MSc Macroeconomics

Problem set 4

Deadline: Friday 9th November 2007, 10:00

Each student must hand in an answer sheet. Answer sheets should be written legibly and unreadable scribblings will be ignored. Answer sheets returned after the deadline will be awarded a zero grade.

Problem 1 (compulsory)

This exercise focus on the continuous-time model of investment seen during the lecture. This model is based on a system of two differential equation given by

$$\dot{K}(t) = f(q(t) - 1) \tag{1}$$

$$\dot{q}(t) = rq(t) - \pi(K(t)) \tag{2}$$

Construct the related phase diagram and describe the effects of a war that destroys half of the capital stock on the $\dot{K} = 0$ curve and the $\dot{q} = 0$ curve, on K and q at the time of the change, and on their behaviour over time. Assume that K and q are initially at their long-run equilibrium values.

Problem 2 (compulsory)

This exercise studies the Cagan (1956) model of money and prices. Cagan (1956) assumed that real money demand is a function of expected inflation. Higher expected inflation lowers the demand for real money holdings by raising the opportunity cost of holding money. The model is written in log-linear form as

$$m_t^d - p_t = -\eta E_t(p_{t+1} - p_t) \tag{3}$$

where m_t^d is the (log of the) nominal demand for money at time t, and p_t is the (log of the) price level at time t. The condition for money market equilibrium requires that the nominal supply of money, denoted as m_t, is equal to the demand for money, denoted as m_t^d.

(a) Insert the money market equilibrium condition into the money demand equation, and solve the resulting expression for p_t as a function of m_t and $E_t p_{t+1}$.

(b) Using the appropriate transversality condition, solve this stochastic first-order difference equation forward for the price level at time t and show that

$$p_t = \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta} \right)^{s-t} E_t m_s$$

What does this result say about the determination of the price level?

(c) What is the appropriate transversality condition? Provide for an interpretation of the transversality condition.

(d) Suppose that the money supply process is governed by an autoregressive process given by

$$m_t = \rho m_{t-1} + \varepsilon_t$$

where ε_t is zero on average, and $0 < \rho < 1$. Show that the price level is now equal to

$$p_t = \frac{m_t}{1 + \eta - \eta\rho}$$

What is the solution if money supply shocks are expected to be permanent?

(e) (Harder, extra free points if solved!) Assume now perfect foresight, so that $E_t m_s$ in your solution at point (b) is replaced by m_s. Suppose that at time zero the central bank announces unexpectedly that the money supply will be raised at time T permanently. In other words, suppose that at time zero, it becomes known that $m_t = \bar{m}$, $t < T$, and $m_t = \bar{m}'$, $t \geq T$, with $\bar{m}' > \bar{m}$. What happens to the price level: (i) at time 0; (ii) after time T; (iii) between time 0 and time T?