MSc Macroeconomics

Problem set 1

Deadline: Friday 19th October 2007, 10:00

Each student must hand in an answer sheet. Answer sheets should be written legibly and unreadable scribblings will be ignored. Answer sheets returned after the deadline will be awarded a zero grade.

Problem 1 (compulsory)

Hall (1978) extends the life-cycle permanent-income model to account for uncertainty. This exercise focuses on the derivation of the famous result that consumption should follow a random walk in such a framework. It also examines the relationship between utility levels under certainty and uncertainty.

An infinitely-lived representative consumer maximizes his expected lifetime utility under a lifetime budget constraint. The optimization problem is written as

$$\max E\left[\sum_{t=1}^{\infty} \frac{1}{(1+\rho)^t} u(C_t)\right] \tag{1}$$

subject to

$$\sum_{t=1}^{\infty} \frac{1}{(1+r)^t} C_t = A_0 + \sum_{t=1}^{\infty} \frac{1}{(1+r)^t} Y_t \tag{2}$$

(a) What is the intuitive interpretation of the budget constraint?

(b) Write down the Lagrangian function for this problem.

(c) Compute the first-order conditions corresponding to time periods t and $t+1$, and show that

$$(1+\rho)\frac{u'(C_t)}{E_t u'(C_{t+1})} = (1+r) \tag{3}$$

(d) Suppose the (i) $\rho = r$, (ii) linear marginal utility, so that $u'(C) = \phi - \gamma C$, and (iii) rational expectations, so that the forecast errors should be unrelated to any information available at the time when the forecast is made. Show that consumption follows a random walk.

(e) Hall (1978) estimates the following regression model:

$$C_{t+1} - C_t = \beta X_t + e_{t+1} \tag{4}$$

What is the null hypothesis according to which consumption follows a random walk?

(f) Suppose now that $\rho = 0$. Using the random walk computed at point (d) and assuming that $u(C) = C - \frac{a}{2}C^2, a > 0$, what is the expected lifetime utility equal to? Is this the same level of lifetime utility as in the certainty case?

Problem 2 (optional)

In the life-cycle permanent-income model, the period budget constraint is given by

$$C_t + A_t = (1 + r)A_{t-1} + Y_t$$

Using the transversality condition given by $\lim_{t \to T} \frac{A_t}{(1+r)^t} = 0$, solve this expression by recursive substitution to obtain the lifetime budget constraint written as

$$\sum_{t=1}^{T} \frac{1}{(1+r)^t} C_t = A_0 + \sum_{t=1}^{T} \frac{1}{(1+r)^t} Y_t$$