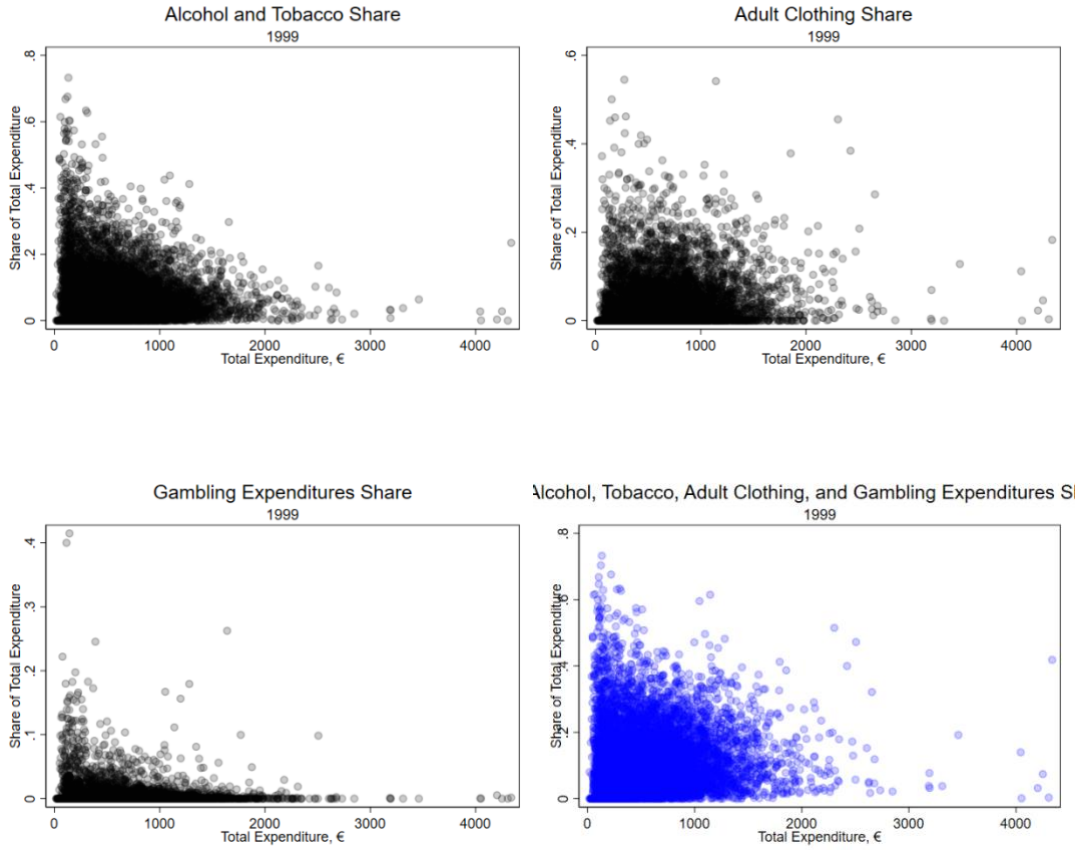
Note: Own calculations using HBS 2009-10. Total expenditure is monthly.

Figure A.8: Expenditure on goods of the Rothbarth method as share of total expenditure, HBS 2004



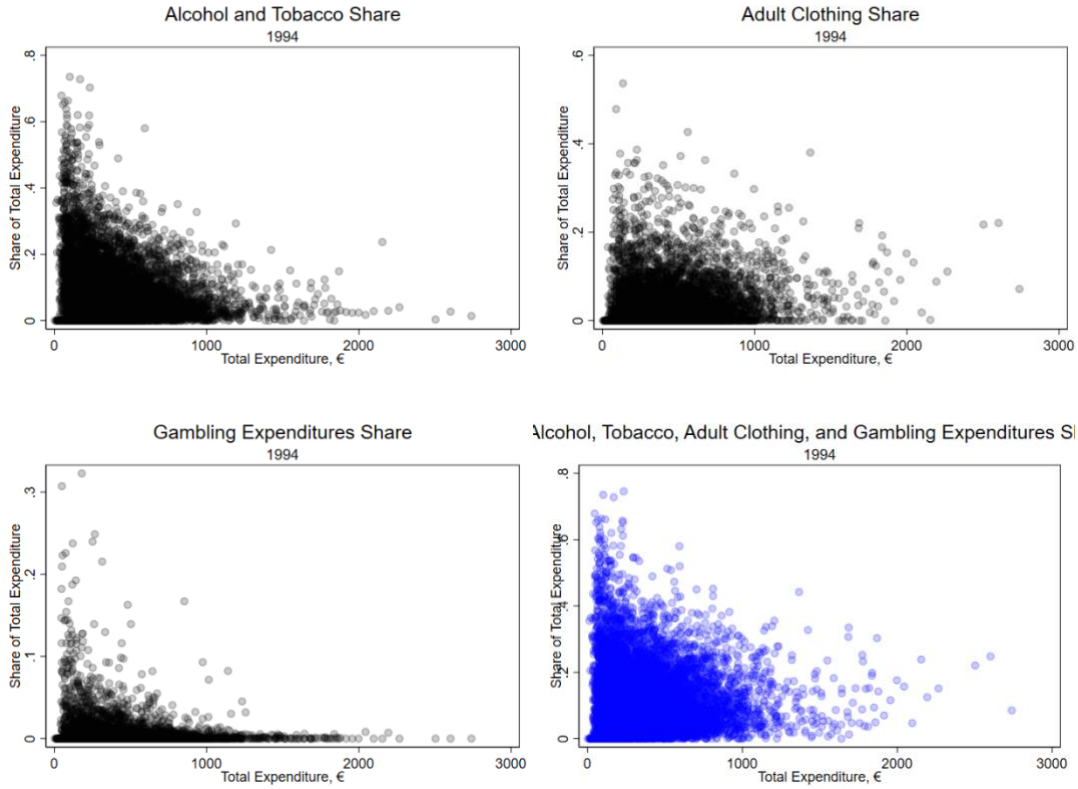
Note: Own calculations using HBS 2004-05. Total expenditure is monthly.

Figure A.9: Expenditure on goods of the Rothbarth method as share of total expenditure, HBS 1999



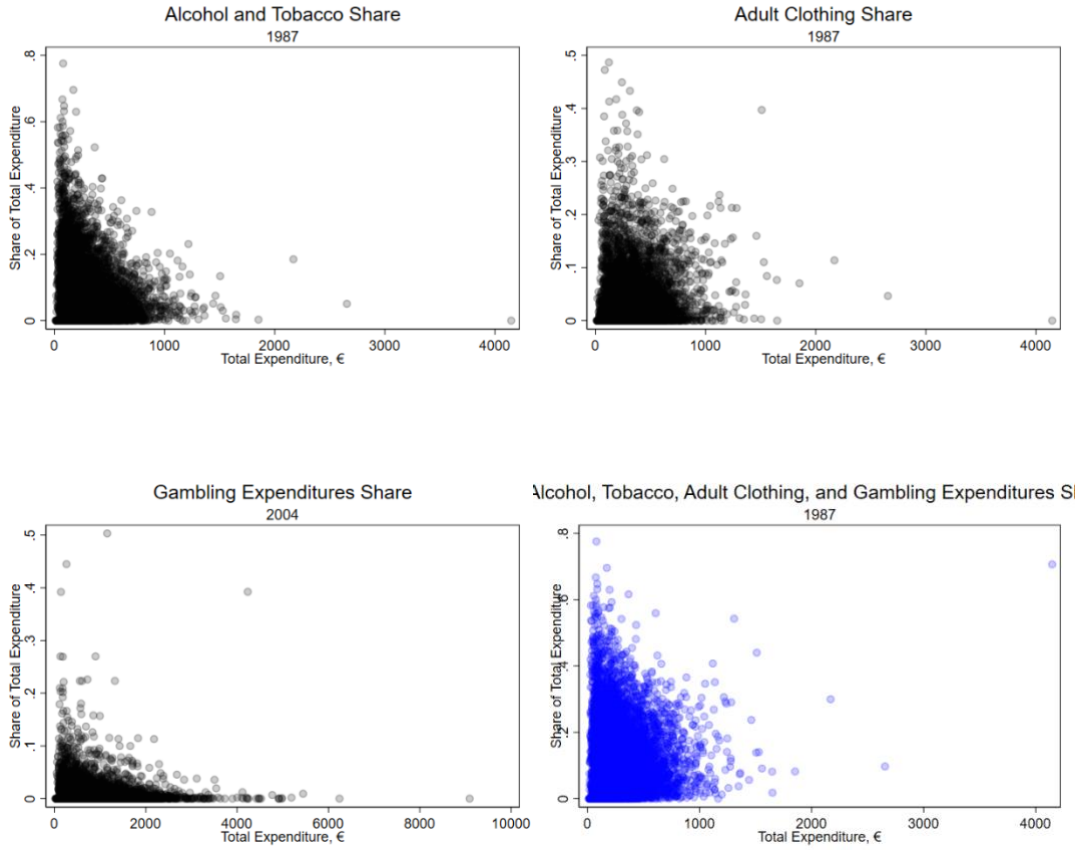
Note: Own calculations using HBS 1999-2000. Total expenditure is monthly.

Figure A.10: Expenditure on goods of the Rothbarth method as share of total expenditure, HBS 1994



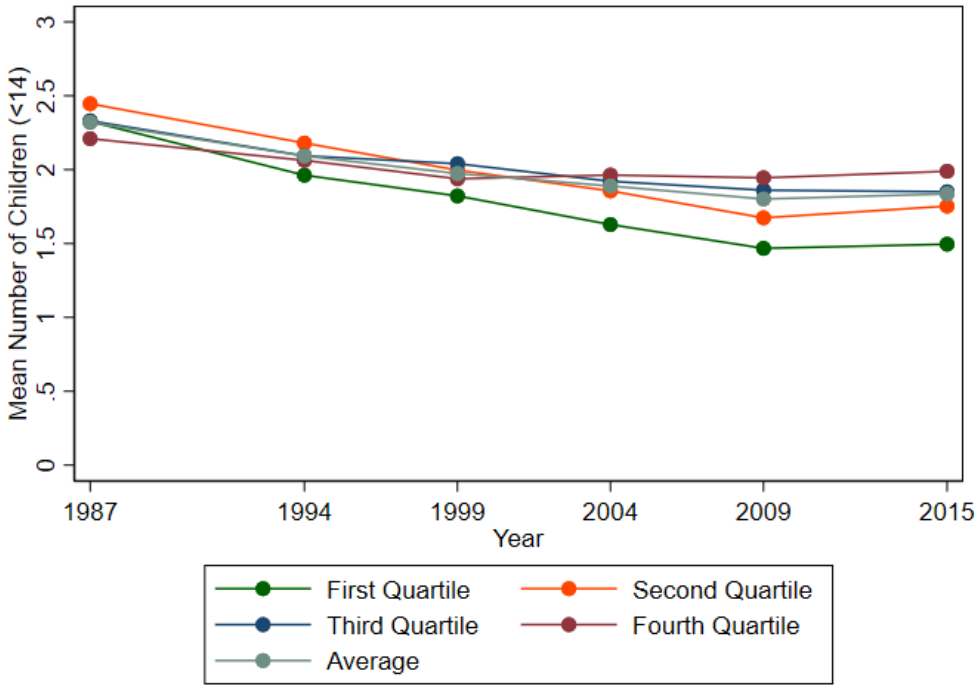
Note: Own calculations using HBS 1994-95. Total expenditure is monthly.

Figure A.11: Expenditure on goods of the Rothbarth method as share of total expenditure, HBS 1987



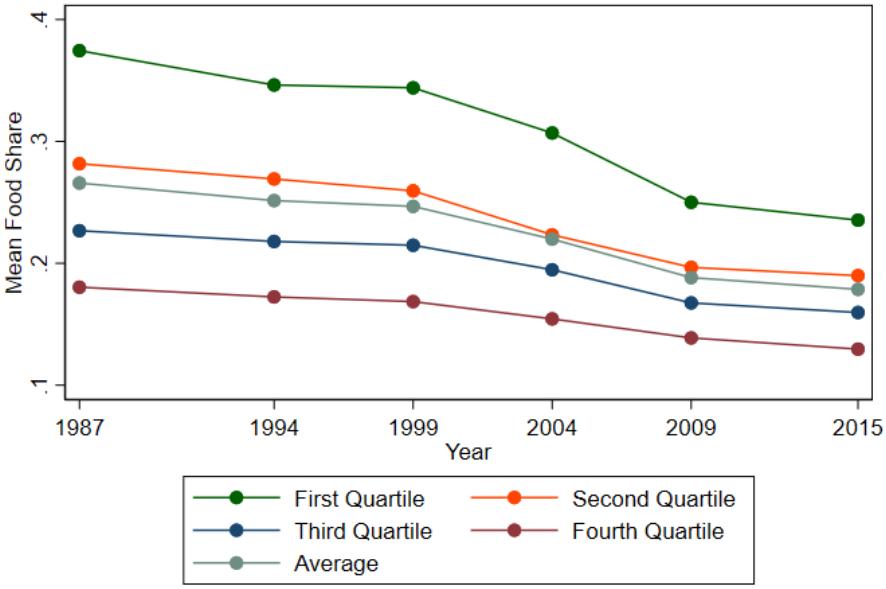
Note: Own calculations using HBS 1987-88. Total expenditure is monthly.

Figure A.12. Mean number of children by income quartile



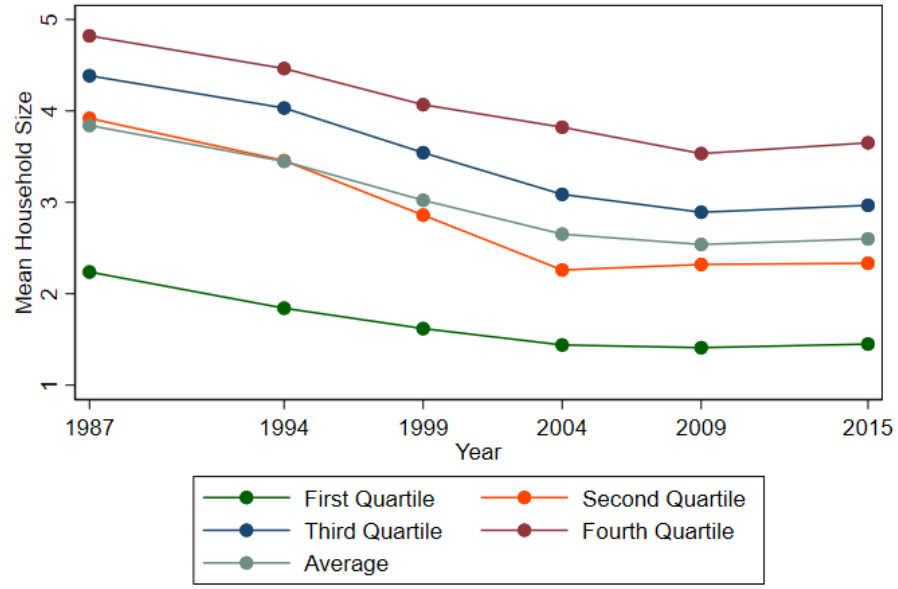
Note: Own calculations using HBS.

Figure A.13. Mean share of expenditure on food by income quartile



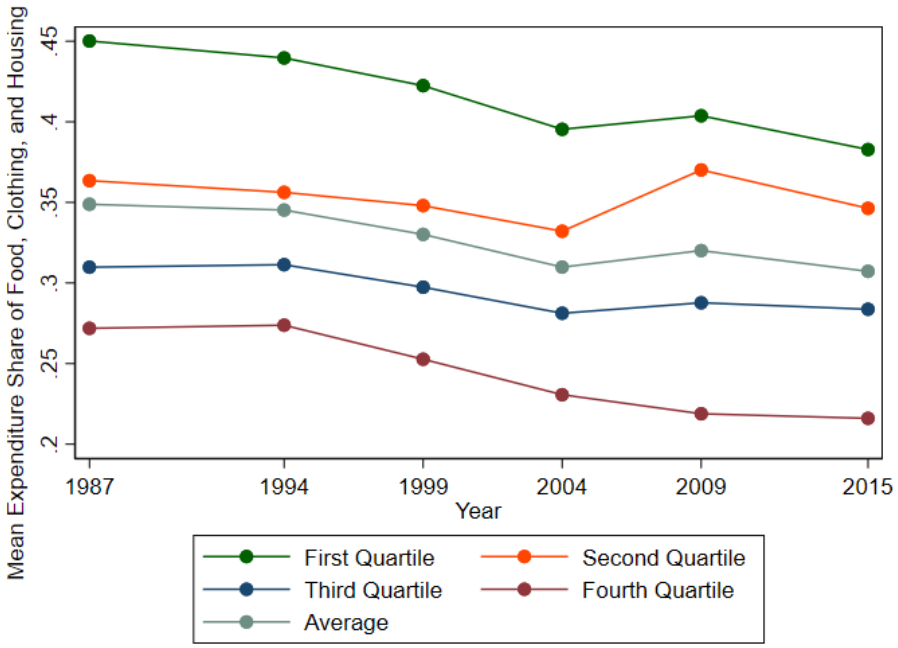
Note: Own calculations using HBS.

Figure A.14. Mean household size by income quartile



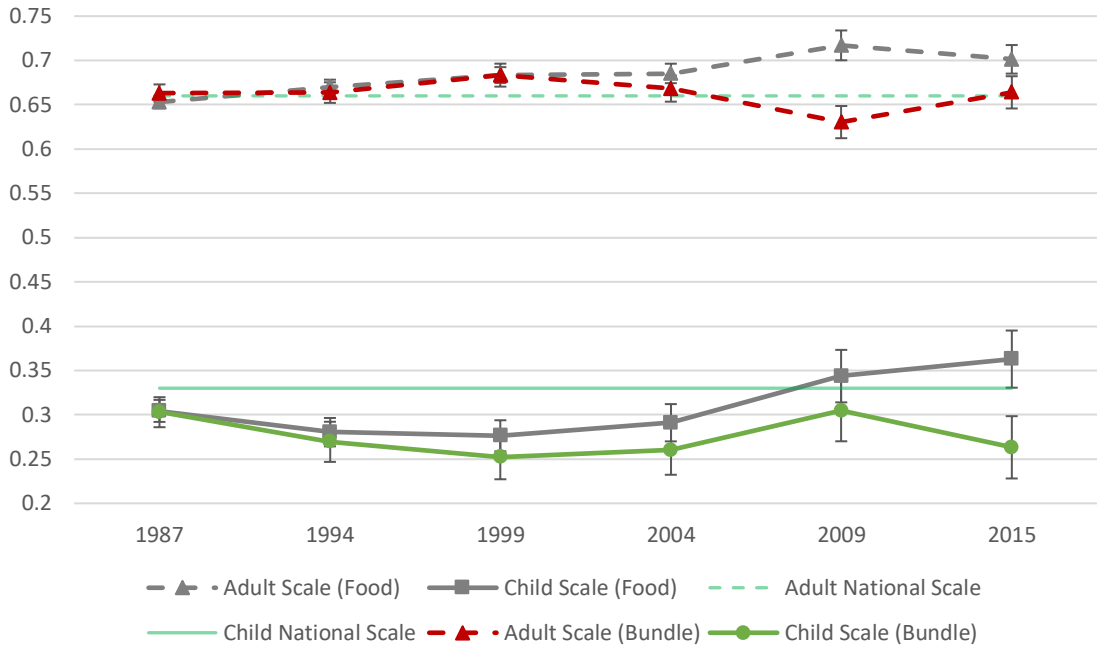
Note: Own calculations using HBS.

Figure A.15. Share of expenditure on adults goods by quartile



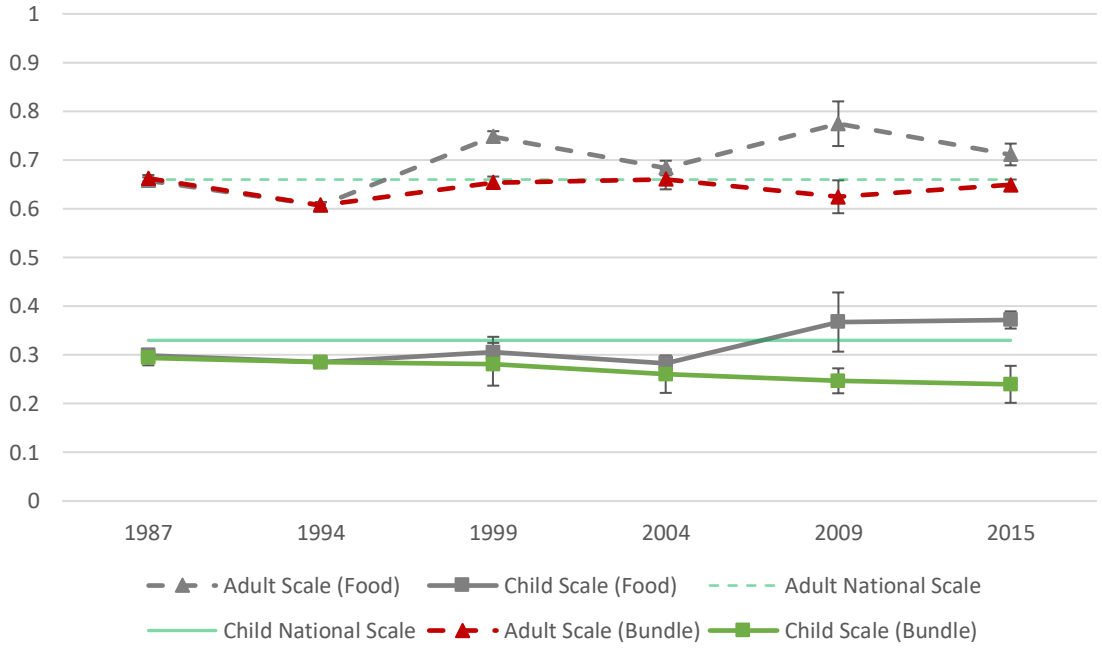
Note: Own calculations using HBS.

Figure A.16. Engel Scales, Working-Leser Form, 1987 - 2015



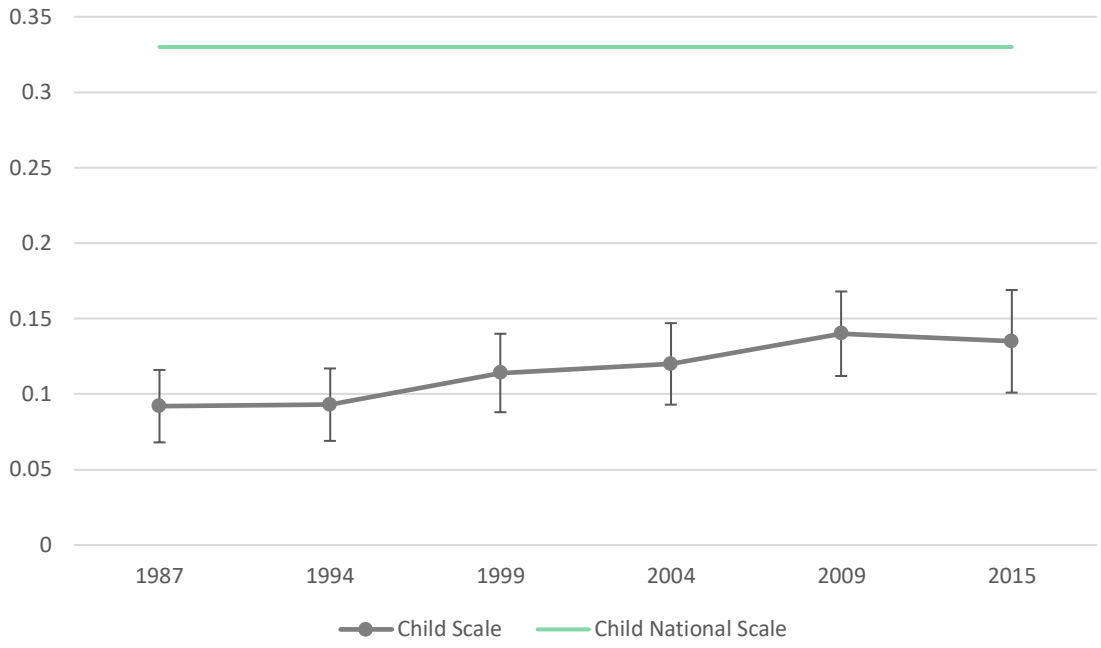
Note: Own calculations using HBS.
Bars represent 95% confidence intervals. Standard errors computed using the delta method.

Figure A.17. Engel Scales, Kernel Regression, 1987 – 2015



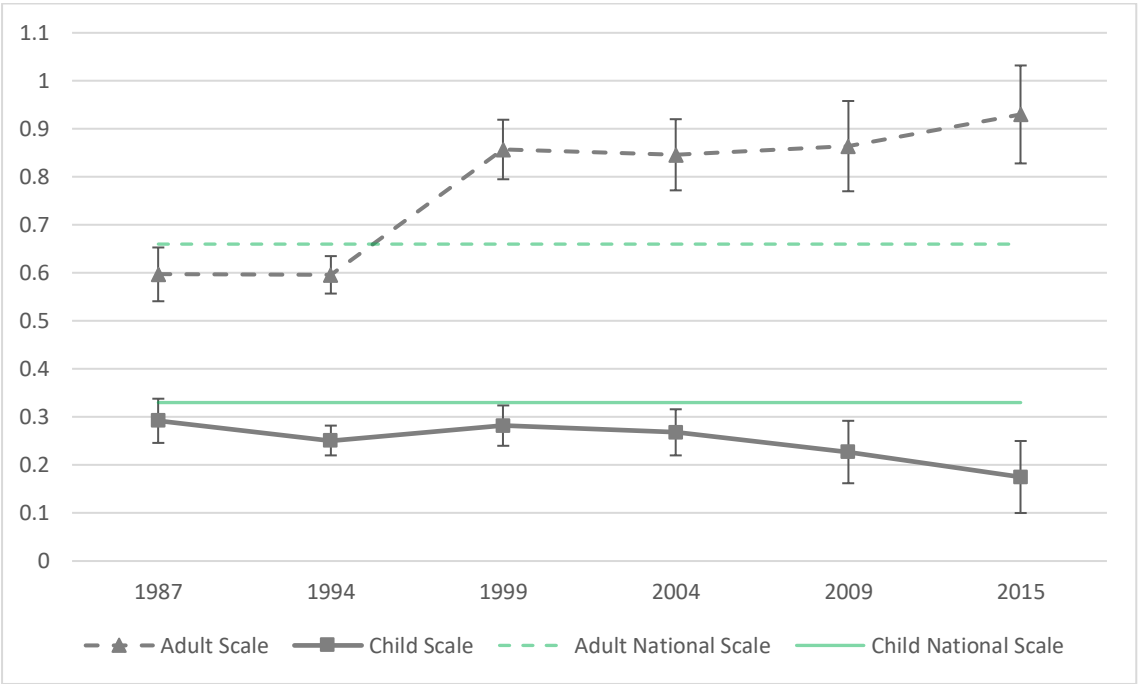
Note: Own calculations using HBS.
Bars represent 95% confidence intervals. Standard errors computed using the delta method.

Figure A.18. Rothbarth Scales, 1987 – 2015



Note: Own calculations using HBS.
Bars represent 95% confidence intervals. Standard errors computed using the delta method.

Figure A.19. Almost Ideal Demand System Scales, 1987 – 2015



Note: Own calculations using HBS.
Bars represent 95% confidence intervals. Standard errors computed using the delta method.