

Automation, New Technology, and Non-Homothetic Preferences

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Automation, New Technology, and Non-Homothetic Preferences*

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Abstract

This paper provides a microfoundation of the neoclassical growth theory. To rationalize a substantial share of labor in income despite ongoing automation of tasks, we present a simple model in which demand shifts toward goods of increasing sophistication along a vertically differentiated production structure. Automation of more advanced goods requires increasingly sophisticated capital which remains scarce along the growth path. This is why labor maintains a substantial share in income independent of core parameter assumptions. While our model features an entirely different mechanism, we show that its aggregate representation is the one of a neoclassical model with labor-augmenting technical change.

JEL: E23, E25, J24, O14, O33

Keywords: Uzawa's theorem, automation, goods quality, structural change, reallocations, growth, non-homothetic preferences, hierarchical demand

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1 Introduction

In this paper we offer a thorough reinterpretation of the neoclassical growth theory. In particular, we provide answers to two outstanding research questions. 1) *Why does labor still earn a substantial share in income despite progressive task automation over the centuries?*¹ 2) *Why does the neoclassical model imply that productivity growth is labor-augmenting despite the significant improvements in capital productivity?* We show that technical change is manifested as labor-augmenting because of an asymmetry in the manner that the neoclassical growth model treats capital and labor. We contend that the same asymmetry can also explain why labor maintains a substantial share in income.

Based on the evidence of Figure 1 and recent studies which illustrate that goods with higher income elasticities are more labor intensive, we develop a micro-founded model of the aggregate economy to address the two aforementioned questions.² In this model, a hierarchy in demand induces continuous reallocations of capital and labor toward more sophisticated goods that become increasingly difficult to produce but provide higher quality.^{3,4} Initially, labor has a comparative advantage over capital in the production of these complex goods. Subsequently, however, as technology progresses, capital replaces labor in these tasks and labor moves on to produce even more sophisticated goods. Importantly, automation of more advanced goods requires increasingly sophisticated capital which remains scarce along the growth path.

We derive two parallel aggregate representations of the production side of the economy in our model. The first representation is the neoclassical production function with labor-augmenting technical change. The second representation is the neoclassical production function with factor-neutral technical change. In the standard neoclassical model which is akin to the first representation, capital is treated as a quality-adjusted variable but labor is *not* adjusted for quality.⁵ As the quality of both labor and capital improves over time, quality-adjusted capital becomes abundant relative to quality-unadjusted labor. The productivity residual that measures how the quality of factors improves therefore indicates that the productivity of labor grows at a higher rate than the pro-

¹While a few recent studies such as [Elsby et al. \(2013\)](#), [Karabarbounis and Neiman \(2014\)](#), [Piketty and Zucman \(2014\)](#) and [Autor et al. \(2017\)](#) argue that the U.S. labor share has decreased somewhat since the 1980s, other studies argue that this decline is due to measurement problems, see e.g. [Auerbach and Hassett \(2015\)](#) and [Rognlie \(2015\)](#). However, all studies clearly show that the U.S. labor share is still a substantial fraction of GDP despite the ongoing automation of tasks that is described in [Autor \(2015\)](#).

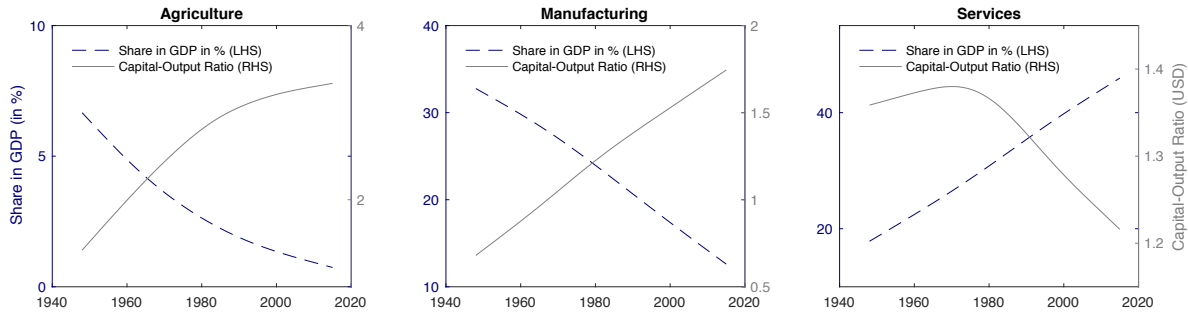
²[Buera and Kaboski \(2012\)](#) and [Caron et al. \(2014\)](#) both link non-homothetic preferences to production characteristics. In particular, they highlight a connection between higher income elasticity goods and the skill-intensity in production. In our model, labor is given by the number of hours/workers as in the neoclassical model. Labor is thus *unadjusted* for quality, hence it incorporates quality/skill. Under our framework, production shifts over time to higher-quality goods which are produced by more skilled or more productive labor.

³Initially observed by [Engel \(1857\)](#), there is now extensive empirical evidence on non-homothetic preferences, see e.g. [Houthakker \(1957\)](#) and [Aguilar and Bils \(2015\)](#). Our modeling of non-homothetic preferences is based on [Laitner \(2000\)](#), [Caselli and Coleman \(2001\)](#), [Matsuyama \(2002\)](#), [Greenwood and Uysal \(2005\)](#), [Foellmi and Zweimüller \(2008\)](#), [Matsuyama \(2009\)](#) and [Fajgelbaum et al. \(2011\)](#).

⁴A large and growing literature documents the vast improvements in product quality. Contributors to this literature are, among others, [Bils and Klenow \(2001\)](#), [Bils \(2004\)](#), [Schott \(2004\)](#) and [Hallak and Schott \(2011\)](#).

⁵In the neoclassical model, “real” capital (real USD capital) is capital adjusted for quality and variety improvements. By contrast, “real” labor (number of hours/workers) is labor *unadjusted* for quality and variety improvements.

Figure 1: Non-homothetic preferences, automation and structural change.



Notes: The Figure shows the decline in Agriculture and Manufacturing shares in the U.S. Private Industries' GDP as well as the rise of the Services' share. Notably, in Agriculture, which experiences the fastest GDP share decline, the capital-output ratio increases from under 2 to around 3.7 over time. In Manufacturing, which experiences the second fastest GDP share decline, the capital-output ratio rises from 0.7 to 1.7. In Services, which experiences a rise in its GDP share, the capital-output ratio falls from 1.3 to 1.2. The data are hp-filtered ($\lambda = 30,000$) and drawn from the U.S. Bureau of Economic Analysis (BEA).

ductivity of capital. Hence, productivity growth is labor-augmenting. In the second representation capital is quality-*unadjusted* and remains as scarce as quality-*unadjusted* labor. With both factors being treated symmetrically in this representation, productivity is factor neutral.

To explain why labor takes up a substantial fraction in income after hundreds of years of automation, we focus on the same two representations of the neoclassical production function. In the first representation, the productivity growth rate of quality-*unadjusted* labor is higher than the productivity growth rate of quality-adjusted capital, i.e. productivity growth is labor-augmenting. Therefore, the share of labor is stable independent of the elasticity of substitution between capital and labor. The economic interpretation of labor-augmenting technical change, as a result of the asymmetry in quality adjustment in this model, is otherwise difficult. Our second representation offers a more meaningful economic interpretation. Specifically, both quality-*unadjusted* capital and quality-*unadjusted* labor remain equally scarce factors in production along the growth path. This is why they both earn a substantial share in income independent of the elasticity of substitution between capital and labor.

Crucially, the main implication of our model is consistent with the data. In [Struck and Velic \(2017\)](#) we show that empirical estimates of productivity growth are higher in more labor intensive industries once the raw data are adjusted for quality and variety measurement errors in goods inflation. That is, we employ standard post-war U.S. data in which capital is quality-adjusted but labor is not. The same paper demonstrates that higher productivity growth in the more labor intensive industries is a direct implication of economy-wide labor-augmenting technical change. Moreover, our interpretation that quality-*unadjusted* capital (quality capital) is scarce is consistent with the empirical evidence from post-war U.S. data: whereas the price of quality-adjusted capital (the real interest rate) has remained flat, the price of quality-*unadjusted* labor (wages per hour) has increased. Although we do not observe a price index for quality-*unadjusted* capital, it is plausible to assume that this price index must have increased given the significant improvements in the quality

of capital over time.

Our paper mainly relates to three different literatures. First, it is a contribution to the literature that attempts to reconcile the overwhelming empirical evidence on structural change with the empirical evidence on balanced growth.⁶ While this literature has featured supply-side mechanisms (Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008)) and demand side mechanisms (Kongsamut *et al.* (2001), Foellmi and Zweimüller (2008)) independently, our paper is in line with recent contributions that attempt to integrate both mechanisms under one framework (Buera and Kaboski (2009), Herrendorf *et al.* (2013), Boppart (2014), Comin *et al.* (2015)). Relative to these contributions our explanation combines a quality hierarchy in demand with industry heterogeneity in labor intensities, factor-neutral productivity growth and automation. In this environment, we derive an endogenously stable labor share in income that is independent of the capital-labor substitution elasticity.

It further differs from those studies in that it seeks to provide a direct microfoundation underlying the aggregate production function and is thus closely related to the approaches of Acemoglu (2002) and Jones (2005). Crucially, we deviate from these two papers by focusing on the asymmetric treatment of quality across factors in standard frameworks. Our approach stresses the possibility that both capital and labor are scarce factors in the economy. Notably, we extend Uzawa (1961)'s analysis by showing that even with factor-neutral technical change balanced growth can arise independent of the capital-labor elasticity if quality capital remains scarce. Furthermore, we manage to reconcile balanced growth with the substantial improvements in capital productivity that have arguably appeared over time. Our approach resembles that of Grossman *et al.* (2017) in that we also focus on escaping the straightjacket of Uzawa's theorem. In contrast to our approach, they focus on endogenous schooling with greater complementary between capital and skilled, rather than raw, labor.

Second, our paper is a contribution to the recent literature that attempts to reconcile the ongoing automation of tasks with the increase in employment over time. A key variable that this literature indirectly attempts to explain is the substantial share of labor in income despite the continuous replacement of labor-intensive tasks by capital. So far, this literature has offered one central explanation: automation leads to new goods/tasks which sustains employment as outlined by Olsen and Hemous (2014) and Acemoglu and Restrepo (2016), among others. The latter paper is closest to our approach in that we also assume that labor has a comparative advantage over capital in the production of new goods/tasks. Instead of an endogenous growth mechanism, the driving force in our framework is a combination of non-homothetic preferences and factor-neutral productivity. Significantly, while Acemoglu and Restrepo (2016, 2017) exogenously assume labor-augmenting technical change, we concentrate on explaining labor-augmenting technical change from within our framework.

Third, our paper is a contribution to the literature that endeavors to empirically identify the elasticity of substitution and the bias of technical change. As Diamond *et al.* (1978) and León-Ledesma

⁶For evidence on balanced growth see Kaldor (1957) and Jones (2015) for example.

et al. (2010) point out, identification is notoriously difficult to achieve. Our theoretical contribution has important implications for this literature. Namely, it highlights a crucial asymmetry in standard estimation procedures which leads to biased empirical estimates that are difficult to interpret economically. Allowing for factor-biased technical change, many papers find quality-adjusted capital and quality-*unadjusted* labor to be complements (Arrow *et al.* (1961), Antras (2004), Klump *et al.* (2007), Oberfield and Raval (2014), Lawrence (2015), Chirinko and Mallick (2016)). We challenge this interpretation of the data by highlighting the possibility that both quality-*unadjusted* capital and quality-*unadjusted* labor remain equally scarce along the growth path. We theoretically outline that using quality-*unadjusted* capital in the estimation procedure most likely alters the bias of technical change from labor-augmenting to factor-neutral. Our point corresponds to the study of Young (2014) who underlines that labor is usually not adjusted for worker quality.

2 Theoretical Model

Our analysis considers a closed economy. In this economy, good $n \in \mathbb{Z} \equiv \{\dots, -2, -1, 0, 1, 2, \dots\}$ is produced by the perfectly competitive firm n . Goods are differentiated by their quality. Higher indexed goods are characterized by a higher quality. Higher quality goods provide greater real output, but are also more difficult to produce. Initially, higher quality goods can only be produced by labor. Subsequently, as technology develops, capital replaces labor in these tasks and labor moves on to produce more sophisticated goods. Importantly, automation of more advanced goods requires more advanced capital. The upper-case variables $Y_t, Y_{n,t}, P_t, P_{n,t}, R_{n,t}, K_t, K_{n,t}, C_t, I_t, I_{n,t}$ are quality-adjusted, while the lower case variables $y_t, y_{n,t}, p_t, p_{n,t}, w_{n,t}, r_{n,t}, k_t, k_{n,t}, l_t, l_{n,t}$ are quality-*unadjusted*. On the demand side, a representative consumer supplies labor inelastically at the aggregate level. Specifically, in our framework, the consumer's preferences are non-homothetic. That is, as the economy develops, demand shifts to higher quality goods.

2.1 Intratemporal Demand

The quality-adjusted output of good n is given by $Y_{n,t} = \left(\frac{1}{\Gamma}\right)^n y_{n,t}$, where $y_{n,t}$ is the corresponding quality-*unadjusted* output and $(1/\Gamma)^n$ is the time invariant quality of good n with $\Gamma \in (0, 1)$. Aggregate output demand Y_t combines the individual goods demands $Y_{n,t}$ according to the constant elasticity of substitution function

$$Y_t = \left[\sum_{n \in \mathbb{Z}} \gamma_{n,t}^{\frac{1}{\theta}} [Y_{n,t}]^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \quad (1)$$

where $\gamma_{n,t}$ is the time-varying weight of good n in total output with $\sum_{n \in \mathbb{Z}} \gamma_{n,t} = 1 \ \forall t$, and $\theta > 1$ is the goods elasticity of substitution. The change in the weight of good n in aggregate output is driven by the level of economic development. In particular, we define

$$\gamma_{n,t} = \frac{e^{-|t-n|}}{z} \quad (2)$$

where $z = 1 + 2 \sum_{i=1}^{\infty} e^{-i}$ ensures that the weights across goods sum to unity. Intuitively, this definition implies that demand is continuously shifting toward higher quality goods over time. Overall, quality-adjusted output Y_t can be used for both consumption C_t and investment I_t , namely

$$Y_t \equiv C_t + I_t. \quad (3)$$

Solving the intratemporal optimization problem, which entails maximizing equation (1) subject to the usual expenditure constraint

$$E = \sum_{n \in \mathbb{Z}} P_n Y_n, \quad (4)$$

yields the relative demand for quality-adjusted goods

$$\frac{Y_{n,t}}{Y_{-n,t}} = \left[\frac{\gamma_{n,t}}{\gamma_{-n,t}} \right] \left[\frac{P_{n,t}}{P_{-n,t}} \right]^{-\theta} \quad (5)$$

where $P_{-n,t}$ and $P_{n,t}$ denote the quality-adjusted prices of goods $-n$ and n respectively. Given that the weights on goods depend on the level of development, and hence income implicitly, preferences are non-homothetic. The link between quality-adjusted and quality-unadjusted prices is straightforward as $P_n Y_n = p_n y_n$. Specifically, it is given by

$$P_{n,t} \left(\frac{1}{\Gamma} \right)^n = p_{n,t}. \quad (6)$$

Therefore, the relative demand for quality-unadjusted goods is given by

$$\frac{y_{n,t}}{y_{-n,t}} = \left[\frac{\gamma_{n,t}}{\gamma_{-n,t}} \right] \left[\frac{p_{n,t}}{p_{-n,t}} \right]^{-\theta} \left[\frac{\Gamma^n}{\Gamma^{-n}} \right]^{1-\theta}. \quad (7)$$

2.2 Firms

The quality-unadjusted output of good n , $y_{n,t}$, is supplied according to the production function

$$y_{n,t} = f_{n,t}(k_{n,t}, l_{n,t}) = \begin{cases} A_t \Gamma^n l_{n,t}, & \text{if } n > t \\ A_t \Gamma^n k_{n,t}, & \text{if } n \leq t \end{cases} \quad (8)$$

where, as before, $\Gamma \in (0, 1)$ is the parameter governing how quality changes across goods, A_t is the level of productivity common to all goods at time t . The evolution of productivity is given exogenously by the process $A_t = (1 + x)^t$, where $x > 0$. To maintain a tractable analysis, we impose the parameter constraint $\Gamma^n = 1/(1 + x)^n$.

Our analysis employs a dichotomous production setup. In particular, higher quality goods production is purely labor-intensive. Conversely, lower quality goods production is purely capital-intensive. The implication of the production structure is that higher quality goods production is eventually automated over time given technological progress, while labor subsequently moves on to produce even higher quality goods. Intuitively, automation of these latter goods is not possible in the early stages of the product life-cycles due to a lag in technological progress. Put differently, the relatively more sophisticated capital required for the production of such goods is not available

initially, meaning that labor therefore has to take its place. Finally, equation (8) indicates that a higher quality good, given by a higher n , is more difficult to produce than a lower quality good, given by a lower n , at any point in time t .

Sectoral quality-adjusted capital accumulation is given by

$$K_{n,t+1} = (1 - \delta)K_{n,t} + I_{n,t}, \quad (9)$$

where δ is the capital depreciation rate and $I_{n,t}$ is sectoral investment. Meanwhile, sectoral quality-*unadjusted* capital accumulation is given by

$$k_{n,t+1} = (1 - \delta)k_{n,t} + \Gamma^t I_{n,t}, \quad (10)$$

where $\Gamma^t = 1/(1+x)^t$. Importantly, as the economy shifts to the production of higher quality goods over time, the previous equation indicates that higher quality capital will be necessary.⁷ The sum of sectoral investments equates to aggregate investment, while the sum of sectoral capital stocks equates to aggregate capital. Specifically, given our production structure, the following equalities must hold

$$I_t = \sum_{n \leq t} I_{n,t} \quad \text{and} \quad K_t = \sum_{n \leq t} K_{n,t} \quad \text{and} \quad k_t = \sum_{n \leq t} k_{n,t}. \quad (11)$$

2.3 Intertemporal Demand

The representative consumer maximizes the present discounted value of lifetime utility from consumption

$$U_0 = \sum_{t=0}^{\infty} \beta^t \frac{[C_t]^{1-\phi}}{1-\phi} \quad (12)$$

subject to the standard per period budget constraint

$$P_t C_t + P_t I_t = \sum_{n > t} w_{n,t} l_{n,t} + \sum_{n \leq t} r_{n,t} k_{n,t} \quad (13)$$

where β is the subjective discount factor, ϕ is a parameter governing the intertemporal elasticity of substitution, $P_t = [\sum_{n \in \mathbb{Z}} \gamma_{n,t} [P_{n,t}]^{1-\theta}]^{1/(1-\theta)}$ is the welfare based quality-adjusted aggregate price index, $l_{n,t}$ is quality-*unadjusted* sectoral labor, $k_{n,t}$ is quality-*unadjusted* sectoral capital, $w_{n,t}$ is the sectoral wage rate, and $r_{n,t}$ is the sectoral rental rate. The consumer supplies labor inelastically at the aggregate level with $l_t = 1$. Labor market clearing implies that the sum of labor supplies allocated across industries equals the aggregate labor supply, namely

$$l_t = \sum_{n > t} l_{n,t}. \quad (14)$$

⁷We have also modeled the evolution of quality-*unadjusted* capital as $k_{n,t+1} = (1 - \delta)k_{n,t} + \Gamma^n I_{n,t}$. However, this alternative specification complicates the solution without adding any additional insight.

2.4 Equilibrium

In period t , the representative household maximizes the Lagrangian function

$$\begin{aligned} \mathcal{L} = \sum_{s=0}^{\infty} \beta^{t+s} & \left[\frac{C_{t+s}^{1-\phi}}{1-\phi} + \lambda_{t+s} \left(\sum_{n>t+s} w_{n,t+s} l_{n,t+s} + \sum_{n \leq t+s} r_{n,t+s} k_{n,t+s} - \sum_{n \leq t+s} P_{t+s} I_{n,t+s} - P_{t+s} C_{t+s} \right) \right. \\ & + \sum_{n \leq t+s} q_{n,t+s} ((1-\delta)k_{n,t+s} + \Gamma^{t+s} I_{n,t+s} - k_{n,t+1+s}) \\ & \left. + \chi_{t+s} (l_{t+s} - \sum_{n>t+s} l_{n,t+s}) \right], \end{aligned}$$

where χ_{t+s} , $q_{n,t+s}$, and λ_{t+s} denote Lagrange multipliers. Optimization with respect to $k_{n,t+s+1}$, $I_{n,t+s}$, C_{t+s} , and $l_{n,t+s}$ respectively yields the first-order conditions

$$\mathcal{L}_{k_{n,t+s+1}} = (1-\delta)\beta q_{n,t+s+1} + \beta \lambda_{t+s+1} r_{n,t+s+1} - q_{n,t+s} = 0, \quad (15)$$

$$\mathcal{L}_{I_{n,t+s}} = -\lambda_{t+s} P_{t+s} + \Gamma^{t+s} q_{n,t+s} = 0, \quad (16)$$

$$\mathcal{L}_{C_{t+s}} = C_{t+s}^{-\phi} - P_{t+s} \lambda_{t+s} = 0, \quad (17)$$

$$\mathcal{L}_{l_{n,t+s}} = \lambda_{t+s} w_{n,t+s} - \chi_{t+s} = 0. \quad (18)$$

The representative firm in sector n maximizes profits, $\pi_{n,t}$, given by

$$\pi_{n,t} = \begin{cases} p_{n,t} y_{n,t} - w_{n,t} l_{n,t}, & \text{if } n > t \\ p_{n,t} y_{n,t} - r_{n,t} k_{n,t}, & \text{if } n \leq t. \end{cases} \quad (19)$$

Optimization with respect to $l_{n,t}$ and $k_{n,t}$ leads to the first-order conditions

$$w_{n,t} = p_{n,t} \Gamma^n A_t \quad \text{if } n > t \quad (20a)$$

$$r_{n,t} = p_{n,t} \Gamma^n A_t \quad \text{if } n \leq t. \quad (20b)$$

As there is a lot happening simultaneously in our model, we highlight our main results in the simplest way possible by comparing two static equilibria in the neighborhood of the balanced growth path. One state is characterized by low productivity, state t , and the other is characterized by high productivity, state $t+h$. We solve the model under the assumption that productivity is fixed in each state, i.e. $A_t = A_{t+1}$ and $A_{t+h} = A_{t+h+1}$, although it differs across states with $A_{t+h} > A_t$. Similarly, $\Gamma^t = \Gamma^{t+1}$ and $\Gamma^{t+h} = \Gamma^{t+h+1}$, but $\Gamma^t > \Gamma^{t+h}$. Demand is also fixed in each state, i.e. $\gamma_{n,t} = \gamma_{n,t+1}$ and $\gamma_{n,t+h} = \gamma_{n,t+h+1}$, but differs across states, i.e. $\gamma_{n,t} \neq \gamma_{n,t+h}$.⁸

Combining equations (15), (16) and (17) yields the Euler equation which in equilibrium is

$$R_{n,t} = r_{n,t} \Gamma^t = P_t \left(\frac{1}{\beta} - 1 + \delta \right) \quad \forall n \leq t, \quad (21)$$

⁸In Appendix A we show that the main results can be illustrated using a dynamic equilibrium.

where $R_{n,t}$ denotes the rental rate of quality-adjusted capital of firm n in period t ($R_{n,t}$ is the real interest rate in the standard neoclassical model). The returns to quality-*un*adjusted capital are equalized across industries, as are the returns to quality-*un*adjusted labor. Namely, we have

$$r_{n,t} = r_{-n,t} \quad \forall n, -n \leq t \quad \text{and} \quad w_{n,t} = w_{-n,t} \quad \forall n, -n > t. \quad (22)$$

Substituting the first-order conditions from the firm problem, equations (20), into equations (22) leads to an expression for goods prices in equilibrium,

$$\frac{p_{n,t}}{p_{-n,t}} = \frac{\Gamma^{-n}}{\Gamma^n}, \frac{P_{n,t}}{P_{-n,t}} = 1 \quad \forall n, -n > t \quad \text{and} \quad \forall n, -n \leq t. \quad (23)$$

Combining the first equation in (23) with equations (7) and (8) yields the sectoral quality-*un*adjusted labor and capital allocations

$$\frac{l_{n,t}}{\sum_{n>t} l_{n,t}} = \frac{l_{n,t}}{l_t} = \frac{\gamma_{n,t}}{\sum_{n>t} \gamma_{n,t}} \quad \forall n, -n > t \quad (24)$$

$$\frac{k_{n,t}}{\sum_{n \leq t} k_{n,t}} = \frac{k_{n,t}}{k_t} = \frac{\gamma_{n,t}}{\sum_{n \leq t} \gamma_{n,t}} \quad \forall n, -n \leq t. \quad (25)$$

Employing equations (1), (8), (24), (25) and quality-adjusted capital in equilibrium, $K_t = (1/\Gamma^{t+s})k_t$, we obtain our first representation of aggregate production, namely

$$Y_t = [\gamma^{\frac{1}{\theta}} [K_t]^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} [A_t l_t]^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}}, \quad (26)$$

where $\gamma = \sum_{n \leq t+s} \gamma_{n,t+s}$. We note that in this first representation there is an asymmetry on the right-hand side of the equation: capital K is adjusted for quality while labor l is not.

Proposition 1 (Aggregate Production - Representation 1). *Aggregate production takes the form of a Neoclassical Production Function with labor-augmenting productivity. ■*

Turning to our second representation, we simply substitute out quality-adjusted capital for quality-*un*adjusted capital in equation (26). Doing so delivers

$$Y_t = [\gamma^{\frac{1}{\theta}} [A_t k_t]^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} [A_t l_t]^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}}. \quad (27)$$

We note that in this second representation there is symmetry on the right-hand side of the equation: capital k is *un*adjusted for quality and so is labor l .

Proposition 2 (Aggregate Production - Representation 2). *Aggregate production takes the form of a Neoclassical Production Function with factor-neutral productivity. ■*

We now shift focus to quality-*un*adjusted capital. In particular, we demonstrate that quality-*un*adjusted capital remains as scarce as quality-*un*adjusted labor across the two states of nature t and $t+h$. To see this, transform equation (5) to

$$\frac{Y_{K,t}}{Y_t} = \gamma \left[\frac{P_{K,t}}{P_t} \right]^{-\theta}, \quad (28)$$

where $Y_{K,t} = \sum_{n \leq t} Y_{n,t} = A_t \sum_{n \leq t} k_{n,t} = A_t k_t = K_t$ and $P_{K,t} = P_{n,t} \quad \forall n \leq t$. Inserting the first aggregate representation of production, equation (26), into equation (28) and solving for K_t/l_t using $P_{K,t} = P_{n,t} = \Gamma^n p_{n,t} = r_{n,t}/A_t$ and equation (21) yields

$$\frac{K_t}{l_t} = A_t \left(\frac{\left(\gamma^{-\frac{1}{\theta}} [1/\beta - 1 + \delta] \right)^{\theta-1} - \gamma^{\frac{1}{\theta}}}{(1-\gamma)^{\frac{1}{\theta}}} \right)^{\frac{\theta}{1-\theta}}. \quad (29)$$

Therefore, quality-adjusted capital becomes abundant relative to quality-*un*adjusted labor over time, as $A_{t+h} > A_t$. As in equilibrium $k_t = \Gamma^t K_t$ and $A_t = 1/\Gamma^t$, equation (29) can be rewritten as

$$\frac{k_t}{l_t} = \left(\frac{\left(\gamma^{-\frac{1}{\theta}} [1/\beta - 1 + \delta] \right)^{\theta-1} - \gamma^{\frac{1}{\theta}}}{(1-\gamma)^{\frac{1}{\theta}}} \right)^{\frac{\theta}{1-\theta}} \quad (30)$$

which now corresponds to the aggregate production representation of equation (27). Importantly, in equation (30), quality-*un*adjusted capital does *not* become abundant relative to quality-*un*adjusted labor as we move from state t to state $t+h$, i.e. $k_t/l_t = k_{t+h}/l_{t+h}$. Instead, both factors remain scarce.

Proposition 3 (Scarcity of Quality-*un*adjusted Capital). *Over time, quality-adjusted capital (real capital) becomes abundant relative to quality-*un*adjusted labor, but quality-*un*adjusted capital remains as scarce as quality-*un*adjusted labor. ■*

Having shown that quality-*un*adjusted capital and quality-*un*adjusted labor remain equally scarce over time, we next concentrate on the labor share in income. Defining the labor income share as 1 minus the capital income share and using equation (28) we obtain

$$\frac{P_{l,t} Y_{l,t}}{P_t Y_t} = 1 - \frac{P_{K,t} Y_{K,t}}{P_t Y_t} = 1 - \gamma \left[\frac{P_{K,t}}{P_t} \right]^{1-\theta}, \quad (31)$$

where $P_{l,t} = P_{n,t} \quad \forall n > t$ and $Y_{l,t} = \sum_{n > t} Y_{n,t} = A_t \sum_{n > t} l_{n,t} = A_t l_t$. Substituting the Euler equation (21) into the right-hand side of equation (31) and noting that $P_{K,t} = r_{n,t}/A_t$ gives

$$\frac{P_{l,t} Y_{l,t}}{P_t Y_t} = 1 - \gamma (1/\beta - 1 + \delta)^{1-\theta}. \quad (32)$$

This expression indicates that the share of labor in income depends only on exogenously given parameters. Therefore, the labor income share is stable across the two states t and $t+h$, i.e. $P_{l,t} Y_{l,t} / (P_t Y_t) = P_{l,t+h} Y_{l,t+h} / (P_{t+h} Y_{t+h})$.

Proposition 4 (Share of Labor in Income). *Over time, the share of quality-*un*adjusted labor in income is stable because quality-*un*adjusted capital remains equally scarce. ■*

Moving on, we now highlight how labor and capital continuously reallocate across sectors within the economy over time. In order to do so, we define two industry-weighted average quality indexes,

$F_{l,t}$ for the position of labor in the quality ladder and $F_{k,t}$ for the position of capital in the quality ladder. These two indexes are given by

$$F_{l,t} = \sum_{n>t} n \frac{\gamma_{n,t}}{\sum_{j>t} \gamma_{j,t}} \quad \text{and} \quad F_{k,t} = \sum_{n \leq t} n \frac{\gamma_{n,t}}{\sum_{j \leq t} \gamma_{j,t}}. \quad (33)$$

Applying formulas for geometric series leads to the expressions

$$F_{l,t} = t + \frac{e^{-1}}{1 - e^{-1}} \frac{e^{-1}}{(1 - e^{-1})^2} \quad \text{and} \quad F_{k,t} = t - \frac{1}{1 - e^{-1}} \frac{e^{-1}}{(1 - e^{-1})^2}. \quad (34)$$

Consequently, it is easy to see that labor and capital reallocate along the quality dimension toward higher quality goods as $F_{l,t+h} > F_{l,t}$ and $F_{k,t+h} > F_{k,t}$.

Proposition 5 (Structural Change). *Labor and capital continuously reallocate toward higher-quality firms (higher indexed firms) over time. ■*

To highlight how overall average product quality in the economy evolves with time, we define the industry-weighted average index

$$Q_t = \sum_{n \in \mathbb{Z}} \frac{1}{\Gamma^n} \frac{P_{n,t} Y_{n,t}}{P_t Y_t} \quad (35)$$

which can be rewritten as

$$Q_t = \frac{1}{\Gamma^t} \left[\frac{1}{1 - e^{-1}} \frac{1}{1 - \Gamma e^{-1}} \gamma (1/\beta - 1 + \delta)^{1-\theta} + \frac{e^{-1}}{1 - e^{-1}} \frac{\Gamma^{-1} e^{-1}}{1 - \Gamma^{-1} e^{-1}} (1 - \gamma (1/\beta - 1 + \delta)^{1-\theta}) \right] \quad (36)$$

where $\Gamma^{-1} e^{-1} < 1$ is assumed. Given that the term in brackets is just a positive constant, equation (36) implies that $Q_{t+h} > Q_t$.

Proposition 6 (Quality Evolution). *Over time, the average quality of goods produced rises. ■*

Finally, we highlight the ongoing automation of tasks within our framework. This is fairly simple as it follows directly from the way we have set up the production side of the model. Over time the number of tasks executed by the factor capital expands, i.e. $B_{t+h} > B_t$ where B_t denotes the subset of \mathbb{Z} containing all elements $n \leq t$.

Proposition 7 (Automation). *Over time, the number of goods produced by capital rises. ■*

3 Conclusion

Why is the share of labor in income still substantial despite hundreds of years of ongoing automation of tasks? Why does the neoclassical model imply that productivity growth is labor augmenting despite significant improvements in capital productivity? In this paper, we provide a microfoundation of the neoclassical growth model that offers a thorough reinterpretation of the standard theory. We then use this framework to address the two aforementioned questions.

Our microfounded model features a hierarchical goods demand structure. As general productivity in the economy improves, demand shifts from lower-quality to higher-quality tasks. Higher quality tasks offer more real output but are also more difficult to produce. Labor and capital continuously reallocate along the quality dimension toward goods of increasing sophistication. Initially, labor has a comparative advantage over capital in the production of higher quality goods. Subsequently, however, as technology progresses, capital replaces labor in these tasks and labor moves on to produce even more sophisticated goods. Notably, automation of more advanced goods requires increasingly sophisticated capital. Such advanced capital remains as scarce as labor along the growth path.

Importantly, our model highlights two parallel aggregate representations of the production side of the economy. The first one is akin to the production function of a neoclassical growth model with labor-augmenting technical change. It emphasizes the asymmetric treatment of the two factors, capital and labor, in the neoclassical growth model. While capital is quality-adjusted in this representation, labor is not adjusted for quality improvements. Over time, quality-adjusted capital becomes abundant, with quality-*un*adjusted labor remaining relatively scarce. In this case, productivity growth must therefore be labor-augmenting. The second one is akin to a neoclassical model with factor-neutral technical change. It treats both factors symmetrically in the sense that both factors are quality-*un*adjusted. Within this setting, productivity growth is factor neutral as both the quality of capital and labor improve over time. Given that quality capital and quality labor remain equally scarce along the growth path, labor endogenously acquires a substantial share in income.

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A Appendix - Dynamic Solution

To show that the economy is on a balanced growth path, consider the Euler equation (21) in the dynamic equilibrium,

$$R_t = r_{n,t}\Gamma^t = r_t\Gamma^t = P_t \left(\Gamma \left(\frac{C_t}{C_{t-1}} \right)^\phi \frac{1}{\beta} - 1 + \delta \right), \quad (\text{A.1})$$

where r_t and R_t are the interest rates of any industry n that uses capital. Suppose now that quality-adjusted consumption, C , grows at a constant rate g , i.e. $C_t/C_{t-1} = 1 + g$. Employing this equation, we can show that the quality-adjusted capital to labor ratio is a constant,

$$\frac{k_t}{l_t} = \left(\frac{\left(\gamma^{-\frac{1}{\theta}} [\Gamma(1+g)^\phi 1/\beta - 1 + \delta] \right)^{\theta-1} - \gamma^{\frac{1}{\theta}}}{(1-\gamma)^{\frac{1}{\theta}}} \right)^{\frac{\theta}{1-\theta}}. \quad (\text{A.2})$$

Given the assumption of a constant l_t , it directly follows that k_t is constant too. Given that k_t and l_t are both constant, the second production function (27) implies that

$$\frac{Y_t}{Y_{t-1}} = \frac{A_t}{A_{t-1}}. \quad (\text{A.3})$$

In other words, quality-adjusted output, Y , grows at the constant rate x . Given that k_t is constant, the capital accumulation equation (10) implies that

$$\frac{I_t}{I_{t-1}} = \frac{\Gamma^{t-1}}{\Gamma^t}. \quad (\text{A.4})$$

Thus, quality-adjusted investment, I , also grows at the constant rate x . Employing the resource constraint (3), we conclude that quality-adjusted consumption, C , must also grow at the constant rate x . Given that we know the growth rate of C , it is relatively easy to show that the shares of capital and labor, respectively, are constant in the economy in this dynamic equilibrium. Specifically, substituting the Euler equation (A.1) into Eq. (31) yields the constant share

$$\frac{P_{l,t}Y_{l,t}}{P_tY_t} = 1 - \gamma(\Gamma(1+x)^\phi 1/\beta - 1 + \delta)^{1-\theta}. \quad (\text{A.5})$$