Rationally Expected Externalities: The Implications for Optimal Waste Discharge and Recycling

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Abstract

What if consumers' actions reveal concern for contributing to an externality, even without a pecuniary incentive? Within a two-level model, a policymaker prices disposal of waste, and a representative consumer chooses a consumption level for a dirty good and a division of the consequent waste between recycling and disposal; only disposal creates an externality. In the special case of rational expectations, each consumer accepts full responsibility for his contribution to the externality. A first-best optimum is then achieved by a form of Pigouvian pricing, assuming unconstrained income taxes/transfers. Otherwise, Pigouvian pricing is second-best, unless individuals disclaim all responsibility for the externality and utility has a separable form. The model explains why recycling may occur even with free waste-disposal.

Keywords: externality; Pigouvian tax; separable utility; rational expectation; recycling.

JEL classification: D11, D21, H23, Q5.

1. Introduction

The essence of Pigouvian pricing is that, in the interest of Pareto efficiency, price should reflect external as well as private marginal costs, the element corresponding to external cost being a Pigouvian tax. This has been a major theme in the economics of externalities since Pigou (1912, pp. 148-171);[1] it remains influential, and is a major focus of such authorities as, for example, Baumol and Oates (1988), Stern (2007) and Fullerton *et al*. (2008). Coase (1960) proposes an alternative approach based on bargaining as a route to efficient allocation, but here there will be too many agents here for Coasian bargaining, and we follow Pigou in exploring the internalization of externalities through the price system.

Many recent contributions to the literature on Pigouvian taxation of externalities focus on second-best policymaking for an economy that has prior tax distortions, and it is typically assumed that consumers take the level of an externality as exogenous: for example, see the survey by Bovenberg and Goulder (2002), and parts I and II of Goulder (ed.) (2002). The research reported here deviates from that mainstream: primarily in recognizing that consumers may accept some responsibility for production of an externality; secondly in being concerned with the pricing of unrecycled waste by a policymaker, rather than with the taxation of an underlying waste-generating consumption good; and thirdly in allowing waste to have alternative destinations: either disposal, with a consequent disposal cost, or alternatively recycling or abatement, with a consequent cost. The externality is assumed to arise when waste disposal occurs, but is avoided when waste is recycled or abated.

It is certainly the case that many people undertake recycling of waste products, and in other ways show an active concern for the environment, even in the absence of pecuniary incentives. This paper shows that when consumers recognize some part of their own responsibility for an externality we are then in a second-best world, regardless of other distortions. A central contribution is to show that Pigouvian pricing satisfies the necessary conditions for a first-best social optimum only in either of two limiting cases. In one a very strong condition must prevail: specifically, rational expectations on the part of consumers, in terms of how they visualize the production of the externality. In that case, the optimal Pigouvian tax is zero and waste should be priced at marginal disposal cost. Alternatively, when consumers are completely oblivious of the environmental impact of their actions the

[1] Pigou (1912) is the precursor of Pigou's *The Economics of Welfare* of 1920. In those works Pigou does not use the terms 'externality' or 'external effect'.

standard Pigouvian prescription is for the price of waste to equal marginal disposal-plus-environmental costs, the environmental component being the Pigouvian tax; however, this is first-best optimal only if the externality is weakly separable from the consumption goods in the utility function – hereinafter referred to as 'the separability condition'. Between these limiting cases we are in a world of second-best optima in which the tax should reflect the proportion of their own environmental impact for which consumers do not recognize liability.

Given its objectives, this paper avoids consideration of general equilibrium and of pre-existing distortions. A two-level optimizing procedure is adopted, embracing individual consumers and a policymaker who all utilize asymmetrically-held information. At the lower level, consumers respond to price signals, as modified by the policymaker. At the upper level, the policymaker determines an optimal policy, given his knowledge of lower-level responses. A maintained assumption is that social objectives are based on individual utility, using a representative agent approach. This is potentially controversial. In fact, individuals may differ in terms of how an external effect affects utility: for example, the impact of noise may depend on one's acuteness of hearing. Some externalities may cause problems that are uncertain and may only develop fully in the long run, and people may differ in the extent to which they believe that the effects will arise and in their discounting of the welfare of future generations: climate change is an obvious example.

We explore the following questions for a world in which waste may be abated or recycled (at a cost) or alternatively disposed of directly (also at a cost):–

What may be concluded about the optimal pricing of waste, and the optimal mix of recycling and waste?

How does the outcome of two-level decision compare with the true social optimum?

Why do consumers undertake recycling even when waste disposal is free?

What is the significance of the extent to which the consumer recognizes his own contribution to environmental degradation?

What if unrestricted income taxes and transfers are not available to the policymaker?

Section 2. sets out a framework for two-level optimization. Section 3. introduces the externality problem and establishes a benchmark first-best optimum. Section 4. deals with the consumer. Section 5. covers the policy-level part of the process. In Section 6., the core of the paper, the results of Sections 4. and 5. are brought together and conclusions are drawn. Section 7. presents comparative static results, and concluding remarks are in Section 8.

2. Two-level optimization: general principles

Before addressing the externality problem itself, we begin with a demonstration that a formalization of the two-level optimization procedure that was introduced in Section 1. is capable of achieving optimality. Consider the general problem for a consumer: $\underset{x}{Max}\ u(\boldsymbol{x})$, subject to a set of constraints, where $u(\boldsymbol{x})$ is a continuous utility function and $\boldsymbol{x}$ is a vector of instruments. A vector $\boldsymbol{\pi}$ of parameters bears on the consumer and may be varied by the policymaker, and the consumer maximizes u with $\boldsymbol{\pi}$ taken as given. Other parameters are exogenous for all parties and may be ignored. The feasible set for the economy is the set $\boldsymbol{X}$, which is assumed to be compact.

In the two-level problem, $\boldsymbol{X}(\boldsymbol{\pi})$ (a subset of $\boldsymbol{X}$) is the feasible set for $\boldsymbol{x}$, given $\boldsymbol{\pi}$, and $\boldsymbol{\Pi}$ is the feasible set for $\boldsymbol{\pi}$. Assume that $\boldsymbol{\Pi}$ and each $\boldsymbol{X}(\boldsymbol{\pi})$ are all compact. Then the objectives are:

– for the individual: $\underset{\boldsymbol{x}}{Max}\ u(\boldsymbol{x})$ subject to $\boldsymbol{x}\in\boldsymbol{X}(\boldsymbol{\pi})$, given a vector $\boldsymbol{\pi}\in\boldsymbol{\Pi}$, with at least one solution $\boldsymbol{x}(\boldsymbol{\pi})$;

– and for the policymaker: $\underset{\boldsymbol{\pi}}{Max}\, u(\boldsymbol{x}(\boldsymbol{\pi}))$ subject to $\boldsymbol{\pi}\in\boldsymbol{\Pi}$, with at least one solution $\boldsymbol{\pi}^*$.

Define $\boldsymbol{X}_{\cup} = \bigcup_{all\,\boldsymbol{\pi}\in\boldsymbol{\Pi}} \boldsymbol{X}(\boldsymbol{\pi})$, and then:

(1) $u(\boldsymbol{x}(\pi^*))\geq u(\boldsymbol{x})$ for any $\boldsymbol{x} \in \boldsymbol{X}_{\cup}$.

On the other hand, consider the single-level constrained maximization problem:

– for the policymaker: $\underset{x}{Max}\, u(\boldsymbol{x})$ subject to $\boldsymbol{x}\in\boldsymbol{X}$ with at least one solution $\tilde{\boldsymbol{x}}$.

In this problem we have

(2) $u(\tilde{\boldsymbol{x}})\geq u(\boldsymbol{x})$ for any $\boldsymbol{x} \in \boldsymbol{X}$.

If $\boldsymbol{X}_{\cup}=\boldsymbol{X}$, then clearly $u(\tilde{\boldsymbol{x}})=u(\boldsymbol{x}(\boldsymbol{\pi}^*))$. Now let us assume that u is a strictly quasi-concave function of $\boldsymbol{x}$, and that $\boldsymbol{X}$ is a convex set. In that case both problems have unique solutions so that we now have:

Lemma 1. Assume that the consumer and the policymaker have the same continuous and strictly quasi-concave utility function $u(\boldsymbol{x})$,[2] that the opportunity sets $\boldsymbol{X}$, $\boldsymbol{X}(\boldsymbol{\pi})$ and $\boldsymbol{\Pi}$ are compact, that $\boldsymbol{X}$ is convex, and that $\bigcup_{all\,\boldsymbol{\pi}\in\boldsymbol{\Pi}} \boldsymbol{X}(\boldsymbol{\pi}) = \boldsymbol{X}$. Then $\tilde{\boldsymbol{x}}$ and $\boldsymbol{x}(\boldsymbol{\pi}^*)$ are identical, where these are the solutions to the one- and two-level problems, respectively.

The Lemma requires that the function $u(\boldsymbol{x})$ be the same for all parties, or at least that the condition stated in note 2 holds. A crucial departure from this will arise in following sections, where consumer and policymaker may take different views about the extent to which the consumer's actions bear on the production of the external good. In that case, views will differ on the manner in which individual production of waste affects individual utility: consequently policymaker and consumer will not share a common perception of the true objective function, and it will then not generally be true that single- and two-level optimization will have identical outcomes, *ceteris paribus*. A similar consequence follows when constraints on income taxes and transfers drive a wedge between $\boldsymbol{X}$ and $\boldsymbol{X}_{\cup}$.

3. The problem described formally

3.1 *Preliminaries*

This paper uses the simplest possible approach, based on an exchange economy in which there are two consumption goods: a 'dirty' good d that produces a negative externality, and a 'clean' good c. All the functions introduced in the following paragraphs (i.e. g, u, C, D and f) are assumed to be continuously twice differentiable.

The individual agent is a price-taking consumer whose net income is m after any direct taxes. He chooses quantities x_d and x_c, where d and c are assumed to be in infinitely elastic supply at positive market prices p_d and p_c respectively. Consumption of d produces waste according to a convex function $g(x_d)$ that satisfies $g(0)\geq 0$, and $g'(x_d)>0$ for all $x_d\geq 0$.[3] Individual utility is a function of x_d and x_c and also of the total quantity E of a negative externality that results ultimately from the total (market) consumption of the dirty good, by way of the impact of

[2] Or at least that the policymaker maximizes $v(\boldsymbol{x})$ where $v=\psi(u)$ and ψ is a strictly increasing transformation, because in that case if $u(\boldsymbol{x}(\boldsymbol{\pi}))\geq u(\boldsymbol{x})$ then $v(\boldsymbol{x}(\boldsymbol{\pi}))\geq v(\boldsymbol{x})$, (1) and (2) above are still satisfied if we replace u with v, and the stated conclusions stand.

[3] Primes denote derivatives of this and other functions of a single variable. Convexity of g allows for two possibilities at the margin. In one case g may be linear (or more generally affine), when a change in x_d has a constant marginal impact on waste production. In the other case, where g is strictly convex, the marginal impact rises with x_d.

unrecycled waste. We assume the utility function $u(E, x_d, x_c)$ to be strictly quasi-concave. Using subscripts to u to denote partial derivatives, we assume that u_E<0, u_d>0, and u_c>0.

Examples of the externality are: smoke produced by domestic heating, resulting in air pollution; and pollution resulting from the use of landfill sites or incinerators to dispose of domestic waste. The analysis may also be applied to more global and long-run effects such as climate-change caused by the burning of fossil fuels, where the waste in that case is the discharge of carbon dioxide.

The output of waste generated by the dirty good may be disposed of or may alternatively be recycled by the consumer, the respective quantities being w and R. Conceptually, recycling may be interpreted broadly to include all actions that achieve the consumption objective at a reduced discharge of waste, and the term 'recycling' will cover all such actions, including abatement: thus, carbon dioxide discharged into the atmosphere is clearly waste, whereas carbon capture and storage fall within the definition of recycling. Total recycling costs are $C(R)$, which is assumed to be a convex function with C'>0 and C''>0.

Disposal of unrecycled waste is charged for at a nonnegative price q per unit: for example, q might be a carbon tax, or it might be a charge for disposal of domestic garbage. Its value is parametric for the consumer and is determined by a policymaker, along with that of a second policy variable, a tax (or transfer) T on gross income M. Ultimately, disposal of unrecycled waste is a responsibility of the policymaker, and $D(\sum w_h)$ gives the cost of waste disposal, where h indexes consumers. It is assumed to be a convex function with D' >0 and D''>0.

There are n identical consumers, and the total quantity of unrecycled waste determines the level of the externality, so $E = f(\sum w_h)$. Initially we make no assumption about the form of f, other than that f'>0. The individual impact on E of a marginal change in w_h is given by $\frac{\partial E}{\partial w_h}$ $=f'(\sum w_h)>0$, whereas if all consumers behave identically then $E = f(nw)$ and the total impact is given by $\frac{\partial E}{\partial w} = nf'(nw)$.

3.2 *The benchmark: optimal pricing of waste in a first-best 'planning' context*

Our central concern is with a world in which outcomes depend on interactions between a policymaker and individual agents. We need criteria of optimality, so before proceeding further we explore a benchmark where outcomes are determined by a hypothetical planner.

Consider the single-level planning problem in which the Lagrangian is

(3) $\mathcal{L}=u(f(nw), x_d, x_c)+ \alpha_1[n[M-p_dx_d-p_cx_c-C(R)]-D(nw)] + \alpha_2[n[R+w-g(x_d)]]$

where M is the consumer's gross income and α_1 and α_2 are Lagrange multipliers. The planner chooses x_d, x_c, w and R to maximize the utility of the representative consumer, subject to aggregate income equalling expenditure, and to total waste generation equalling the amount recycled plus the amount discharged. Assuming that all instruments are positive at the unique optimum, the first-order conditions (FOCs) with respect to them may be combined to give:

(4) $u_d \quad = \quad n\alpha_1[p_d + C'g'] = \dfrac{u_c[p_d + C'g']}{p_c}$

or equivalently, writing $\mathcal{M}$ for the marginal rate of substitution $\dfrac{u_d}{u_c}$,[4]

(4') $\mathcal{M}_{cd} \quad = \quad \dfrac{p_d + C'g'}{p_c}$ with

(5) $-nu_Ef' \; = \; n\alpha_1[C'-D'] \; = \; nu_c[C'-D']/p_c$ or equivalently

(5') $C'-D' = \; -\dfrac{nu_Ef'p_c}{u_c}$

to which we shall return in due course.

4. Private optimality

4.1 *Externality seen as exogenous*

In Section 2. a general structure was set out that is now applied to the externality problem, focusing in this sub-section on a consumer who regards the level of externality as exogenous to his actions: i.e., from the individual's perspective, E is a datum and individual actions are not seen individually as influencing E.

The Lagrangian for the consumer is

(6) $\mathcal{L} = u(E, x_d, x_c)+ \lambda_1[m-p_dx_d-p_cx_c-C(R)-qw] + \lambda_2[R+w-g(x_d)]$

where λ_1 and λ_2 are Lagrange multipliers. The first constraint is the budget, and the second relates the total of recycled plus unrecycled waste to the amount generated. The consumer's

[4] Marginal rates of substitution are defined to be positive throughout this paper.

decision variables are x_d, x_c, w and R, but not (at least in this section) E. Both constraint-sets are convex in the decision variables, in view of the convexity of the functions $C()$ and $g()$.

Using subscripts to $\mathcal{L}$ to denote partial derivatives, the first-order conditions include the following, assuming that x_d>0 and x_c>0:

(7) $\mathcal{L}_w = -\lambda_1 q + \lambda_2 \leq 0;$

(8) $\mathcal{L}_d = u_d - \lambda_1 p_d - \lambda_2 g' = 0;$

(9) $\mathcal{L}_c = u_c - \lambda_1 p_c = 0;$

(10) $\mathcal{L}_R = -\lambda_1 C' + \lambda_2 \leq 0.$

Given $u_c>0$ it follows from (9) that $\lambda_1>0$ at the optimum: therefore the budget constraint binds. Moreover at least one of (7) and (10) must be satisfied as a strict equality at the optimum, otherwise $w=R=0$, which would violate the second constraint.

If (10) is satisfied as a strict inequality, then R=0. In that case $q<C'(0)$, from (7) and (10), and it is cheaper to pay for disposal of the entire quantity of waste $g(x_d)$. A particular, and important, case is where q=0, when it is clear from (7) that $\lambda_2 = 0$ also; from the conditions on C, (10) is then satisfied as a strict inequality and R=0. Thus when q=0 there is no recycling, and all of $g(x_d)$ is disposed of at zero private cost. In fact, individuals do not always behave like this: some show a concern for environmental effects, and bear the private cost of recycling and waste avoidance even without a pecuniary incentive. Section 4.2 explores this, but clearly, according to the model as it stands, $q>0$ is a necessary condition for $R>0$.

If (7) is satisfied as a strict inequality at the optimum, then w=0. Using (10), which must then be satisfied as an equality, we have $q>C'$: the disposal charge exceeds $C'(g(x_d))$ and the optimal choice is to recycle all of the quantity $g(x_d)$.

The final possibility to consider is a solution at which $w>0$ and $R>0$. In such a case we have $C'=q$: i.e. the optimal choice is to recycle up to the point at which the marginal recycling cost equals q, and to dispose of the balance of the quantity $g(x_d)$ at the price q.

From (7), (8), (9) and (10) we have for the marginal rate of substitution ($\mathcal{M}$) at the optimum:

(11) $\mathcal{M}_{cd} = \frac{u_d}{u_c} = \frac{p_d}{p_c} + \frac{g'}{p_c}\frac{\lambda_2}{\lambda_1} = \frac{p_d}{p_c} + \frac{g'}{p_c}.min\{q, C'\}$

with $q=C'$ if w and R are both positive at the optimum. When marginal recycling costs are non-zero, then levying a charge for disposal in effect raises the relative price of good d. We may interpret this as incorporating some part of the environmental cost of consuming good d into its price, thus forcing the consumer to bear part of the environmental impact of his actions: part of the externality is internalized. If good c is a gross substitute for good d, the effect of a higher relative price of good d is to shift the balance of consumption towards c, in the sense that the ratios $p_c x_c/m$ and x_c/x_d will both rise. The ratio $p_c x_c/m$ obviously rises, since x_c rises in this case; the second ratio certainly rises also if x_c rises and x_d falls. We explore comparative static effects of parameter changes in some detail in Section 7. below.

The Pigouvian price of discharged waste is given by $q = \tilde{D}' - \frac{n\tilde{u}_E \tilde{f}' p_c}{\tilde{u}_c}$, i.e. the sum of marginal disposal cost and marginal environmental cost, the latter being the Pigouvian tax. The tildes indicate functions evaluated at first-best optimal values of the relevant variables, consistent with (4) and (5) in Section 3.2. Focusing for clarity on the case where all variables take positive values at the optimum with all FOCs satisfied as equalities, then with Pigouvian pricing , (7) and (10) now imply $C' = \tilde{D}' - \frac{n\tilde{u}_E \tilde{f}' p_c}{\tilde{u}_c}$. Comparing this with (5') we see that this value for q induces individual decision to arrive at the first-best value for C' and hence for R.

However, the Pigouvian price does not in general induce the first-best optimal values of all variables unless we assume that the utility function satisfies the separability condition. This may be seen by returning to (11), written as $\mathcal{M}_{cd} = \frac{p_d}{p_c} + \frac{g'}{p_c}\tilde{C}'$. If we substitute the first-best values $\tilde{x}_d$ and $\tilde{x}_c$ into this, then the right-hand side will be the same as in (4') but without separability the left-hand side will differ from (4'), in general, unless the individual expects E to have its first-best optimal value. Therefore in general, without separability, $\tilde{x}_d$, $\tilde{x}_c$ and $\tilde{R}$ cannot satisfy (11), and these values cannot be induced by Pigouvian pricing – which must therefore produce a second-best result when the individual expectation for E differs from the first-best value. However, if the separability condition holds, then these considerations do not apply. The values of E and of $\mathcal{M}_{cd}$ are then independent; by setting q as above we ensure that the FOCs of the consumer are identical with those of the planner, in particular in containing identical functions, so that they have the same solution.

We may set this in the framework of Section 2. In general, the planner's and consumer's objective functions differ, being $u(f(nw), x_d, x_c)$ and $u(E, x_d, x_c)$ respectively, and we may not apply Lemma 1. except in the special case where E is set equal to the (first-best) optimal value $f(n\tilde{w})$. However, notwithstanding the different objective functions, it is the case that when $\mathcal{M}_{cd}$ is independent of E then, by setting q appropriately, the FOCs for the consumer can be made identical with those for the planner and the two sets of solutions will be identical. It should be apparent that outcomes depend in part on consumers' expectations concerning the generation of the externality, and the analysis from this point allows for that dependence.

4.2 *Green consumers*

We now suppose that consumers recognize that their own actions influence the level E of the externality: without such an assumption it is impossible to explain the observable fact that many people voluntarily bear the costs of recycling waste and of mitigating the effects of residual waste, even without a pecuniary incentive. We write the production function for the externality as seen by consumer h as: $E=f(\theta_h w_h+\bar{W}_h)$, $\theta_h>0$. The consumer assesses total waste output by all other consumers as $\bar{W}_h$, or forms this expectation, and the nonnegative psychological parameter θ_h reflects his beliefs about the environmental effect of his own actions. In principle $\bar{W}_h=\sum_{k\neq h} w_k$, but the consumer only needs to evaluate the aggregate $\bar{W}_h$.

Subjectively, $\frac{\partial u_h}{\partial w_h}=\theta_h\frac{\partial u_h}{\partial E}f'(\theta_h w_h+\bar{W}_h)$. If consumer h has $\theta_h=0$ as in Section 4.1, then he sees the externality as everyone else's fault. If $\theta_h=1$, he fully recognizes his own contribution and $\frac{\partial u_h}{\partial w_h}=\frac{\partial u_h}{\partial E}f'$. Beyond this there is the further possibility that the representative consumer holds a rational expectation and believes not merely that $\theta_h=1$ but also that he is representative, in which case from his perspective $E=f(nw)$, and $\frac{\partial u_h}{\partial w_h}=n\frac{\partial u_h}{\partial E}f'$: then, in effect, it as if $\theta_h=n$ with other consumers' waste output ($\bar{W}_h$) internalized through θ_h. This is the rational expectation because if $\theta_h=n$ then consumer h's decisions are based on the correct economic model in a world in which he is representative: when he changes his production of waste by Δw in response to some event, that event causes everyone else to behave identically; agent h recognizes this, and believes correctly that the total amount will change by $n\Delta w$.

Suppressing the h subscripts for clarity, the Lagrangian for consumer h is now

(6') $\mathcal{L} = u(f(\theta w+\overline{W}), x_d, x_c)+ \lambda_1[m-p_d x_d-p_c x_c-C(R)-qw] + \lambda_2[R+w-g(x_d)]$

and the first-order conditions are (8), (9), (10) and

(7') $\mathcal{L}_w = \theta u_E f' - \lambda_1 q + \lambda_2 \leq 0.$

If θ=0, (7') reduces to (7) and the results of Section 4.1 stand. Here we suppose that θ>0. If w>0 and R>0, then from (7'), (8), (9) and (10) we have

(12) $u_d = \lambda_1[p_d + C'g'] = u_c[p_d + C'g']/p_c$ and

(13) $-\theta u_E f' = \lambda_1[C' - q] = u_c[C' - q]/p_c$ or

(13') $C' - q = -\dfrac{\theta u_E f'}{\lambda_1} = -\dfrac{\theta u_E f' p_c}{u_c}.$

With u_E<0 it follows that C'>q at the optimum:[5] facing q as a parameter, the internalization of the externality pushes the consumer to recycle beyond the point at which the marginal cost of recycling equals q: given q, the amount of recycling exceeds the level identified in Section 4.1, where with θ=0 we had C'=q optimally, when R and w were both positive.[6] Here, from (13'), recycling is pushed to the point where its marginal cost equals the sum of the pecuniary cost plus the internalized portion of the external cost of waste discharge, so that even if q=0, an optimal programme may include positive recycling. The consumer recognizes some part of his responsibility for the external cost of his production of waste, and this is sufficient to provoke some recycling even when disposal is free. The term λ_1 is the marginal utility of $1 from the consumer's budget constraint. Thus $\frac{1}{\lambda_1}$ is the optimal dollar price of a marginal util, so that the excess of C' over q at the optimum is just the value, at that price, of the marginal disutility of the externality as perceived by the consumer. The full extent of the marginal disutility of a unit of individually-generated waste is $-u_E f'$, but the individual only perceives a responsibility for the fraction θ of this.

Equation (11) now becomes

(11') $\mathcal{M}_{cd} = \dfrac{u_d}{u_c} = \dfrac{p_d}{p_c} + \dfrac{g'}{p_c}\dfrac{\lambda_2}{\lambda_1} = \dfrac{p_d}{p_c} + \dfrac{g'}{p_c}.min\{q - \dfrac{\theta u_E f'}{\lambda_1}, C'\}.$

[5] Here and later we draw inferences from expressions containing u_E. Such expressions always contain u_E in the format u_E/u_c: i.e. the (negative of the) marginal rate of substitution, which is independent of the particular utility function that is used to represent the underlying preference ordering.

[6] Just as in Section 4.1, so also here we could have a solution with R=0, when $C'(0)>q-\theta u_E f'\frac{1}{\lambda_1}$. Alternatively, we could have $C'(g(x_d))< q -\theta u_E f' \frac{1}{\lambda_1}$, in which case $R= g(x_d)$ and w=0. The second case may possibly arise even if q=0, provided $\theta \neq 0$.

When w and R are both positive, $q - \frac{\theta u_E f'}{\lambda_1} = C'$ from (13').

The Pigouvian price that was introduced in Section 4.1 becomes

(14) $q = \tilde{D}' - (\frac{n-\theta}{n})(\frac{n\tilde{u}_E\tilde{f}'p_c}{\tilde{u}_c})$ where the tilde indicates first-best optimal values.

The absolute value of the marginal externality expressed in monetary units is $-\frac{n\tilde{u}_E\tilde{f}'p_c}{\tilde{u}_c}$, the fraction of this that is not internalized by the representative consumer is $\frac{n-\theta}{n}$, and their product is the optimal value of the Pigouvian tax, to be levied on top of the marginal disposal cost. Further consideration of this may conveniently be deferred until Section 6.4, where it is discussed in the context of two-stage optimization.

5. Conditions for socially optimal pricing given individual behaviour, when $w > 0$, $R > 0$

Consider a single representative consumer. Given any values for prices and net income m, the consumer chooses w, x_d, x_c and R to maximize individual utility, and at any prices the indirect utility function, giving the consumer's maximized utility, is

$V(q, p_d, p_c, m) \equiv u\big(f(\theta w(q, p_d, p_c, m) + \overline{W}), x_d(q, p_d, p_c, m), x_c(q, p_d, p_c, m)\big)$.

However, these individual actions relate to a belief that $E = f(\theta w + \overline{W})$, whereas the policymaker knows that this should be $E = f(nw)$, where in either case the utility-maximizing value of w is a function of q, p_d, p_c and m. We take the social objective function to be $\hat{V}(q, p_d, p_c, m) \equiv u\big(f(nw(q, p_d, p_c, m)), x_d(q, p_d, p_c, m), x_c(q, p_d, p_c, m)\big)$ where w, x_d and x_c are the utility-maximizing demands as derived in Section 4., which respect the first-order conditions (7'), (8), (9) and (10). In general V and $\hat{V}$ differ, and a modified version of Roy's identity is valid for $\hat{V}$. We confine attention to the case where $w>0$ and $R>0$ at the consumer's optimum, in which case

$\frac{\partial \hat{V}}{\partial q} = (n-\theta)u_E f' \frac{\partial w}{\partial q} - \lambda_1 w(q, p_d, p_d, m)$ and $\frac{\partial \hat{V}}{\partial m} = (n-\theta)u_E f' \frac{\partial w}{\partial m} + \lambda_1$. These expressions are derived in Appendix A., but the intuition is obvious. They are the familiar expressions for Roy's identity, modified respectively by terms $(n-\theta)u_E f' \frac{\partial w}{\partial q}$ and $(n-\theta)u_E f' \frac{\partial w}{\partial m}$ that correct for the extent $(n-\theta)$ to which the individual fails to take full account of the external effect of his actions.[7] Obviously these terms vanish if $\theta = n$ or $u_E f'=0$.

The problem is to find values for q and an income tax T (which, paid out of gross income M, determines m) that maximize maximized utility $\hat{V}$, subject to three constraints: total disposal costs $D(nw)$ must be covered by revenue from both sources, tax and charge, and the tax is subject to upper and lower bounds $\overline{T}$ and $\underline{T}$ where $\overline{T} > \underline{T}$. Unless we specify $\underline{T} \geq 0$, the optimal value of T may be negative, i.e. a subsidy. We could in addition or alternatively consider levying a sales tax on the dirty good, but we focus on an income tax alone, for the familiar reason that such a tax raises a given amount of revenue more efficiently than a sales tax.

Because we are interested in the case $E>0$, we assume that $w>0$. Writing the Lagrange multipliers as μ_1, μ_2 and μ_3, the Lagrangian is:[8]

$$(15)\quad \mathcal{L} = \hat{V}(q, p_d, p_c, m(T))$$
$$+ \mu_1[nqw(q,m(T),...) + nT - D(nw(q,m(T),...))] + \mu_2[\overline{T}-T] + \mu_3[T-\underline{T}].$$

The first-order conditions, using the modified version of Roy's identity and assuming nonzero optimal values for q and T, are:

$$(16)\quad \mathcal{L}_q = (n-\theta)u_E f'\frac{\partial w}{\partial q} - \lambda_1 w + n\mu_1[w + q\frac{\partial w}{\partial q} - D'\frac{\partial w}{\partial q}] = 0;$$

$$(17)\quad \mathcal{L}_T = -(n-\theta)u_E f'\frac{\partial w}{\partial m} - \lambda_1 + n\mu_1[-q\frac{\partial w}{\partial m} + 1 + D'\frac{\partial w}{\partial m}] - \mu_2 + \mu_3 = 0, \quad \text{using } \frac{dm}{dT} = -1.$$

Combining (16) and (17),[9] we have

$$((n-\theta)u_E f' + n\mu_1[q-D'])(\frac{\partial w}{\partial q} + w\frac{\partial w}{\partial m}) + (\mu_2-\mu_3)w = 0 \text{ so that}$$

$$(18)\quad ((n-\theta)u_E f' + n\mu_1[q-D'])\frac{\delta w}{\delta q} + (\mu_2-\mu_3)w = 0,$$

writing $\frac{\delta w}{\delta q}$ for the substitution term, after using the Slutsky equation. Throughout the following we take it that $\frac{\delta w}{\delta q} < 0$.[10]

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[7] A similar modification of Roy's identity is implicit in Atkinson and Stiglitz (1980, p. 452).

[8] Note that this Lagrangian does not satisfy the concavity/convexity requirements for an application of Kuhn-Tucker: $\hat{V}$ is quasi-convex in its arguments, and the shape of the constraint-set depends *inter alia* on the function $w()$, which is not necessarily concave.

[9] Multiply (17) by w, subtract from (16), and collect terms.

It makes no qualitative difference to the conclusions drawn above if we replace the tax T with a proportionate tax $t[M–A]$, where t is the tax rate, M is gross income and A is a tax-free allowance, where $M>A\geq 0$. Where m is net income we then have $\frac{dm}{dt}=A–M$, and equation (18) is replaced by

$$(19) \quad \left((n-\theta)u_E f' + \mu_1(q-D')\right)\frac{\delta w}{\delta q} - (\mu_2-\mu_3)w/(A-M) = 0.$$

Because $A–M<0$, the qualitative conclusions to be drawn from (18) would be unaltered; however, replacing the lump-sum tax with an income tax, or altering an existing income tax, would have distortionary implications that could not be neglected.

6. Two-level optimality

We now explore the interaction between the set of optimality conditions derived in Sections 4.2 (for the green consumer) and 5. (for the policymaker), in the case where w and R are both positive. The externality is significant at the margin if $u_E f'<0$, which is the case unless we alter one of the assumptions $u_E<0$, $f'>0$, to permit a zero value.

The consumer pays q for discharge of waste and bears the cost of recycling, the policymaker receives the revenue from waste and bears the cost of disposal, and income taxes or transfers will be required for budget-balance at each level. To see this, note that for the first constraints in the Lagrangians (6') and (3) both to be satisfied, it must be that $D(nw)=n[qw+T]$, where $T=M–m$, and T may be positive, negative or zero. There are three possibilities to consider, according as the tax constraints are slack or binding at the optimum. We assume that $\mu_1>0$ always, in order to rule out an excess of revenue over disposal costs; at most one of μ_2 and μ_3 may be nonzero.

6.1 *Slack tax constraints*

If both constraints on the tax T are slack, then $\mu_2=\mu_3=0$. Equation (18) is a necessary condition for social optimality, based on a given pattern of consumption behaviour at the individual level. When there is no effective constraint on the tax levied then (18) becomes

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[10] $\frac{\delta w}{\delta q}\leq 0$ from the standard property of a compensated demand function; a sufficient condition to exclude $\frac{\delta w}{\delta q}=0$ is that the indifference surfaces be differentiable. See Somerville (2007).

(18') $(n-\theta)u_E f' + n\mu_1[q-D'] = 0$, assuming that $\frac{\delta w}{\delta q} \neq 0$, which may be written

$$q = D' - (\frac{n-\theta}{n})(\frac{nu_E f'}{n\mu_1})$$

However, (16) may be written:

(16') $w[n\mu_1-\lambda_1] + \left((n-\theta)u_E f'+n\mu_1[q-D']\right)\frac{\partial w}{\partial q} = 0,$

which together with (18') implies that $n\mu_1-\lambda_1=0$: because T is effectively unconstrained, it takes a value that ensures that the public and private marginal utilities of \$1 are equated. Therefore:

(14') $q=D' -(\frac{n-\theta}{n})(\frac{nu_E f' p_c}{u_c}) =D' - (\frac{n-\theta}{n})(\frac{nu_E f'}{\lambda_1})= D' - (\frac{n-\theta}{n})(\frac{nu_E f'}{n\mu_1}),$

which differs from (14) in that (14) is evaluated at the first-best optimum.

If the externality is significant (i.e. $u_E f'<0$) and if also $n>\theta$ then $q-D' >0$ is a necessary condition for optimality. Alternatively, if the externality is insignificant, or if $\theta=n$, or both, then the necessary condition is $q-D'=0$. These possibilities are summarized in Table 1.

Table 1. *Optimal price q of waste relative to marginal disposal cost D', when the tax constraints are both slack so that* $\mu_2=\mu_3=0$.

	If $0\leq\theta < n$:	*If* $\theta = n$:
Externality significant, i.e. $u_E f' <0$:	$q > D'$	$q = D'$
Externality insignificant, i.e. $u_E f' =0$:	$q = D'$	$q = D'$

Possibly the optimal solution for T may be negative, i.e. T may be a subsidy. The rationale for this may be explored by combining the first constraint in (15) with the constraint in the individual problem to obtain: $M-p_d x_d-p_c x_c-C= D/n= qw+T$. In the case where optimal recycling costs are relatively large and disposal costs are relatively small, it may be optimal in effect to repay to the consumer part of the revenue qw. As an example suppose that optimality involves $q=D'$, and consider the case where D has the form, or is approximated by, $D(w)=\beta_1 w+½\beta_2 w^2$, where the β_i are constants and $\beta_2>0$. Without loss of generality let $n=1$. Then $D' =\beta_1+\beta_2 w$ so that $qw>D$ when $q=D'(w)$, and optimally $T<0$. However if the total cost function $D(w)$ is modified to contain a positive element β_0 of fixed costs, then for β_0 sufficiently large, T will be positive optimally.

We bring in recycling costs by combining (18') with the individual condition (13'), eliminating q and using $n\mu_1=\lambda_1$ to obtain

$$(20) \quad C'-D' = -\frac{nu_E f'}{\lambda_1} = -\frac{nu_E f' p_c}{u_c}$$

so that if $u_E\, f'<0$, then $C'-D'>0$.

This has the same structure as first-order condition (5') in the planning problem of section 3.2, but is identical with it only in the case of $\theta=n$ and rational expectations, when the consumer and the policymaker have the same objective function $u(.)$. In that special case the first-best optimum may be reached by the two-level route, provided that unrestricted taxes or transfers are possible. Otherwise, if $\theta\in(0, n)$, the two-level procedure has a second-best outcome because the objective functions differ at each level.

The three equations that characterize the optimum are (20), (14'), and (13') written as $C' - q = -nu_E f' \frac{\theta}{n}\frac{1}{\lambda_1}$, which give us the optimal relationships between q, D' and C' that are set out in Figure 1.

Figure 1. here – see page 30

While the value of θ does not directly enter the optimal relationship between D' and C', it directly determines where q lies in the interval $[D', C']$, so that the excess of q over D', i.e. the Pigouvian tax, is the non-internalized portion $\frac{n-\theta}{n}$ of marginal external cost, $-\frac{nu_E f'}{\lambda_1}$. If expectations are rational and $\theta=n$, then $q=D'$: the optimal price of waste is just the marginal disposal cost, and the Pigouvian tax is zero. The smaller θ is, the larger q needs to be relative to D' and C', and in the limiting case $\theta=0$ we have $q=C'$ where the Pigouvian tax is the full marginal environmental cost of the externality; strictly speaking it is only in the case $\theta=0$ that the full Pigouvian tax is appropriate. However, remarkably, this case for the full Pigouvian tax is approximated in the important case $\theta=1$: then with n large, $n-\theta\approx n$ and $\frac{n-\theta}{n}\approx 1$, with equality in the limit. Such an individual may be characterized as 'socially responsible': he recognizes his own contribution to the externality, but does not hold a rational expectation and recognize his representativeness.

6.2 *Binding upper constraint on T*

The second possibility is that $\mu_2>0$ at the optimum so that the upper constraint on the tax T binds, and therefore $\mu_3=0$. Since $w>0$ by assumption and $\frac{\delta w}{\delta q}<0$, condition (18) becomes:

(18") $(n-\theta)u_E f' + n\mu_1[q-D']>0$

so that whether the externality is significant ($u_E f'<0$) or not ($u_E f'=0$), and whether θ is equal to or less than n, a necessary condition for optimality is $q-D'>0$, with $q-D'$ exceeding the non-internalized part of external cost. We have a requirement to price waste-disposal above marginal cost even if the externality is 'insignificant': but of course it is only privately insignificant at the private margin in the sense that $u_E f'=0$. The policymaker knows that aggregate waste must be disposed of, and because the required revenue cannot be raised entirely by the income tax, the shortfall must be made up via appropriate pricing.

The first-best optimum of Section 3.2 is not achievable: combining (18") with (13') we obtain

(20') $C'-D' > -nu_E f'[\frac{\theta}{n}\frac{1}{\lambda_1} + \frac{n-\theta}{n}\frac{1}{n\mu_1}]$.

Again we have $C'-D'>0$, but the gap exceeds the value of the external cost.

Moreover if $\theta=n$, $C'-D' > -\frac{nu_E f'}{\lambda_1} = -\frac{nu_E f' p_c}{u_c}$ so that (5') is not satisfied and the outcome is second-best even in this rational expectations case, unlike the case of slack tax constraints where T was able to adjust to equate private and social marginal utility. Here the upper constraint on T binds: private expenditures (C and C') are higher than otherwise while those of the public sector (D and D') are lower, so that $C'-D'$ exceeds the value of the externality $-nu_E f'[\frac{\theta}{n}\frac{1}{\lambda_1} + \frac{n-\theta}{n}\frac{1}{n\mu_1}]$, taken as a weighted average over public and private valuations. The social marginal utility of \$1 exceeds the private value, i.e. $\lambda_1<n\mu_1$, so that $\frac{1}{\lambda_1} > \frac{1}{n\mu_1}$,[11] and if $0<\theta<n$, then $\frac{1}{\lambda_1} > \frac{\theta}{n}\frac{1}{\lambda_1} + \frac{n-\theta}{n}\frac{1}{n\mu_1} > \frac{1}{n\mu_1}$.

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[11] From (16') and (18") it follows that $n\mu_1-\lambda_1>0$, if we assume (reasonably) that the uncompensated term $\frac{\partial w}{\partial q}$ is negative. Therefore $\frac{1}{\lambda_1} > \frac{1}{n\mu_1}$, and a weighted average of these lies between them. The inequalities should be reversed to cover Section 6.3, but the conclusion is the same.

In Section 6.1 (using equation (14')) we concluded that when $\mu_2 = \mu_3 = 0$ then $q=D'$ is necessary for optimality when $\theta = n$, with the condition becoming $q>D'$ when $\theta < n$. Here, we require $q>D'$ for optimality for all $\theta \in [0, n]$, as is clear from (18").

6.3 *Binding lower constraint on T*

The final possibility is that $\mu_3>0$ at the optimum so that the lower constraint on the tax binds, and therefore $\mu_2=0$. In this case the optimality condition is.

(18''') $(n-\theta)u_E f' + n\mu_1[q-D'] < 0$.

If $\theta=n$ then $q<D'$: here T is constrained to be above its first-best optimal level, and to offset this q is depressed below the value of D'. However if $\theta<n$, we have the interaction of two effects: one (binding constraint on T from below) tending to reduce q, and the other (departure from rational expectations) tending to raise it, and the solution could have q above, or below, or equal to D'.

Combining (18''') with (13') gives

(20") $C'-D' < -nu_E f'[\frac{\theta}{n}\frac{1}{\lambda_1} + \frac{n-\theta}{n}\frac{1}{n\mu_1}]$

so that $C'-D'$ is below the value $-nu_E f'[\frac{\theta}{n}\frac{1}{\lambda_1} + \frac{n-\theta}{n}\frac{1}{n\mu_1}]$ of the externality, again taking a weighted average over private and public valuations. Here T hits its lower bound and the second-best optimum has less private expenditure (lower C and C') and more public expenditure (higher D and D') than if T were unconstrained.

6.4 *Necessary condition for optimality: conclusions, when* $w>0$ *and* $R>0$

From the discussion in Section 6.1, a necessary condition for optimality if the tax constraints are slack is (20):

$C'-D' = -\frac{nu_E f'}{\lambda_1} = -\frac{nu_E f'}{n\mu_1} = -\frac{nu_E f' p_c}{u_c}$, whether or not expectations are rational: i.e., independent of the value of θ. For optimality in this case, marginal recycling costs should equal the sum of marginal waste-disposal costs plus the dollar-equivalent of the externality at the margin: recycling is pushed to the point at which its marginal cost just equals the full marginal opportunity cost of waste, including disposal and external costs. The price q of waste should exceed D' by the uninternalized fraction $\frac{n-\theta}{n}$ of the gap between D' and C'. The policymaker's and consumer's objective functions differ unless $\theta=n$, being respectively $u(f(nw),...)$ for the policymaker, and $u(f(\theta w+\overline{W}),...)$ for the consumer. It is only when $\theta=n$, where the consumer has a rational expectation, that these functions coincide. It is easy to show that the constraint-set for the single-level planning problem is the union of all possible constraint sets in the two-level problem (see Appendix B.), provided that the upper and lower tax constraints do not bind at the solution and may therefore be ignored; moreover the constraint set is convex.[12] Consequently, Lemma 1. may be applied to conclude that the two-level outcome is first-best when $\theta=n$. The maximized value of u is the same, whether it is maximized on two levels, or directly. The optimal values of w, x_d, x_c and R will be the same under either mode of optimization, so it must be that $\frac{1}{n\alpha_1}=\frac{1}{\lambda_1}\ (=\frac{1}{n\mu_1})$, and the social and private prices of a marginal util are equated.

Because the objective functions differ when $\theta<n$, we should expect that in general the two-level procedure would not lead to the first-best outcome, because a condition for Lemma 1. is not satisfied. We may confirm this by repeating and appealing to the FOCs:[13]

(13') $\quad C'-q \ = \ -\frac{\theta u_E f' p_c}{u_c}=\frac{\theta}{n}nf'p_c\mathcal{M}_{cE}$ and

(11') $\quad \mathcal{M}_{cd} \ = \frac{p_d}{p_c}+\frac{g'}{p_c}.\, C'$

which must be satisfied by the solution to the consumer's optimization problem. The Pigouvian value of q is given by (14)

$q=\tilde{D}'-(\frac{n-\theta}{n})(\frac{n\tilde{u}_E\tilde{f}'p_c}{\tilde{u}_c})=\tilde{D}'+(\frac{n-\theta}{n})(np_c\,\tilde{f}'\tilde{\mathcal{M}}_{cE})$, and with this value of q (13') becomes

(13'') $\quad C'-\tilde{D}' \ = \ \frac{\theta}{n}(np_cf'\mathcal{M}_{cE})+(\frac{n-\theta}{n})(np_c\,\tilde{f}'\tilde{\mathcal{M}}_{cE})$.

Bearing in mind that the vector $(\tilde{w},\ \tilde{x}_d,\ \tilde{x}_c,\ \tilde{R})$ satisfies (5'), it follows that it satisfies (13'') if and only if $f'(\theta w+\overline{W})\mathcal{M}_{cE}=f'(n\tilde{w})\tilde{\mathcal{M}}_{cE}$ at that point. A sufficient condition for this is that $\theta\tilde{w}+\overline{W}=n\tilde{w}$, i.e. $\overline{W}=(n-\theta)\tilde{w}$, so that the representative consumer expects E to have its first-best optimal value $\tilde{E}$. Given that $\overline{W}$ is an *ex ante* parameter for the consumer, this is extremely unlikely to hold. The sufficient condition is also necessary if f' is constant. Otherwise, it is conceivable that $f'(\theta\tilde{w}+\overline{W})\mathcal{M}_{cE}=f'(n\tilde{w})\tilde{\mathcal{M}}_{cE}$ without $\theta\tilde{w}+\overline{W}=n\tilde{w}$, so (13'')

[12] Given the assumptions that C, D and g are convex functions, and given that the instruments otherwise enter the constraints linearly.

[13] The marginal rates of substitution are defined as absolute values of ratios of marginal utilities. Because $u_E<0$, $\mathcal{M}_{cE}$ is defined as $-u_E/u_c$. The sign adjustment is unnecessary for $\mathcal{M}_{cd}$.

would be satisfied at $(\tilde{w}, \tilde{x}_d, \tilde{x}_c, \tilde{R})$; but then (11') would be satisfied only if utility satisfied the separability condition. Thus, we conclude that in general the first-best optimum is not achieved by Pigouvian pricing when $\theta \in (0, n)$.

In the particular case θ=0, policymaker and consumer have differing objective functions, and (as in section 4.1) we may appeal to (11') to conclude that the first-best optimum is not in general a solution to the problem when q is assigned its Pigouvian value. However, under the separability condition the lack of congruence between the two objective functions becomes irrelevant when θ=0 because the consumer's utility is then in effect a function of two rather than three variables. The consumer's FOCs then coincide with those of the policymaker, and the two-level solution is first-best in this special case.

When $\mu_2 > 0$, so that the upper tax constraint binds, we have condition (20') whereby recycling is pushed beyond the point at which its marginal cost equals the full marginal cost of waste, including disposal and external costs. Finally that inequality is reversed according to (20'') when $\mu_3 > 0$ so that the lower tax constraint binds. In either case we have a solution that does not satisfy the first-best condition (5').

Because $\frac{\theta}{n}\frac{1}{\lambda_1} + \frac{n-\theta}{n}\frac{1}{n\mu_1}$ is in effect the optimal dollar price of a unit of utility at the margin, the expression $-nu_E f'[\frac{\theta}{n}\frac{1}{\lambda_1} + \frac{n-\theta}{n}\frac{1}{n\mu_1}]$ is the dollar-equivalent of the externality at the margin, and it becomes $-nu_E f'\frac{1}{\lambda_1}$ either if both tax constraints are slack (when $\lambda_1 = n\mu_1$) or if expectations are rational (when $\theta = n$). A binding tax constraint prevents the equalization of λ_1 and $n\mu_1$, and the marginal util is then valued at the average $\frac{\theta}{n}\frac{1}{\lambda_1} + \frac{n-\theta}{n}\frac{1}{n\mu_1}$ unless $\theta = n$.

The smaller the value of θ, the larger q must be in the second-best optimum (see Figure 1.), and the smaller in consequence will be the consumer's own perception of maximized utility, given $\frac{\partial V}{\partial q} = -\lambda_1 w < 0$. This failure of the two-level procedure occurs even if each individual fully recognizes the significance of his own actions but ignores the fact that he is 'representative': i.e. if θ =1. This case approximates to the case θ = 0 for large n.

We have seen how the value of θ determines the optimal location of q in the interval [D', C'].[14] Under rational expectations (θ=n) and effectively unconstrained taxes we have q=D'. For lower values of θ, the individual does not take full account of the implications of his actions, and q needs to be correspondingly further above D' and closer to C' in order to achieve the correct balance of waste and recycling. If additionally a tax constraint binds, then $q \neq D'$ even if θ=n. In the extreme case (corresponding to Section 4.1) where θ=0, then the optimum has $q = C'$ (from (13')) whether or not the tax is effectively constrained.

6.5 *The cases where* w=0 *or* R=0

So far in Section 6. we have focused on solutions where w and R are both positive, and we have found that optimality requires a division of $g(x_d)$ between disposal and recycling such that $C'=D' - \frac{nu_E f'}{\lambda_1}$. The parameters of the problem may preclude such a solution, as Figure 2. illustrates for a given value of x_d in the case μ_2=μ_3=0 (i.e. ineffective tax constraints).

Figure 2. here – see page 30

Depending on the configuration of the functions D' and C', there are three possible cases. First we may have an all-disposal solution, when $C'(0) > D_1'(g(x_d)) - \frac{nu_E f'}{\lambda_1}$, and replacing some disposal of $g(x_d)$ by recycling would raise total social cost. Secondly, we may have an all-recycling solution, when $D_2'(0) - \frac{nu_E f'}{\lambda_1} > C'(g(x_d))$, and replacing some recycling of $g(x_d)$ by disposal would raise total social cost. In the intermediate case when the C' and D' curves intersect as indicated along $D_3'(w)$, we have a solution where disposal and recycling both take place, and C' exceeds D_3' by exactly the amount of the marginal social external cost of waste.

6.6 *Prices versus quantities as policy instruments*

We have seen that, given unrestricted taxes or transfers, optimality could in principle be achieved using price (i.e. q) as the policy instrument; in certain circumstances, optimality achieved in this way would be first-best. Alternatively, quantity (i.e. the solution (w, x_d, x_c, R) to the problem posed in Section 3.2) could be used, and the policymaker would need the same information set in either case (Weitzman, 1974, p. 478): that point is visible above, in that to compute q from equation (14), Section 4.2, for example, knowledge of socially optimal

[14] See Figure 1.

quantities would be necessary. In either case computation will be problematical, given the asymmetry of information between policymaker and consumer, and this provides the rationale for explicitly developing a two-level approach to optimizing, whereby policymaker and consumer each uses his own particular information-set.

The preceding sections have explored a procedure in which price is used as the policy instrument, for two reasons. The first is that instruments in price-domain (e.g. carbon taxes) have for long been receiving considerable attention as possible mechanisms for controlling externalities (for example, Weitzman, 1974, p. 477). Secondly, there are general arguments in favour of price. Compared with a quantity target, the use of price may be more practical and cheaper, with lower enforcement costs, when the number of consumers is large and (relaxing an assumption made earlier) have heterogeneous preferences. However, a reading of Weitzman (1974, p. 479) indicates that caution is appropriate. For further support for the use of prices see Kaplow and Shavell (2002).

7. Comparative static analysis

We may use standard comparative static analysis to explore the effects of varying the parameters: in particular θ, but also p_d and q. Most of the required analysis is fairly technical: much of it is set out in Appendix C., and only a summary is presented in this section.

The comparative statics are most easily handled by reducing the dimension of the green consumer's problem. As it stands there are four instruments (w, x_d, x_c, R) and two constraints, which generate a 6×6 bordered Hessian matrix. However, from the constraints we may write
$R = g(x_d) - w$ and $x_c = [m - p_d\, x_d - C(g(x_d) - w) - qw]/p_c \equiv \zeta(w, x_d)/p_c$,
and then we have a simpler unconstrained problem of maximizing
$u(E, x_d, x_c) \equiv u\big(f(\theta w + \overline{W}), x_d, \zeta(w, x_d)/p_c\big)$, with respect to two instruments w and x_d. The Hessian matrix $\boldsymbol{H}$ derived from the two first-order conditions is now 2×2. From quasi-concavity of u, we have $H_{dd} < 0$ and also, assuming f'' to be negligible, $H_{ww} < 0$.[15]

The sign of H_{wd} depends on the compensated cross-price effects between E and d. The case being analysed has two goods and one bad. In the absence of c the compensated cross-price

[15] We must also appeal to assumptions made about the signs of certain other functions and parameters: see Appendix C. The condition on f'' is only needed if $f''<0$.

effects between E and d would have unambiguous signs (see Appendix C. and in particular Figure C.1.), and the sign of H_{wd} would be known. If we assume that the signs of the compensated cross-price effects are the same when the total number of goods and bads N=3 as when N=2, then $H_{wd}>0$. Finally, we assume that the second-order conditions for a maximum are satisfied so that $|\boldsymbol{H}|>0$ in the reduced optimizing problem.

The expression from which we derive the comparative static results is

$$(21)\qquad \begin{bmatrix} H_{ww} & H_{wd} \\ H_{wd} & H_{dd} \end{bmatrix} \begin{bmatrix} dw \\ dx_d \end{bmatrix} = \begin{bmatrix} r_w^{\varphi} \\ r_d^{\varphi} \end{bmatrix} d\varphi = \boldsymbol{r}^{\varphi} d\varphi$$

where φ denotes one of the parameters θ, q and p_d, and the vector $\boldsymbol{r}^{\varphi}$ depends on which one of these is to be varied. We obtain expressions for dw and dx_d as $\frac{|\boldsymbol{H}_i|}{|\boldsymbol{H}|}d\varphi$, i=1, 2, respectively, where $\boldsymbol{H}_i$ is the matrix formed by replacing the ith column of $\boldsymbol{H}$ with the vector $\boldsymbol{r}^{\varphi}$.

For $\varphi = \theta$, and writing $\mathcal{M}_{ij}$ for the absolute value of the marginal rate of substitution, we have the following, where the Φs are positive functions:

$r_w^{\theta} = \Phi_1 \frac{\partial \mathcal{M}_{cE}}{\partial E} + \Phi_2$, neglecting a term in f'', and $r_d^{\theta} = -\,\Phi_3 \frac{\partial \mathcal{M}_{cd}}{\partial E}$,

For variations in q and p_d we have right-hand-side vectors $\boldsymbol{r}^{\varphi}$ with components:

for $\varphi = q$: $\quad r_w^{q} = -\,\Phi_4 \frac{\partial \mathcal{M}_{cE}}{\partial x_c} + \Phi_5$ and $r_d^{q} = \Phi_6 \frac{\partial \mathcal{M}_{cd}}{\partial x_c}$;

for $\varphi = p_d$: $\quad r_w^{d} = -\,\Phi_7 \frac{\partial \mathcal{M}_{cE}}{\partial x_c}$ $\quad$ and $r_d^{d} = \Phi_8 \frac{\partial \mathcal{M}_{cd}}{\partial x_c} + \Phi_5$.

The signs of these depend on the signs of the partial derivatives of the $\mathcal{M}_{ij}$, all the Φ terms being positive. A discussion of this is set out in Appendix C., where it is argued that it is reasonable to assume that $\frac{\partial \mathcal{M}_{cE}}{\partial E}>0$, $\frac{\partial \mathcal{M}_{cd}}{\partial E} \leq 0$, $\frac{\partial \mathcal{M}_{cE}}{\partial x_c}<0$ and $\frac{\partial \mathcal{M}_{cd}}{\partial x_c}>0$. In that case $r_i^{\varphi}>0$, all $\varphi=\theta$, q, p_d and $i=w$, d, except that $r_d^{\theta} \geq 0$.

We may now explore the differentials for the comparative static effects:

$$(22)\qquad dw = \frac{(H_{dd}\, r_w^{\varphi} - H_{wd}\, r_d^{\varphi})}{|\boldsymbol{H}|}\, d\varphi \qquad \text{and} \qquad dx_d = \frac{(H_{ww}\, r_d^{\varphi} - H_{wd}\, r_w^{\varphi})}{|\boldsymbol{H}|}\, d\varphi$$

for φ specified to be θ or q or p_d. Given our assumptions about the nature of compensated substitution between E and d, and about the reactions of marginal rates of substitution to *ceteris paribus* changes in consumption levels of individual goods, the sign pattern in equation (21) is

$$\begin{bmatrix} - & + \\ + & - \end{bmatrix} \begin{bmatrix} dw \\ dx_d \end{bmatrix} = \begin{bmatrix} + \\ + \end{bmatrix} d\varphi$$ except for the possibility that $r_d^{\theta} = 0$. We conclude from (22) that changes in θ, q or p_d, *ceteris paribus*, all have unambiguously negative impacts on the consumption of the dirty good and the output of unrecycled waste. The 'greener' the consumer is, and the higher the prices that he faces for waste-disposal and for the dirty good, the cleaner will his behaviour be. The impact on recycling levels cannot be established at this level of generality: a movement in either direction is consistent with the predictable movements of w and x_d in response to parameter changes.

8. Concluding remarks

Detailed conclusions have been presented already, in particular in Section 6.4, so these final remarks will be confined to observations concerning the most important results. The contribution of this paper is to bring out the significance of the fact that many individuals reveal a concern for the environment even when pecuniary incentives are absent. A crucial element is the parameter θ, which measures the extent to which the consumer recognizes his own responsibility for the externality; with full recognition by the consumer of this and also of the fact that he is representative, then $\theta = n$ and we have the extreme case in which expectations are said to be rational. By allowing the consumer to exhibit some recognition of his responsibility for the consequences of his actions (i.e. letting $\theta > 0$), we explain the phenomenon of voluntary recycling even when waste discharge is unpriced. This result is interesting in itself, and it also provides a support for the assumption that $\theta > 0$. Changes in θ or the price of waste q or the price p_d of the dirty good all have an unambiguously negative effect on consumption x_d of the dirty good and the discharge w of unrecycled waste, under reasonable assumptions concerning preferences. Thus, the 'greener' the consumer is, and the higher the prices that he faces, the cleaner will be his behaviour.

The two-stage process will achieve the benchmark first-best outcome in the extreme case of rational expectations, provided that unrestricted income taxes and transfers are possible. Otherwise, the outcome will be second-best unless $\theta = 0$ and the externality is weakly separable from the set of consumption goods in the utility function. When $\theta = 1$ the consumer is 'socially responsible' in that he recognizes his own contribution to the externality, without recognizing his representativeness. As the number of consumers increases, optimal pricing of waste in this case converges to that for θ=0.

The first-best optimum involves marginal recycling costs C' exceeding marginal waste-disposal costs D' by the marginal disutility of the externality (in absolute value), aggregated over consumers, that is, the absolute amount $-\frac{nu_E f' p_c}{u_c}$. This condition also appears in the two-stage procedure with slack tax/transfer constraints; the optimal price of waste q then lies between D' and C' and exceeds D' by the uninternalized portion of the marginal disutility of the externality, aggregated over consumers: i.e. $-\frac{n-\theta}{n}\frac{nu_E f'}{\lambda_1}$, which is the Pigouvian tax. In the case θ=n, the tax should be zero, with waste priced at marginal disposal cost D'. This provides a general prescription for policy, i.e. that the Pigouvian tax should be inversely related to the extent to which consumers accept their own responsibility for the externality. With a binding constraint on taxes and transfers, the condition $C'-D' = -\frac{nu_E f'}{\lambda_1}$ is replaced by an inequality, and it is no longer the case in general that the (second-best) optimal value of q exceeds D' by the uninternalized portion of the marginal disutility of the externality.

The one-stage planning model of Section 3.2 is to be seen as a benchmark, and not as the basis of practical policy-implementation. However, moving closer to practicality, the two-stage procedure still places considerable demands on the policymaker, in terms of the assembly and utilization of information concerning consumers' responses to price signals. A further development of this research might explore how the policymaker might develop optimal values of policy instruments through successive approximations in an iterative process. It could also explore how the consumer might revise unfulfilled expectations of the level of the externality, this being an important question when the externality is not separable from other goods in the utility function.

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Appendix A. The modified indirect utility function when *w* and *R* are positive

Let $V(q, p_d, p_d, m)$ be the indirect utility function derived from individual optimization, where the Lagrangian is $L=u\big(f(\theta w+\overline{W}), x_d, x_c\big)+\lambda_1[m-p_dx_d-p_cx_c-C(R)-qw]+\lambda_2[R+w-g(x_d)]$.

By Roy's identity, $\frac{\partial V}{\partial q}=-\frac{\partial V}{\partial m}w(q, p_d, p_c, m)$. This takes account of the dependence of maximized utility on θw. However, from the policymaker's perspective, the utility function is $u\big(f(nw), x_d, x_c\big)$. Define $\hat{V}=u\big(f(nw(q, p_d, p_c, m)), x_d(q, p_d, p_c, m), x_c(q, p_d, p_c, m)\big)$ where w, x_d and x_c satisfy the individual first-order conditions for the Lagrangian set out above. Then

$$
\begin{aligned}
\frac{\partial \hat{V}}{\partial q} &= nu_Ef'\frac{\partial w}{\partial q}+u_d\frac{\partial x_d}{\partial q}+u_c\frac{\partial x_c}{\partial q}\\
&= ((n-\theta)+\theta)u_Ef'\frac{\partial w}{\partial q}+u_d\frac{\partial x_d}{\partial q}+u_c\frac{\partial x_c}{\partial q}\\
&= (n-\theta)u_Ef'\frac{\partial w}{\partial q}+\theta u_Ef'\frac{\partial w}{\partial q}+u_d\frac{\partial x_d}{\partial q}+u_c\frac{\partial x_c}{\partial q}\\
&= (n-\theta)u_Ef'\frac{\partial w}{\partial q}+(\lambda_1 q-\lambda_2)\frac{\partial w}{\partial q}+(\lambda_1 p_d+\lambda_2 g')\frac{\partial x_d}{\partial q}+\lambda_1 p_c\frac{\partial x_c}{\partial q}
\end{aligned}
$$

from the FOCs (7'), (8) and (9)). Using FOC (10) we then have

$$
\text{(A.1)}\quad \frac{\partial \hat{V}}{\partial q} = (n-\theta)u_Ef'\frac{\partial w}{\partial q}+\lambda_1[q-C']\frac{\partial w}{\partial q}+\lambda_1[p_d+C'g']\frac{\partial x_d}{\partial q}+\lambda_1 p_c\frac{\partial x_c}{\partial q}.
$$

We have assumed that all these FOCs hold as strict equalities.

Now differentiate across the constraints with respect to q:

$$
-p_d\frac{\partial x_d}{\partial q}-p_c\frac{\partial x_c}{\partial q}-C'\frac{\partial R}{\partial q}-q\frac{\partial w}{\partial q}-w=0 \text{ and } \frac{\partial R}{\partial q}+\frac{\partial w}{\partial q}-g'\frac{\partial x_d}{\partial q}=0.
$$

Multiplying the second by C' and adding the first to it,

$$
\begin{aligned}
-p_d\frac{\partial x_d}{\partial q}-p_c\frac{\partial x_c}{\partial q}-q\frac{\partial w}{\partial q}-w+C'[\frac{\partial w}{\partial q}-g'\frac{\partial x_d}{\partial q}] &= 0 \text{ or}\\
(p_d+C'g')\frac{\partial x_d}{\partial q}+p_c\frac{\partial x_c}{\partial q}+(q-C')\frac{\partial w}{\partial q} &= -w
\end{aligned}
$$

and then from (A.1) above, $\frac{\partial \hat{V}}{\partial q} = (n-\theta)u_Ef'\frac{\partial w}{\partial q}-\lambda_1 w$

In a similar fashion it may be shown that $\frac{\partial \hat{V}}{\partial m} = (n-\theta)u_Ef'\frac{\partial w}{\partial m}+\lambda_1$.

We briefly consider the cases of boundary solutions for w or R. If (7') is a strict inequality at the optimum, then w=0. In that case small changes in parameters will not move the solution off the boundary so that $\frac{\partial w}{\partial q}=\frac{\partial w}{\partial m}=0$. Then, $\frac{\partial \hat{V}}{\partial q}=\frac{\partial V}{\partial q}=0$ and $\frac{\partial \hat{V}}{\partial m}=\frac{\partial V}{\partial m}=\lambda_1$. If (10) is a strict inequality at the optimum, then R=0 and the expressions for $\frac{\partial \hat{V}}{\partial q}$ and $\frac{\partial \hat{V}}{\partial m}$ include additional terms, $(\lambda_1 C' - \lambda_2)(\frac{\partial w}{\partial q} - g'\frac{\partial x_d}{\partial q})$ and $(\lambda_1 C' - \lambda_2)(\frac{\partial w}{\partial m} - g'\frac{\partial x_d}{\partial m})$ respectively, where $\lambda_1 C' - \lambda_2 > 0$.

Appendix B. The identity of the constraint sets for the two- and one-level problems

For the two-level problem the constraint sets are defined:

– for the consumer, by: $m - p_d x_d - p_c x_c - C(R) - qw = 0$ and $R + w - g(x_d) = 0$,
  assuming binding constraints, which combined are $M - T - p_d x_d - p_c x_c - C(g(x_d) - w) - qw = 0$;
– for the policymaker, by: $nqw(q,m(T),...) + nT - D(nw(q,m(T),...)) = 0$.

Combining these gives the union of all possible constraint-sets for the consumer:

$$M - T - p_d x_d - p_c x_c - C(g(x_d) - w) - (D(nw)/n - T) = 0,$$

i.e. $$n(M - p_d x_d - p_c x_c - C(g(x_d) - w) - D(nw) = 0,$$

which defines the constraint-set for the one-stage problem.

Appendix C. Comparative static analysis

Having reduced the four-instrument two-constraint problem to an unconstrained one with two instruments (see Section 7.), we have a 2×2 Hessian $\boldsymbol{H}$, derived from the two FOCs, where

$$H_{ww} = (\theta^2 f'^2/u_c^2)[\, u_c^2 u_{EE} - 2\, u_c u_E u_{cE} + u_E^2 u_{cc}] + \theta^2 u_E f'' - u_c C''/p_c;$$

$$H_{wd} = H_{dw} = (\theta f'/u_c^2)[\, u_c^2 u_{Ed} - u_c u_d u_{cE} - u_c u_E u_{cd} + u_d u_E u_{cc}] + u_c C'' g'/p_c;$$

$$H_{dd} = (1/u_c^2)[\, u_c^2 u_{dd} - 2\, u_d u_c u_{cd} + u_d^2 u_{cc}] - (C'' g'^2 + C' g'') u_c / p_c.$$

When signing these, we use $u_E < 0$, $u_c/p_c > 0$, $C'' > 0$, $\theta f' \geq 0$, $g' > 0$, $C' > 0$, and $g'' \geq 0$.

If we define the bordered Hessian matrix relating to the utility function as

$$\boldsymbol{B} = \begin{bmatrix} 0 & \nabla u' \\ \nabla u & \nabla^2 u \end{bmatrix} = \begin{bmatrix} 0 & u_E & u_d & u_c \\ u_E & u_{EE} & u_{Ed} & u_{Ec} \\ u_d & u_{Ed} & u_{dd} & u_{dc} \\ u_c & u_{Ec} & u_{dc} & u_{cc} \end{bmatrix}$$

, then for the terms in [square] brackets in H_{ww}, H_{dd} and H_{wd} we have respectively:

in H_{ww}: $[\, u_c^2 u_{EE} - 2\, u_c u_E u_{Ec} + u_E^2 u_{cc}] = -B_{dd}$;

in H_{dd}: $[\, u_c^2 u_{dd} - 2\, u_d u_c u_{cd} + u_d^2 u_{cc}] = -B_{EE}$;

in H_{wd}: $[\, u_c^2 u_{Ed} - u_c u_d u_{cE} - u_c u_E u_{cd} + u_d u_E u_{cc}] = B_{Ed}$;

where B_{ij}, is the (signed) cofactor determinant of the element subscripted i, j in $\boldsymbol{B}$.

Because u is assumed quasi-concave, then that $B_{EE} \geq 0$ and $B_{dd} \geq 0$ (Berck and Sydsæter, 1991, p. 63) so that $H_{dd} < 0$, and if we assume that f'' is either nonnegative or negligible, $H_{ww} < 0$ also.

The sign of B_{Ed} depends on the compensated cross-price effects between E and d. If we conceptualize the standard problem of maximizing $u(.)$ subject to a budget constraint as reflecting a payment *to* the consumer of a price p_E per unit of E consumed, then we have $\frac{\partial x_d(\boldsymbol{p},u)}{\partial p_E} = -\frac{\partial E(\boldsymbol{p},u)}{\partial p_d} = -\lambda B_{Ed}/|\boldsymbol{B}|$ where λ is a Lagrange multiplier, assumed positive, and $|\boldsymbol{B}| <$ 0 for the total number of goods and bads N=3.[16] The modified version of Slutsky symmetry should be noted: the negative sign on $\frac{\partial E(\boldsymbol{p},u)}{\partial p_d}$ arises because E is a bad, and the consumer pays to avoid it or is paid to consume it. If N=2 and i=E, d, then it is clear from Figure C.1. that $\frac{\partial x_d(\boldsymbol{p},u)}{\partial p_E} > 0$ and $\frac{\partial E(\boldsymbol{p},u)}{\partial p_d} < 0$ (it is advisable to avoid the terminology of substitute and complement in this context). We assume that the partial derivatives have these signs respectively in the case N=3, in which case $-\lambda B_{Ed}/|\boldsymbol{B}| > 0$, so that $B_{Ed} > 0$, from which it follows that $H_{wd} > 0$.

Figure C.1. here – see page 31

Finally we derive and discuss the components of the vector $\boldsymbol{r}^{\varphi}$ (see equations (21) and (22)). For $\varphi = \theta$, we have

$$r_w^{\theta} = -\theta w f'^2[u_c u_{EE} - u_E u_{cE}]/u_c - u_E(f' + \theta w f'') = -\theta w f'^2[u_c u_{EE} - u_E u_{cE}]/u_c - u_E f'$$

$$= u_c \theta w f'^2 \frac{\partial \mathcal{M}_{cE}}{\partial E} - u_E f' = \Phi_1 \frac{\partial \mathcal{M}_{cE}}{\partial E} + \Phi_2,$$ neglecting the term in f'', and noting that the marginal rate of substitution $\mathcal{M}_{cE}$ is defined as $-u_E/u_c > 0$ (see note 13).

$$r_d^{\theta} = -wf'[u_c u_{dE} - u_d u_{cE}]/u_c = -u_c w f' \frac{\partial \mathcal{M}_{cd}}{\partial E} = -\Phi_3 \frac{\partial \mathcal{M}_{cd}}{\partial E}.$$

For variations in p_c and q we have:

$$r_w^{q} = \frac{w}{p_c u_c} \theta f'[u_c u_{Ec} - u_E u_{cc}] + \frac{u_c}{p_c} = -\left(\frac{u_c w}{p_c} \theta f'\right)\frac{\partial \mathcal{M}_{cE}}{\partial x_c} + \frac{u_c}{p_c} = -\Phi_4 \frac{\partial \mathcal{M}_{cE}}{\partial x_c} + \Phi_5;$$

$$r_d^{q} = \frac{w}{p_c u_c}[u_c u_{dc} - u_d u_{cc}] = \left(\frac{u_c w}{p_c}\right)\frac{\partial \mathcal{M}_{cd}}{\partial x_c} = \Phi_6 \frac{\partial \mathcal{M}_{cd}}{\partial x_c};$$

$$r_w^{d} = \frac{x_d}{p_c u_c} \theta f'[u_c u_{Ec} - u_E u_{cc}] = -\left(\frac{u_c x_d}{p_c} \theta f'\right)\frac{\partial \mathcal{M}_{cE}}{\partial x_c} = -\Phi_7 \frac{\partial \mathcal{M}_{cE}}{\partial x_c};$$

$$r_d^{d} = \frac{x_d}{p_c u_c}[u_c u_{dc} - u_d u_{cc}] + \frac{u_c}{p_c} = \left(\frac{u_c x_d}{p_c}\right)\frac{\partial \mathcal{M}_{cd}}{\partial x_c} + \frac{u_c}{p_c} = \Phi_8 \frac{\partial \mathcal{M}_{cd}}{\partial x_c} + \Phi_5.$$

[16] Along with the expressions for the cross-price effects, for each i it is the case that the own-price substitution effect for good i is a multiple of $B_{ii}/|\boldsymbol{B}|$. (Strict) quasi-concavity of u implies that $|\boldsymbol{B}| \leq 0$ for n=3, but the case $|\boldsymbol{B}|$=0 is clearly pathological for the values of these price effects so we assume that $|\boldsymbol{B}| < 0$.

The signs of the r_i^{φ} terms depend on the signs of the partial derivatives of the $\mathcal{M}_{ij}$, all the Φ terms being positive.

As regards $\frac{\partial \mathcal{M}_{cE}}{\partial E}$ we assume that as E rises, making c relatively scarce, the more c one would want at the margin to compensate for a unit rise in E, so $\frac{\partial \mathcal{M}_{cE}}{\partial E}>0$. As regards $\frac{\partial \mathcal{M}_{cd}}{\partial E}$ one might assume that E is separable from the group $\{d, c\}$, so that $\frac{\partial \mathcal{M}_{cd}}{\partial E}=0$. Alternatively, the externality might have a strong locational connection with consuming d, rather than being widely dispersed: for example, driving on a congested motorway is less pleasant at the margin when the externality rises, so that one requires less of c at the margin to compensate for x_d, so $\frac{\partial \mathcal{M}_{cd}}{\partial E}<0$. As regards $\frac{\partial \mathcal{M}_{cE}}{\partial x_c}$, we assume that the larger is x_c, the less will be required of c at the margin to compensate for a unit of E, so $\frac{\partial \mathcal{M}_{cE}}{\partial x_c}<0$. Finally, as regards $\frac{\partial \mathcal{M}_{cd}}{\partial x_c}$ we assume that as x_c rises, and consumption of d becomes smaller relatively, then the more of c one would forego for a unit rise in d: $\frac{\partial \mathcal{M}_{cd}}{\partial x_c}>0$.

Making these assumptions about the signs of the partial derivatives of the $\mathcal{M}_{ij}$, we have $r_i^{\varphi}>0$, all $\varphi=\theta$, q, p_d and $i=w$, d, except that $r_d^{\theta} \geq 0$. Then from equation (22), for all φ we have $dw<0$ and $dx_d<0$. Even if $r_d^{\theta}=0$, these qualitative conclusions are unaltered.

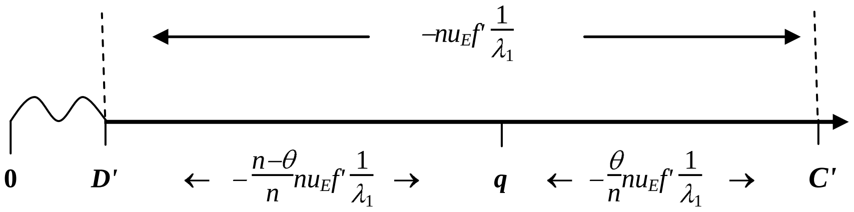
The signs of the partial derivative of the $\mathcal{M}_{ij}$ depend only on the underlying preference ordering, and not on the particular representation by a utility function, being invariant to a monotone increasing transformation of u. The assumptions that have been made concerning these signs are similar to assumptions about the signs of income or substitution effects. In the case N=2, there is a one-to-one relationship between income effects and the two expressions of the form $u_i u_{ji} - u_j u_{ii}$, because these are the cofactors B_1 and B_2. For example, in the case where d and c are the only goods, with no externality, there is an equivalence between the assumption that $\frac{\partial \mathcal{M}_{cd}}{\partial x_c}>0$ and the assumption that good d is normal with respect to income changes. For $N\geq3$, groups of such expressions turn up in a more complicated manner in the cofactors B_j, B_{ij}, i, j =1..., and consequently in the expressions for substitution and income effects.

Figures

Figure 1. Optimal configuration of marginal costs D' and C', and price q,

– *with* $w>0$ *and* $R>0$, *assuming* $u_E<0$;

– *case of slack tax-constraints.* [a]

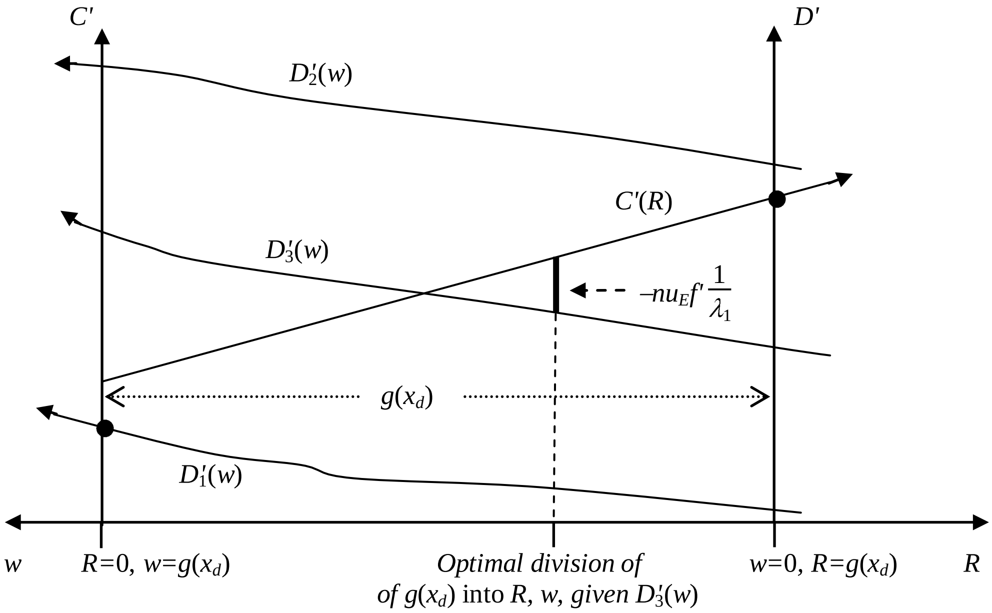


[a] so that $\mu_2=\mu_3=0$

Figure 2. Optimal disposal and recycling at a given value of x_d

– *for various functions* $D'(w)$ (*right axis*), *given* $C'(R)$ *(left axis)*;

– *case of slack tax constraints.* [a]



Given $D_1'(w)$, $R=0$: there will be no recycling, because $C'(0) > D_1'(g(x_d)) - nu_E f' \frac{1}{\lambda_1}$. [b]

Given $D_2'(w)$, $w=0$: there will be no disposal, because $D_2'(0) - nu_E f' \frac{1}{\lambda_1} > C'(g(x_d))$. [c]

Given $D_3'(w)$: optimally $w>0$, $R>0$ at the point indicated.

[a] so that $\mu_2=\mu_3=0$.

[b] For this case, a necessary but not sufficient condition is that $C'(R)$ should intersect the left-hand vertical axis above $D_1'(w)$. Here the vertical distance $C' - D_1'$ is assumed to exceed $-nu_E f' \frac{1}{\lambda_1}$.

[c] For this case, a sufficient but not necessary condition is that $D_2'(w)$ should intersect the right-hand vertical axis above $C'(R)$, as depicted.

Figure C.1. Indifference map for N=2: one good, one bad, assuming strictly convex preferences.