Inequality Measures, Equivalence Scales and Adjustment for Household Size and Composition

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Abstract

Total household income inequality can be very different from inequality measured at the income per-capita level but only in recent years has the pattern of this divergence been investigated. In this paper, results from Coulter et al. (1992) using a one-parameter equivalence scale are updated using data for Ireland, Italy, the UK and the US. A class of two-parameter equivalence scales, representing relative weights for adults and children, is then analysed. Results are shown to depend on the distribution of household size and composition among deciles of the population. Inequality generally increases with children's weight and decreases with adults' weight. OECD and other two-parameter equivalence scales empirically used show similar results to the one-parameter equivalence scale with elasticity around 0.5.

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Studies of income distribution are heavily affected by the different procedures that researchers can choose to measure inequality. One of the major issues involved here is to assess the direction and the extent of the change in inequality when different adjustments for household size and composition are allowed.[1] Until a few years ago, the relevant unit in inequality studies was chosen between total household income (H), household per-capita income (Y) and individual income. While individual income is the simplest unit of analysis, a better alternative is to consider H, as the household is the locus of decisions on income getting and income spending of individual members. An even better solution is considered Y, in which total household income is shared equally among the household members. Y was the preferred unit taken into consideration until a more precise welfare analysis has been brought about in recent years.

In fact, the analysis of Y rules out the possibility of different economies of scale in households of different size: the underlying assumption of the household per-capita income analysis is that the well-being of an individual sharing £20,000 in a two-person household is the same as the well-being of an individual sharing £40,000 in a four-person household. It seems more reasonable to postulate the existence of positive economies of scale within the household; hence, a consistent measure of individual well-being W can be written as in equation (1):

$$W = H / S^{\varepsilon} \quad \textbf{(1)}$$

where H is the sum of individual incomes in the household (total household income), S is household size and ε is a parameter representing economies of

[1] Other issues refer to i) the modification of extreme incomes (the procedure is known as bottom and top recoding); ii) the comparison across countries of definitions such as gross income, disposable income or household; iii) the indices used to assess inequality (indices are neither ordinally nor cardinally equivalent). On this last point, see Cowell (1995). See also Champernowne (1974), Figini (1997), Bigsten (1991) and

scale within the household. ε ranges from 0 (perfect economies of scale) to 1 (absence of economies of scale). Therefore, household income H (ε = 0) and household income per capita Y (ε = 1) are the two extreme cases of a welfare analysis in which the elasticity of scale ε plays a fundamental role.[2]

Buhmann et al. (1988) find that all the equivalence scales empirically used can be approximated by a single parameter scale as (1) and in recent years one-parameter scales have been directly used (Atkinson et al., 1995 measure inequality in OECD countries setting ε = 0.5). This evolution raises a few questions about the "best" value for ε to be used and about the pattern of inequality change when the value of ε varies. While the former problem invokes thinking about welfare assumptions and economies of scale within households, the latter issue has been tackled theoretically and empirically by Coulter et al. (1992): increasing the value of ε from 0 to 1, inequality first decreases and then increases, thus depicting a U-shape. These general findings are re-assessed in the present paper.

Yet, equation (1) is a simplification of a more general formula in which other household characteristics such as composition, location and age might be considered. This approach in adjusting for household characteristics can be represented in (2):

$$W = \frac{H}{(\alpha_1 N_1 + \alpha_2 N_2 + \ldots + \alpha_k N_k)^{\varepsilon}} \qquad \textbf{(2)}$$

where N_i is the size of each type *k* of components of the household (elderly people, adults, children…), α_ι is the relative weight given to them and ε

Sundrum (1990). On the technical aspects of the management of household surveys an excellent overview is provided in Atkinson et al. (1995).

[2] Actually, household income inequality is not technically equal to equation 1 with a parameter ε = 0 because of the different weighting procedure applied to the sample of data: in the former case we weight according to the number of households, in the latter to the number of individuals.

represents the economies of scale within the household. A particular sub-class of this formula will be analysed throughout the paper.

The rest of this paper is organised as follows: in the next section the Luxembourg Income Study (LIS) database, used in this work, is presented. In Section II, a comparison of inequality considering H and Y inequality, following Sundrum's analysis (Sundrum, 1990) is outlined. Section III follows the procedure of Coulter et al. (1992) comparing inequality using equation (1) in four different countries: Ireland, Italy, UK and US. In Section IV, a particular subclass of formula (2) is considered: a two-parameter equivalence scale, which distinguishes between the household head, other adults and children in the household. A weight of 1 for the household head and weights α_1 and α_2 ranging between 0 and 1 for other adults (N_1) and children (N_2) in the household are respectively used (equation 3).[3]

$$W = \frac{H}{1 + a_1 \mathrm{N}_1 + a_2 \mathrm{N}_2} . \qquad \textbf{(3)}$$

Section IV also provides a comparison between the different scales used while Section V concludes.

Section I: The data

Since the Luxembourg Income Study (LIS) project was founded in 1983, a huge step towards a better understanding of inequality and its measurement has been made. The project has four main goals: i) to create a database containing social and economic data collected in household surveys from different countries; ii) to provide a method allowing researchers to use the data under restrictions required by the countries providing the data; iii) to create a system to

[3] OECD scale is a particular case of equation (3), in which α_1 = 0.7 and α_2 = 0.5. Other scales often used, attach values of 0.6 or 0.5 to α_1 and 0.4 or 0.3 to α_2.

allow remote access and to elaborate data using computer networking and iv) to promote comparative studies on income aggregates. At this stage the LIS database includes about 70 observations for 25 countries, covering the period from 1967 to 1995. For almost each survey there are three different files, the first with data at the household level (allowing sometimes a disaggregation also among multi-family households), the second at the individual level and the third at the child level. One of the main issues in setting up such a database is to elaborate data from single surveys transforming variables and re-weighting single cases so as to allow a satisfactory international comparison. Of course, perfect comparability will never be reached but, at this stage, the LIS database allows a good degree of comparability among countries.[4]

Section II: Household Income and Household Per-capita Income Inequality

How much does inequality change moving from household income (H) to household per-capita income (Y) inequality? Is the change similar for all the indices? Is the change similar for all the countries?

The problem can be represented in the following way: household per-capita income is the ratio of household income over household size (S).

$$Y = H / S. \quad \textbf{(4)}$$

Considering logarithmic values and the coefficient of variation (CV) as a measure of inequality, Sundrum (1990) shows that $CV_Y < CV_H$ if:

$$2rCV_H CV_S > CV_S^2 \text{ or if } r > \frac{CV_S}{2CV_H} \quad \textbf{(5)}$$

[4] Technically, the preliminary stage of the research is to set up some "jobs" (SPSS commands) which are sent via e-mail to the server address in the LIS headquarters in Luxembourg. These jobs are automatically executed and the output is sent back to the original e-mail address in a few minutes. A complete documentation with description, frequencies and labels for each variable of the database is also available online (http://lissy.ceps.lu/) to allow researchers to overcome problems of definition and transformation that their own work can require. Technical assistance is always available from LIS staff.

where ρ is the coefficient of correlation between household size and household income. Since CV_S is usually smaller than CV_H, the right-hand side of equation (5) is sufficiently small compared to ρ if there is strong positive correlation between size and total household income (as there is in household data).[5] Therefore, in the generality of cases, Y inequality would be lower than H inequality. LIS database highlights this decrease in inequality moving from H to Y, as Table 1 shows. The 22 countries for which information on inequality in the period 1987-1992 is available are listed. For each country, three different measures of inequality (Gini, Theil and Atkinson with a sensitivity of 0.5) are computed. Table 1 shows that inequality generally decreases moving from H to Y. The only cases for which inequality increases are Israel, Italy, Poland and US (only for Gini and Theil indices).

Section III: Bringing in Economies of Scale within the Household

The procedure illustrated in the previous section is a necessary step to avoid distortions caused by the fact that rich households have a different size compared to poor households. Accordingly, two new elements of bias have been introduced into the analysis: i) the implicit assumption of no intra-household inequality: H is postulated to be evenly distributed among the components of the household. This is not always true, particularly in the case of multi-family households, but for the rest of the paper we will always rely on this hypothesis. ii) The assumption of not having economies of scale within the household. If two households, one composed of two individuals with a total income of £20,000 and another one of four components and £40,000 are considered equivalent, as

[5] The average household size of the bottom decile of the distribution of total household income is the lowest while for the top decile it is the highest for every country in the LIS database. The number of children increases until the 3rd-7th decile and then decreases and, as a result, also the number of economically active persons (computed as the number of household components minus children) increases with total household

equation (4) assumes, the possibility of having intra-household economies of scale is ruled out.

This latter assumption is now relaxed and households are adjusted in order to catch positive economies of scale as size increases. The Buhmann's scale (Buhmann et al., 1988), represented in equation 1, allows this possibility. Coulter et al. (1992), explain the theoretical relationship between equivalence scales and inequality: the well-being W_i of an individual is a function of four different variables, total household income (H), household size (S), elasticity of scale (ε) and household characteristics (η):

$$W_i = Y(H_i, S_i, \varepsilon_i, \eta_i). \quad \textbf{(6)}$$

The well-being reduces to (4) if household characteristics (such as location, age, health) are normalised and the elasticity of scale is set equal to 1. In this section the analysis is broadened by allowing the parameter ε, representing the intensity of economies of scale, to vary according to the Buhmann's scale. When ε is equal to 0, W reduces to H (perfect economies of scale are assumed); when ε is equal to 1, W reduces to Y as in (4) (economies of scale are ruled out and the well being of each individual is simply equal to household per-capita income). Neither of these two cases are realistic because in each household there are some relatively fixed expenditures that are shared among its components (rent, bills) and the scale of the sharing is a function of the household size. In other words, if there are economies of scale, an individual sharing £20,000 within a two-person household is worse off than an individual who shares £40,000 in a four-person household.

Buhmann et al. (1988) and Coulter et al. (1992) demonstrate that the movement between household income and household per-capita income

income. Data for all the countries are available from the author. Data for Ireland, Italy, UK and US are published in Table 5.

inequality is not linear (as it could be infered from the previous section) but involves a U-shape with respect to ε. Inequality first decreases moving from ε = 0 to a higher value and, from a certain stage up to 1, inequality increases. When households are ranked according to their total income, rich households are the largest ones. Therefore, when income is adjusted via the parameter ε, the denominator in equation (1) increases respectively more than the numerator, thus having an equalising effect on the distribution. But, for high values of ε, the re-ranking process acts to counter-balance this change in inequality: by increasing ε, the possibility of a re-ranking of units in the distribution augments. The total effect on income inequality would depend on the strenght of the two effects. For low values of ε, the re-ranking effect is not strong enough to reverse the equalising effect but, for higher ε, the re-ranking will be strong enough to lead to an increase in inequality. This process can be understood by looking at the example outlined in Table 2.

Using variance as a measure of inequality, the introduction of the parameter ε can be represented as follows:

$$VAR\ (w) = VAR\ (h) + VAR\ (s^{\varepsilon}) - 2\rho\varepsilon\ (VAR\ (h)\ VAR\ (s))^{0.5}. \qquad \textbf{(7)}$$

An increase in ε widens the gap between H and W inequality. But above a threshold level, the rise in ε implies a more likely re-ranking of the households causing an overall decrease in the gap. The result is a composite effect depicted by a U-pattern of inequality.

The above model has been tested by Cowell et al. (1992) on different indices of the General Entropy Measures family (GEM), (including Theil, Atkinson and Coefficient of Variation) and on the Gini index: data from the UK confirmed the U-shape in inequality with respect to ε. They also find a different skewness of the U curve for different indices: keeping everything else constant, indices more sensitive to inequality among high-incomes (such as the Coefficient of Variation) show a U curve skewed to the left, more similar to a J-shape. Indices more sensitive to inequality among low-incomes (as Atkinson) show a U curve more skewed to the right, more similar to an inverted J-curve (for the explanation, see Coulter et al., 1992, p. 1073).

In this paper, LIS data for the UK 1991, the US 1991, Ireland 1987 and Italy 1991 are used. For each country Gini, CV, Theil and Atkinson(0.5) indices are computed for both person and household weighting.[6] The main results of the analysis are listed as follows and the patterns of inequality change are recalled in Figures 1 and 2 (UK 1991), 3 and 4 (US 1991), 5 and 6 (Ireland 1987) and 7 and 8 (Italy 1991).

i) The U shape holds for all the countries and all the indices: inequality is a non-linear function of ε.

ii) Contrary to the remark of Coulter et al. (Coulter et al., 1992, note 12, p.1077) the choice of whether to weight according to the number of individuals (PP curves in the figures) or households (HH curves in the figures) affects the robustness of the results. Coulter et al. use household weights, finding that the McClements equivalence scale used by the British Institute for Fiscal Studies (ε ≅ 0.6) actually minimises the extent of inequality. Using person weights, which seem more appropriate in measuring individual well-being, the minimum of inequality is generally reached at a lower value of ε. For this reason, the U-curve depicted using person weights becomes more skewed to the left than the curve drawn using household weights: the J-shape becomes more evident.

iii) The difference between PP and HH curves is not constant. For some countries, namely the UK, the US and Ireland, the difference between PP and

[6] To have results that are significant for the whole population, single cases from the sample have to be weighted. When H income (ε = 0) is measured, it seems appropriate to weight according to the number of households (HH). With other values of the parameter, since individual well-being, is analysed, it seems more appropriate to weight according to the number of individuals (PP). Here both possibilities are considered.

HH is minimised considering per-capita income (ε = 1) but this is not the only possibility. For Italy the gap lowers until the point of minimum inequality (ε = 0.5) and then it goes up again.

iv) The values of ε for which inequality is minimised are shown in Table 3. Minima for PP are 1-2 tenth of ε lower than HH minima. Gini has the highest minimum (ε is around 0.5/0.6 for HH) together with Atkinson. Theil is minimised for ε around 0.4/0.5 and CV is minimised for ε around 0.4, thus confirming the theoretical discussion by Coulter et al.: patterns for those indices which are more sensitive to high income inequality are more skewed to the left than indices which are more sensitive to low income inequality. For PP weights, the minimum moves down to ε around 0.3 for CV and ε around 0.5 for Atkinson and Gini.

v) The shape of the curves also depends on the country, with the US and Italy being more skewed to the left than the UK and Ireland. Countries with a higher inequality in household size distribution (UK and Ireland) are more likely to have a U-shape skewed to the right while countries with lower inequality in household size distribution (U.S. and Italy) seem to have a U-shape more skewed to the left (see Table 5).

In conclusion, the use of a particular value of elasticity is fundamental in assessing not only the absolute level of inequality and its change, but also the ranking between countries. In fact, when the extent of inequality is similar, as it is in the case of Ireland, the UK and the US, the ranking can be affected. The ranking of these three countries, given the most usual assumptions (person weighting and ε = 0.5) is shown, according to the Gini coefficient, in the second column of Table 4. But using different assumptions (household weighting and ε = 1), as in the third column of the same table, the ranking changes. The same picture, using the Atkinson index and person weighting, is represented in Figure

9. The UK starts as the most unequal country for $\varepsilon = 0$ and becomes the most equal for $\varepsilon = 1$.

Section IV: An Analysis with a Two-Parameter Equivalence Scale

A more precise way to adjust for household characteristics is to measure individual welfare not only with respect to income and size, but also with respect to the number of earners, children and elderly people in the household, as in equation (2). An example of such an equivalence scale is the OECD scale (8):

$$W = H / (1 + 0.7(N_\alpha - 1) + 0.5N_c) \qquad \textbf{(8)}$$

where N_α and N_c are the number of adults and children respectively and a weight of 1 is attached to the household head. Using the same procedure followed for the analysis of one-parameter equivalence scale, the fundamental question that will be tackled in this section is: how do measures of inequality change when parameters α_1 and α_2 in equation 2 vary between 0 and 1?

Considering the variance as a measure of inequality, and the variables in logarithmic terms, we have that:

$$VAR(w) = VAR(h) + VAR \log(1+a_1S_1+a_2S_2) - 2COV(h, \log(1+a_1S_1+a_2S_2)) \qquad \textbf{(9)}$$

with the chage in VAR(w) depending on the distribution of adults and children and on their correlation with household income. To be more precise, the class of Generalised Entropy Measures (*I*) is considered:[7]

$$I = \frac{1}{q(q-1)N}\left[\sum_i \left(\frac{W_i}{\overline{W}}\right)^q - 1\right] \qquad \textbf{(10)}$$

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[7] When $\theta = 1$, *I* is equivalent to the Theil index; when $\theta = 2$ an index cardinally equivalent to the Herfindal index is obtained, when $\theta = 3$ the index is ordinally equivalent to the Coefficient of Variation and when $\theta = 1 - \phi$, *I* is ordinally equivalent to the class of Atkinson indices with parameter ϕ. See Cowell (1995) and Figini (1998).

where θ is a parameter determining the aversion to inequality, N is the number of observations, W_i is the well being of the individual-i and $\overline{W}$ is the average of the measure. W_i is calculated according to (11):

$$W_i = \frac{H_i}{1 + \alpha_1 S_1^i + \alpha_2 S_2^i} \qquad \textbf{(11)}$$

where H_i is total household income, S_1 is the number of adults in the household minus the head, S_2 is the number of children in the household, the weight of the household head is set equal to 1, and α_1 and α_2 are the weights of, respectively, other adults and children.

The differentiation of GEM with respect to changes in α_1 is shown in (12). A similar formula holds for changes in α_2, the only difference being the substitution of S_1^i and S_1^j with S_2^i and S_2^j in the numerator of M.

$$\frac{\partial I}{\partial \alpha_1} = \frac{1}{(\theta - 1)N} \sum_i \left(\frac{W_i}{\overline{W}} \right)^{\theta - 1} M \qquad \textbf{(12)}$$

where M is:

$$M = \frac{\dfrac{\overline{W} S_1^i W_i}{1 + \alpha_1 S_1^i + \alpha_2 S_2^i} - \dfrac{W_i}{N} \sum_j \dfrac{Y_j S_1^j}{\left(1 + \alpha_1 S_1^j + \alpha_2 S_2^j\right)^2}}{\overline{W}^2} . \qquad \textbf{(13)}$$

This formula can be read in this way. The change in inequality depends on θ and on the sign of M which, in turn, depends on the values of S_1 and S_2 in each household. Theoretically any sign can result and empirically this would depend on the type of distribution of adults and children among households. Table 5 shows that the number of adults generally increases along the distribution of total household income. Given a certain weight α_2, an increase in the weight given to the number of adults raises the denominator of equation (11), thus implying an equalising effect. Table 5 also shows that the distribution of children is more heterogeneous. In the four countries under consideration, the number of children increases up to the 3rd decile (Ireland and Italy), to the 5th (US) and to the 7th (UK), declining thereafter. Given α_1, an increase in the weight given to children has instead a disequalising effect because rich households have, generally, less children. The total effect, increasing at the same time α_1 and α_2, is less clear and depends also on the re-ranking effect, the absolute values of S_1 and S_2 and the value of parameter θ in GEM.

Empirically, the complexity of the situation and the possibility of contrasting results is fortunately reduced. Given the similar pattern in adults and children distributions among countries, a few stylised facts can be highlighted (see also figures from 10 to 13).

i) When α_1 is held fixed, inequality increases with α_2; for high values of α_1, however, a J-shape appears with inequality decreasing at low values of α_2.

ii) When α_2 is held fixed, inequality decreases with increases in α_1; for low values of α_2, however, an inverted J-shape appears with inequality increasing at high values of α_1.

iii) When the two weights vary together, the overall trend is depicted by an inclined surface with highest inequality for low values of α_1 and high values of α_2 and lowest inequality for high values of α_1 and low values of α_2. Increasing both weights together we obtain a very slight U shape. Inequality is more sensitive to changes in children's weights than to changes in adults' weights.

iv) Particular scales have remarkable importance: the OECD scale (α_1 = 0.7 and α_2 = 0.5) and other two scales in which weights are α_1 = 0.6 and α_2 = 0.4 and α_1 = 0.5 and α_2 = 0.3 respectively. Among two-parameter scales, their values are in the middle of the range, with OECD showing the highest value of inequality of the three. Compared to one-parameter scales, Table 6 shows that they are very close to the value that we would get using (1) with ε around 0.5-0.6.

Section V: Concluding Remarks about Inequality Comparisons

The extent of inequality is heavily dependent on the type of adjustment for household size and composition and on the value of the parameter used to describe economies of scale within the household. Few remarks can be outlined.

i) Among one-parameter scales, each country and each index has a peculiar way of reacting to the choice of the elasticity of scale. Inequality has a U-shape with respect to the value of ε, but the skewness of the curve, the difference between two alternative weighting procedures (person and household weights) and the value of ε which minimises inequality vary considerably.

ii) These multiple variations affect quite heavily the robustness of inequality measurement. When one single country is analysed, the range of values that the index can have might be larger than changes in "real" inequality: in Table 4, between the highest and the lowest value of Gini for Ireland 1987 there is a difference of 12% which is larger than any "real" change of inequality experienced by that country over time. More important is the fact that the ranking of countries in the "inequality league" can be affected by the parameter chosen.

iii) Using two-parameter equivalence scales, an increase in the weight of the adults decreases inequality while an increase in the weight of children increases inequality very heavily. For extreme values of the weights (high α_1 and low α_2) the change in the other weight can provoke a J or an inverted J-shape. These results are not a "law" but stylised facts due to the particular pattern of distribution of adults and children among households ranked with respect to income. When the two weights are increased together both a J-shape and a linear increase in the measure of inequality can appear.

iv) The two-parameter scales empirically used are very close to the one-parameter scale with a value of ε around 0.5-0.6. OECD scale slightly over-measures inequality compared to one-parameter with $\varepsilon = 0.5$.

v) Also for two-parameter scales, the robustness of the results depends on the index used, the country under examination and the distribution of children and adults within each household. In theory a broad spectrum of heterogeneous results might appear.

vi) Empirically, inequality as measured using the most common equivalence scales does not change considerably. This is not due to an intrinsic robustness of results but is the consequence of the fact that equivalence scales empirically used have very similar underlying assumptions. On the other hand, the use of odd scales (e.g., $\alpha_1 = 0$ and $\alpha_2 = 1$) can produce very particular results. This general conclusion is twinned to another similar conclusion regarding the choice of the index. The most common indices of inequality produce very similar results because they have very similar aversions to inequality. Particular aversions to inequality (as for the Coefficient of variation or for the Atkinson (ε=2) index) produce peculiar measures of inequality.[8]

vii) While we need to be aware of the sensitivity of results to changes in the equivalence scale, in empirical studies there is a tendency towards using ε = 0.5. Yet, a comparison of well-being between countries should allow ε to take different values for each country in order to catch in a more precise way the country's peculiarity in terms of household structure and within-household economies of scale. As we can easily figure out, this might have disruptive

[8] For a study of how inequality changes changing the indices used, see Figini (1998).

consequences on the way in which inequality is measured but, without any doubt, further research is needed in this area.

Table 1: inequality considering different recipient units

Country	Unit	Gini	Theil	Atkinson
AUSTRALIA '89	H	0.354	0.210	0.107
AUSTRALIA '89	Y	0.333	0.191	0.093
BELGIUM '92	H	0.301	0.150	0.081
BELGIUM '92	Y	0.251	0.108	0.057
CANADA '91	H	0.339	0.192	0.097
CANADA '91	Y	0.312	0.167	0.081
CZECH REPUBLIC '92	H	0.297	0.152	0.073
CZECH REPUBLIC '92	Y	0.210	0.086	0.039
DENMARK '92	H	0.342	0.201	0.103
DENMARK '92	Y	0.248	0.120	0.059
FINLAND '91	H	0.313	0.158	0.081
FINLAND '91	Y	0.256	0.114	0.054
FRANCE '89	H	0.390	0.272	0.145
FRANCE '89	Y	0.380	0.263	0.134
HUNGARY '91	H	0.364	0.229	0.115
HUNGARY '91	Y	0.294	0.165	0.081
IRELAND '87	H	0.381	0.252	0.124
IRELAND '87	Y	0.359	0.234	0.111
ISRAEL '92	H	*0.347*	*0.199*	*0.098*
ISRAEL '92	Y	*0.355*	*0.222*	*0.102*
ITALY '91	H	0.330	0.182	0.091
ITALY '91	Y	0.313	0.172	0.082
NETHERLANDS '91	H	0.325	0.191	0.097
NETHERLANDS '91	Y	0.316	0.187	0.091
NORWAY '91	H	0.333	0.189	0.095
NORWAY '91	Y	0.253	0.114	0.055
POLAND '92	H	*0.323*	*0.177*	*0.086*
POLAND '92	Y	*0.326*	*0.184*	*0.088*
RUSSIA '92	H	0.501	0.631	0.230
RUSSIA '92	Y	0.440	0.550	0.187
SLOVAKIA '92	H	0.285	0.135	0.067
SLOVAKIA '92	Y	0.202	0.074	0.035
SPAIN '90	H	0.349	0.211	0.102
SPAIN '90	Y	0.326	0.194	0.091
SWEDEN '92	H	0.329	0.178	0.091
SWEDEN '92	Y	0.251	0.108	0.054
SWITZERLAND '82	H	0.385	0.308	0.137
SWITZERLAND '82	Y	0.361	0.274	0.117
TAIWAN '91	H	0.338	0.203	0.096
TAIWAN '91	Y	0.322	0.194	0.086
UK '91	H	0.389	0.271	0.127
UK '91	Y	0.362	0.245	0.111
USA '91	H	*0.372*	*0.227*	0.117
USA '91	Y	*0.374*	*0.235*	0.117

Notes: H = total household income; Y = household per-capita income. Household inequality measured weighting for the number of households. In italics cases where inequality increases moving from H to Y.

Table 2: extent of inequality (coefficient of variation) of a sample distribution when different adjustments for household size are made

Indiv. component	Individual income when different parameters are considered			
	ε = 0	**ε = 0.3**	**ε = 0.6**	**ε = 1**
1, A	10	10	10	10
2, A	18	14.62	11.88	9
2, B	18	14.62	11.88	9
3, A	26	18.70	13.45	8.67
3, B	26	18.70	13.45	8.67
3, C	26	18.70	13.45	8.67
4, A	35	21.60	13.33	7
4, B	35	21.60	13.33	7
4, C	35	21.60	13.33	7
4, D	35	21.60	13.33	7
4, E	35	21.60	13.33	7
Mean income of the sample	27.181	18.485	12.796	8.092
Coeff. of Var.	**0.324**	**0.210**	**0.086**	**0.137**

Notes: Income distribution adjusted for household size (person weighting). Number of households: 4. Total household income: [10, 18, 26, 35]. Household size: [1, 2, 3, 5]. Households are numbered while individuals are represented by letters. In the first column each individual is named after his/her belonging to one of the four households. In the following columns adjusted income is computed according to the formula:

$$Y = \text{Total Household Income} / \text{Household size}^{\varepsilon}$$

for values of ε respectively of 0, 0.3, 0.6, 1. Inequality decreases from column 2 to 3 and to 4 because of the equalising effect due to increasing ε. Re-ranking effect, which starts in column 3 (members of household Nr. 4 become poorer than members of household 3), becomes more evident in column 4. Its disequalising effect becomes stronger in column 4 driving the overall inequality, as the coefficient of variation shows, up again.

Table 3: values of ε for which inequality is minimised for different choices of country, index and weighting procedure

	Gini_{HH}	Gini_{PP}	CV_{HH}	CV_{PP}	Theil_{HH}	Theil_{PP}	Atk_{HH}	Atk_{PP}
Ireland	0.6	0.4/0.5	0.5	0.3	0.5/0.6	0.4/0.5	0.5/0.6	0.4/0.5
Italy	0.5	0.4/0.5	0.4	0.4	0.4/0.5	0.4	0.5	0.4/0.5
UK	0.6/0.7	0.4/0.5	0.4	0	0.6	0.4/0.5	0.6/0.7	0.4/0.5
US	0.5	0.3/0.4	0.4	0.3	0.4/0.5	0.3/0.4	0.5	0.3/0.4/0.5

Table 4 - Inequality ranking using different assumptions

	Gini$_{PP}$, ε = 0.5	**Gini$_{HH}$, ε = 1**
UK	0.341 (1)	0.363 (3)
US	0.337 (2)	0.364 (2)
Ireland	0.330 (3)	0.375 (1)

Notes: in column 2, Gini is computed using person weights and ε is set equal to 0.5. In column 3, Gini is computed using household weights and ε is set equal to 1.

Table 5: average household size and composition in selected countries

	Ireland (1987)			Italy (1991)			United Kingdom (1991)			United States (1991)		
Deciles	size	adults	childrn	size	adults	childrn	size	adults	childrn	size	adults	childrn
1	2.83	1.74	1.09	2.61	2.02	.59	1.84	1.39	.45	2.46	1.51	.95
2	3.70	2.03	1.67	3.21	2.28	.93	2.42	1.68	.74	2.97	1.82	1.15
3	4.74	2.15	2.59	3.44	2.41	1.03	2.88	1.84	1.04	3.31	1.98	1.33
4	4.74	2.28	2.46	3.54	2.55	.99	3.14	1.97	1.17	3.33	2.09	1.24
5	4.67	2.39	2.28	3.61	2.68	.93	3.24	2.07	1.17	3.61	2.23	1.38
6	4.87	2.64	2.23	3.69	2.79	.90	3.39	2.19	1.20	3.59	2.24	1.35
7	5.15	2.83	2.32	3.75	2.89	.86	3.55	2.31	1.24	3.56	2.36	1.20
8	5.14	2.85	2.29	3.91	3.03	.88	3.42	2.40	1.02	3.66	2.46	1.20
9	5.36	3.37	1.99	4.13	3.29	.84	3.36	2.47	.89	3.80	2.61	1.19
10	5.49	3.90	1.59	4.13	3.48	.65	3.73	2.72	1.01	4.08	2.93	1.15
CV	0.175	0.249	0.227	0.126	0.165	0.162	0.186	0.189	0.246	0.132	0.182	0.102

Notes: average size, number of adults and number of children in each decile of the population. A measure of inequality CV (coefficient of variation) for the variables is calculated in the last row.

Table 6: comparison between different equivalent scales

	One parameter equivalence scale			Two parameter equivalence scale		
	Household	ε = 0.5	H per-cap.	OECD	α_1=.6 α_2=.4	α_1=.5 α_2=.3
Ireland						
CV	0.730	0.716	0.815	0.735	0.721	0.705
Theil	0.207	0.193	0.234	0.198	0.192	0.185
Atk 0.5	0.103	0.094	0.111	0.095	0.092	0.090
Atk 2.0	0.348	0.339	0.399	0.351	0.342	0.332
Gini	0.345	0.330	0.359	0.331	0.326	0.321
Italy						
CV	0.605	0.586	0.679	0.611	0.597	0.584
Theil	0.158	0.144	0.172	0.149	0.145	0.141
Atk 0.5	0.078	0.071	0.082	0.073	0.071	0.069
Atk 2.0	0.268	0.256	0.316	0.272	0.263	0.254
Gini	0.307	0.291	0.313	0.293	0.290	0.287
UK						
CV	0.841	0.871	0.973	0.901	0.886	0.869
Theil	0.229	0.214	0.245	0.216	0.212	0.208
Atk 0.5	0.109	0.099	0.111	0.099	0.097	0.096
Atk 2.0	0.414	0.432	0.486	0.448	0.440	0.430
Gini	0.357	0.341	0.362	0.340	0.337	0.334
US						
CV	0.643	0.638	0.742	0.666	0.651	0.636
Theil	0.195	0.188	0.235	0.198	0.192	0.185
Atk 0.5	0.101	0.096	0.117	0.101	0.098	0.095
Atk 2.0	0.293	0.290	0.355	0.307	0.298	0.288
Gini	0.345	0.337	0.374	0.346	0.340	0.335

Notes: person weights have been used in all the computations.

Fig. 1: Gini index, the UK 1991, for both person and household w eighting

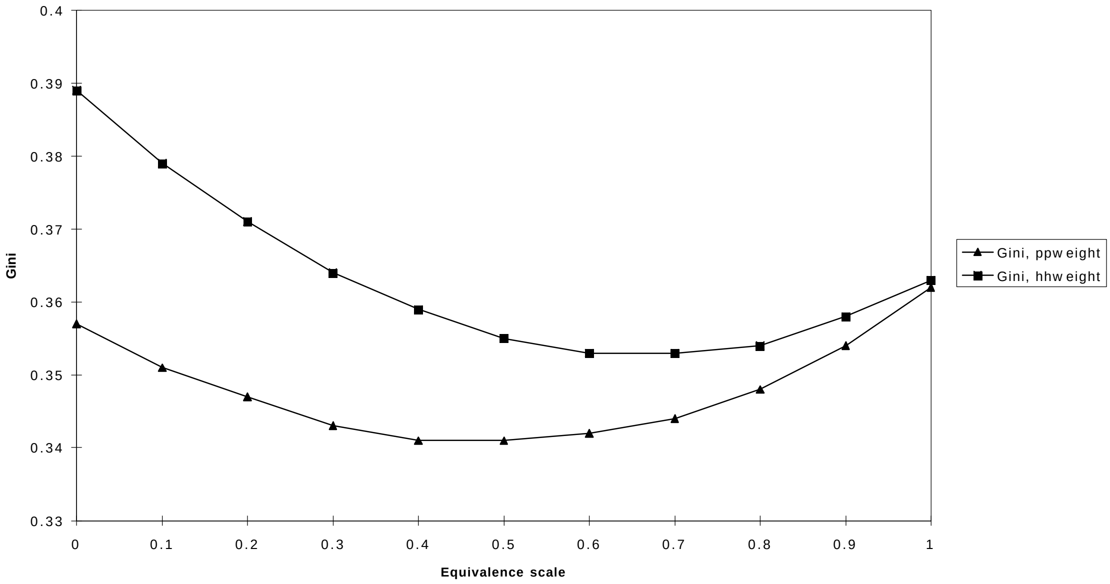


Fig. 2: Coefficient of Variation, the UK 1991, for both person and household w eighting

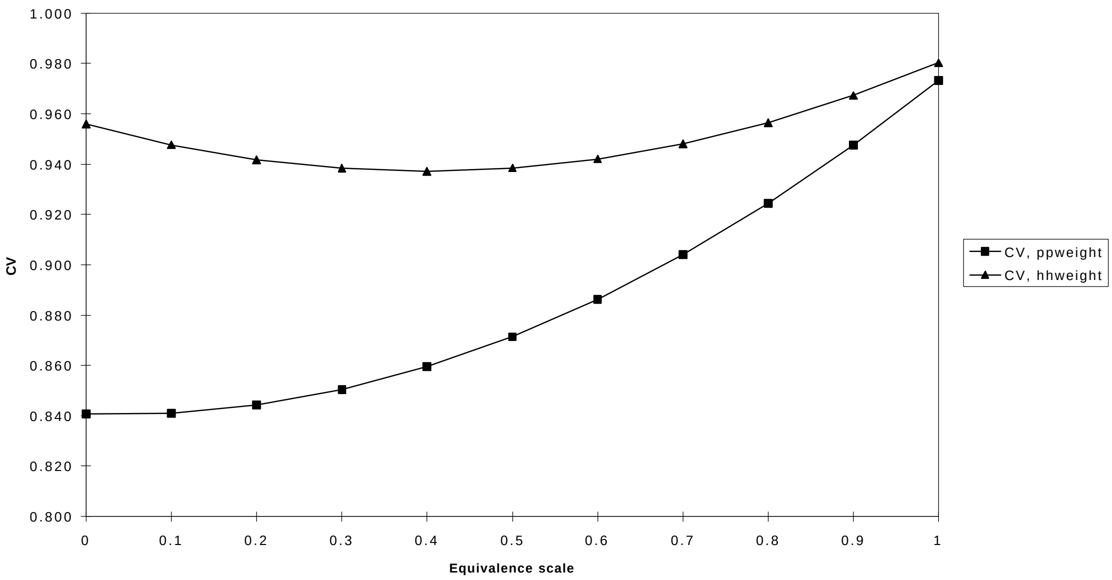


Fig. 3: Theil index, US 1991, Person and household weights

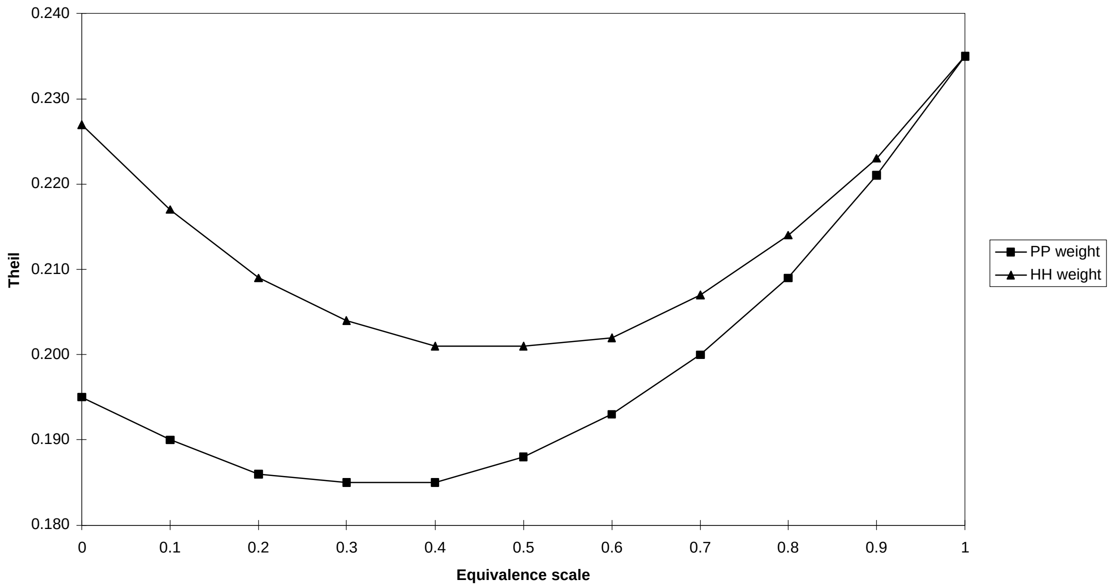


Fig. 4: Atkinson index, US 1991, person and household weights

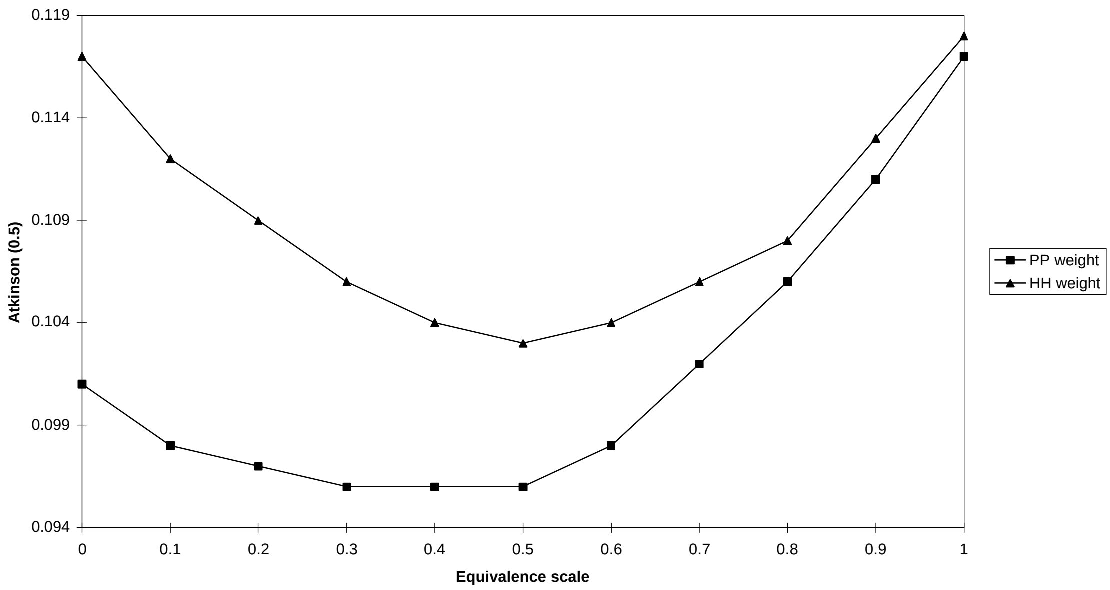


Fig. 5: Gini index, Ireland 1987, person and household w eights

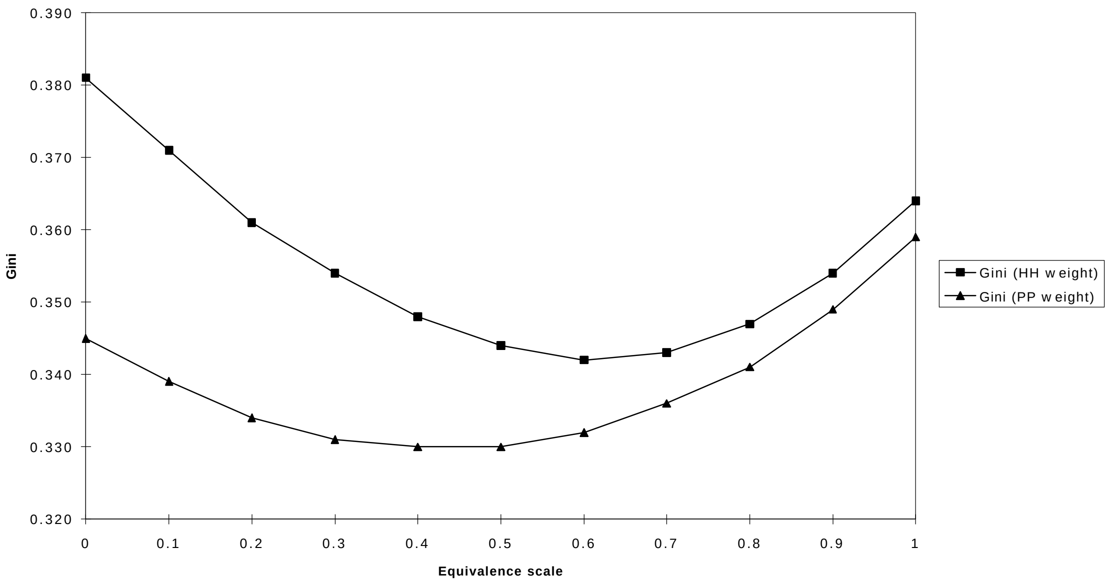


Fig. 6: Atkinson index, Ireland 1987, person and household w eights

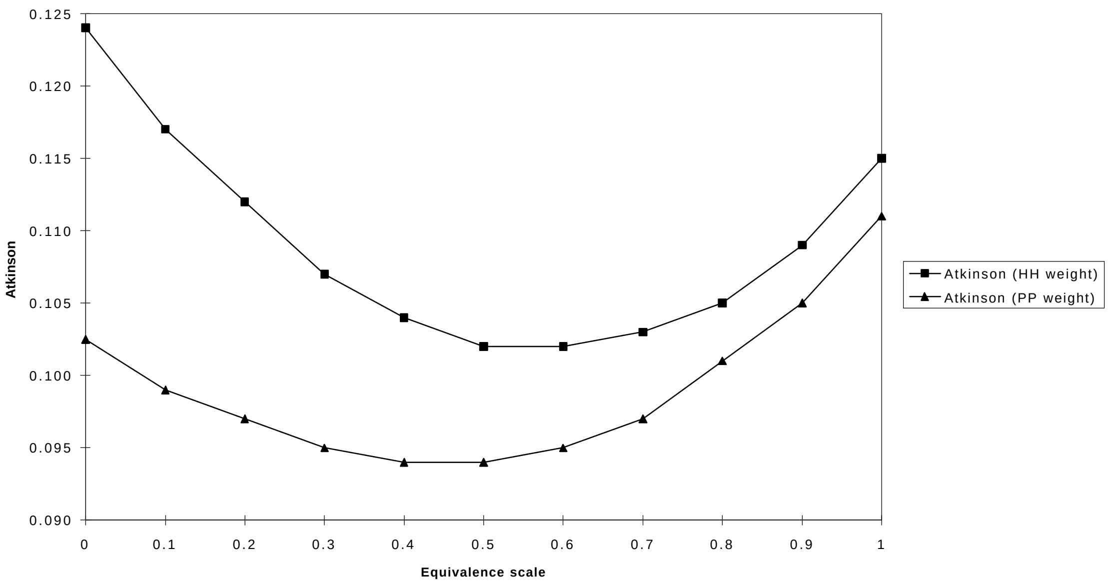


Fig. 7: Coefficient of Variation, Italy 1991, person and household weights

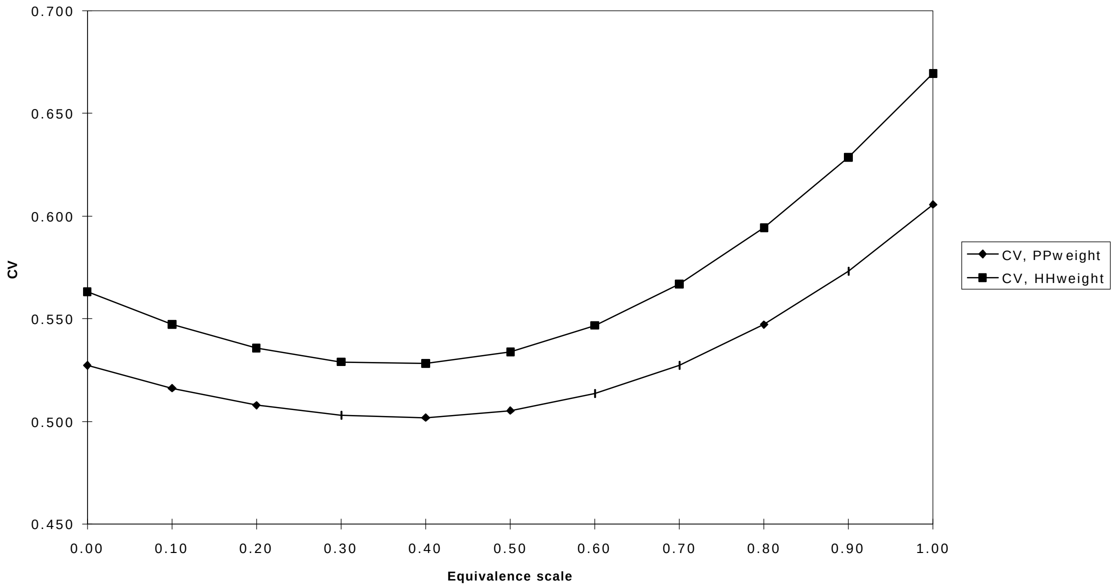


Fig. 8: Atkinson index, Italy 1991, person and household weights

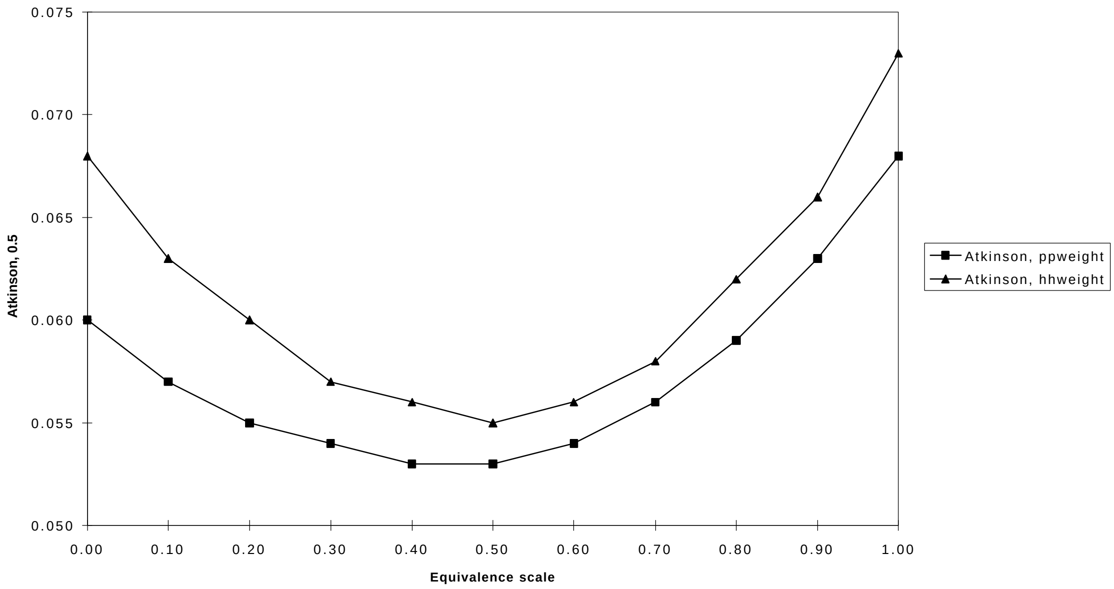


Fig. 9: A comparison of inequality for different values of the parameter ε: Ireland, the UK and the US.

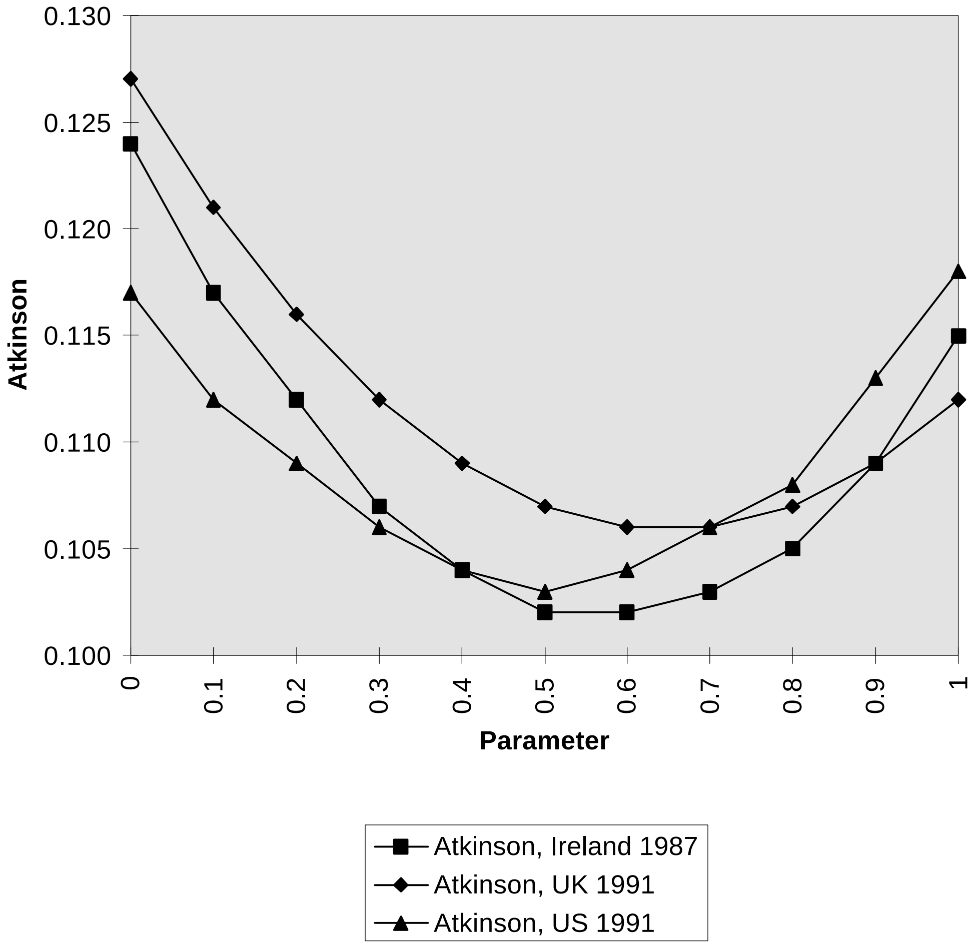


Fig. 10: Theil index, Ireland 1987, two-parameter equivalence scale

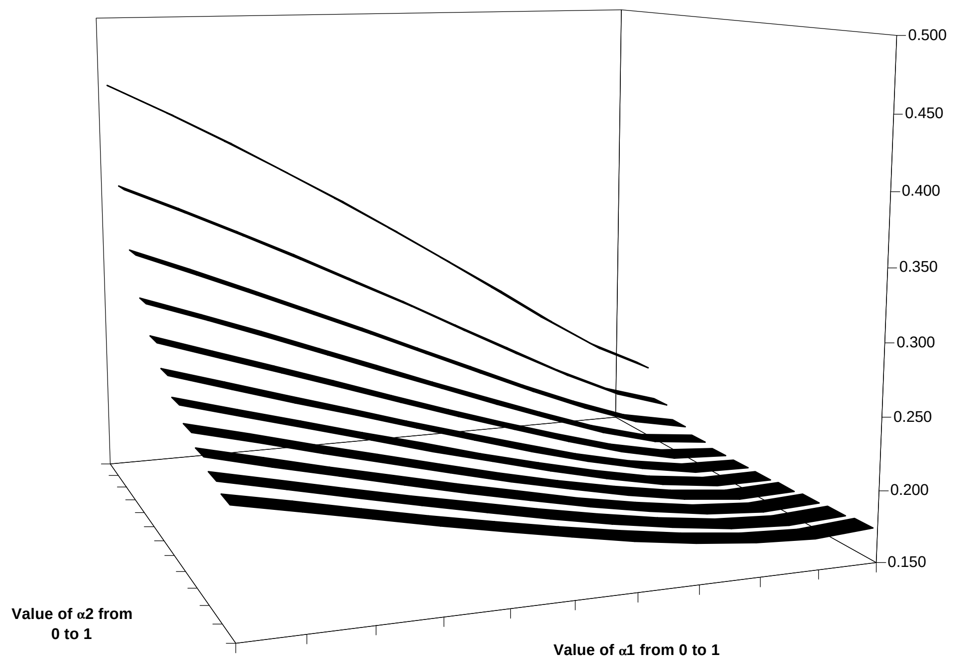


Fig. 11: Coeff. of variation, Ireland 1987, two-parameter equivalence scale

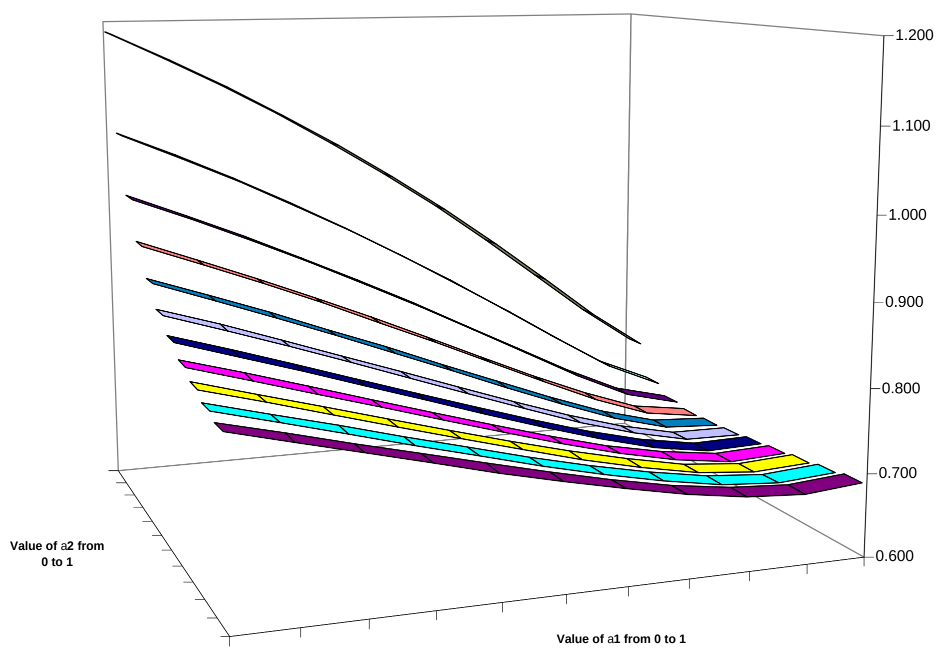


Fig. 12: Coefficient of Variation, US 1991, two-parameter equivalence scale

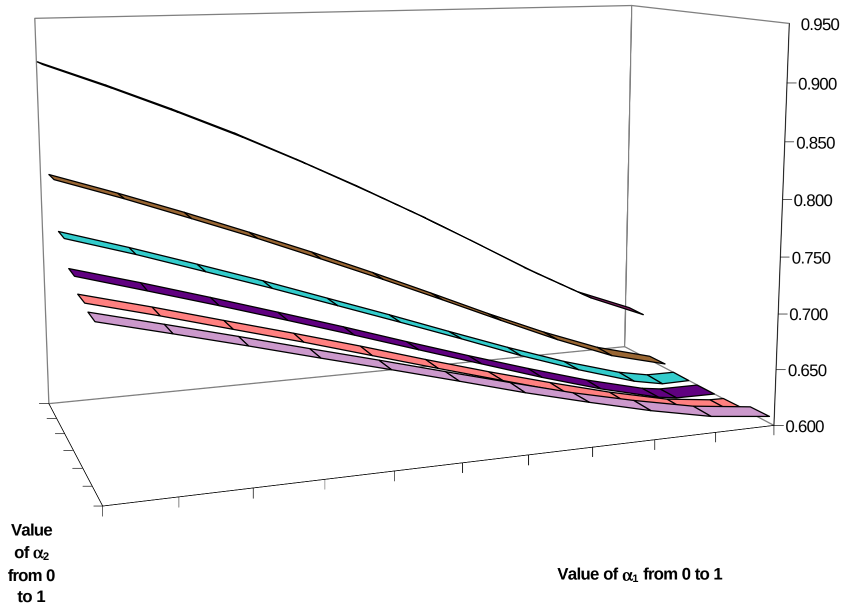
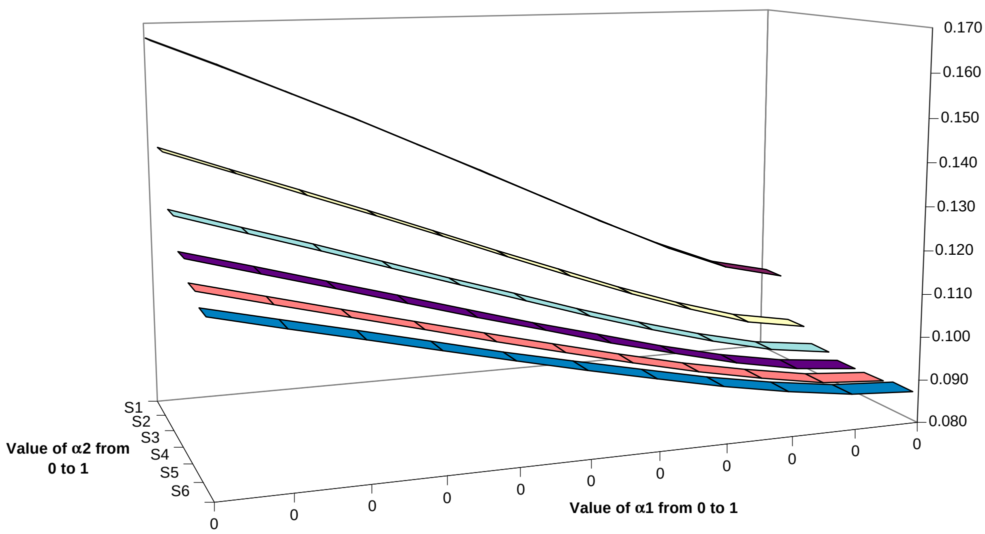


Fig. 13: Atkinson (0.5), US 1991, two-parameter equivalence scale



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