

MSc Macroeconomics

Problem set 2

Deadline: Friday 26th October 2007, 10:00

Each student must hand in an answer sheet. Answer sheets should be written legibly and unreadable scribbles will be ignored. Answer sheets returned after the deadline will be awarded a zero grade.

Problem 1 (compulsory)

This exercise focuses on the determination of expected returns. A representative investor has a utility function defined over the current and future values of consumption, such that

$$U(C_t, C_{t+1}) = u(C_t) + \beta E_t(u(C_{t+1}))$$

At time t , the investor can allocate an exogenous endowment of resources, denoted as e_t , between consumption and assets. The number of assets that he will buy is denoted as ς and the price for each unit of the assets is p_t . At time $t + 1$ assets pay a dividend d_{t+1} that can be used, along with another exogenous endowment of resources, for consumption purposes in period $t + 1$. The problem of the investor is written as

$$\max_{\varsigma} u(C_t) + \beta E_t(u(C_{t+1})) \quad (1)$$

subject to

$$c_t + p_t \varsigma = e_t \quad (2)$$

$$c_{t+1} = e_{t+1} + \varsigma(p_{t+1} + d_{t+1}) \quad (3)$$

(a) Solve this optimisation problem by substituting both constraints into the objective function and show that

$$p_t = \beta E_t \left[\frac{u'(C_{t+1})}{u'(C_t)} (p_{t+1} + d_{t+1}) \right]$$

(b) Denote the ratio of marginal utilities as $m_{t,t+1}$. Denote the gross rate of return of a risky asset j by $(1 + r_{j,t+1}) = \frac{p_{t+1} + d_{t+1}}{p_t}$. Use these notations to obtain the following expression (show all steps):

$$1 = \beta E_t [m_{t,t+1}(1 + r_{j,t+1})]$$

(c) Use the properties of the expectation operator to derive

$$1 = \beta E_t(1 + r_{j,t+1})E_t(m_{t,t+1}) + \beta cov_t(m_{t,t+1}, 1 + r_{j,t+1})$$

Do the same for a risk-free asset f which gross return is denoted as $(1 + r_{f,t+1})$.

(d) Use your results from point (c) to derive the following expression for the excess return:

$$E_t(r_{j,t+1} - r_{f,t+1}) = -\frac{1}{E_t(m_{t,t+1})} cov_t(m_{t,t+1}, r_{j,t+1})$$

(e) Provide an intuitive interpretation of the result in part (d).

Problem 2 (optional)

In the random walk model (see section 7.2 of Romer, 2006), uncertainty about future income does not affect consumption. Does this mean that the uncertainty does not affect expected lifetime utility?