

Problem Set 2

Question 2

We have an IS-LM Model:

$$C = 200 + 0.25Y_D$$

$$I = 150 + 0.25Y - 1000i$$

$$G = 250$$

$$T = 200$$

$$(M/P)^d = 2Y - 8000i$$

$$M/P = 1600$$

(a) We are asked to derive the IS and LM relations.

(i) The IS relation is found using the identity:

$$Y = C + I + G$$

$$Y = 200 + 0.25Y_D + 150 + 0.25Y - 1000i + 250$$

$$Y = 200 + 0.25(Y - T) + 150 + 0.25Y - 1000i + 250$$

$$Y = 200 + 0.25Y - 0.25T + 150 + 0.25Y - 1000i + 250$$

$$Y = 200 + 0.25Y - 0.25(200) + 150 + 0.25Y - 1000i + 250$$

$$Y = 200 + 0.25Y - 50 + 150 + 0.25Y - 1000i + 250$$

$$Y = 0.5Y + 550 - 1000i$$

$$0.5Y = 550 - 1000i$$

$Y = 1100 - 2000i$: Which is the IS relation.

(ii) The LM relation is found by first equating Money Supply with Money Demand:

$$(M/P)^s = (M/P)^d$$

$$1600 = 2Y - 8000i$$

$$1600 - 2Y = -8000i$$

$$8000i = 2Y - 1600$$

$$i = Y/4000 - 1600/8000$$

$$i = Y/4000 - 0.2 : \text{Which is the LM relation.}$$

(b) We are asked to use the IS and LM relations from (a) to solve for the equilibrium levels of output (Y) and interest rate (i). We solve the system of two equations with two unknowns (Y and i):

$$\text{IS: } Y = 1100 - 2000i$$

$$\text{LM: } i = Y/4000 - 0.2$$

We first sub the expression for i from the LM relation into the IS relation:

$$Y = 1100 - 2000(Y/4000 - 0.2)$$

$$Y = 1100 - 0.5Y + 400$$

$$Y = 1500 - 0.5Y$$

$$1.5Y = 1500$$

$$Y = 1000$$

We now sub this value for Y into the LM relation to find the corresponding value for i:

$$i = Y/4000 - 0.2$$

$$i = 1000/4000 - 0.2$$

$$i = 0.25 - 0.2$$

$$i = 0.05$$

$$i = 5\%$$

(c) We are now asked to use our answers from (b) to find values for C (Consumption) and I (Investment) in this equilibrium.

We know that Consumption is given as:

$$C = 200 + 0.25(Y_D)$$

$$C = 200 + 0.25(Y - T)$$

We now sub in our value for Y from (b) and our value for T given in the question:

$$C = 200 + 0.25(1000 - 200)$$

$$C = 200 + 0.25(800)$$

$$\mathbf{C = 400}$$

Investment is given as:

$$I = 150 + 0.25Y - 1000i$$

We sub in our values for i and Y from (b):

$$I = 150 + 0.25(1000) - 1000(0.05)$$

$$I = 150 + 250 - 50$$

$$\mathbf{I = 350}$$

The second part of the question asks us to verify that these figures for I and C are right by using the identity:

$$\mathbf{Y = C + I + G}$$

We sub in the values we have found for C and I and also the value for G given in the question:

$$Y = 400 + 350 + 250$$

$$\mathbf{Y = 1000}$$

This is the same value of Y as we got in (b) so our answers make sense.

(d) We are now asked to look at the effect that a monetary expansion will have on the equilibrium values for Output, Interest Rate, Consumption and Investment in the model.

The monetary expansion takes the form of an increase in the Money Supply. This rises from 1600 in the original model to 1840:

$$\mathbf{(M / P)^s = 1840}$$

Money Demand is still:

$$(M / P)^d = 2Y - 8000i$$

Supply and Demand must be equal for the Money Market to be in equilibrium:

$$1840 = 2Y - 8000i$$

We want to rearrange this equation to give us the new LM relation after the monetary expansion:

$$8000i = 2Y - 1840$$

$$i = Y/4000 - 1840/8000$$

$$i = Y/4000 - 23/100 : \text{This is the new LM relation.}$$

We now sub this new LM relation into the old IS relation. This hasn't changed:

$$\text{IS: } Y = 1100 - 2000i$$

$$Y = 1100 - 2000(Y/4000 - 23/100)$$

$$Y = 1100 - 0.5Y + 46000/100$$

$$Y = 1100 - 0.5Y + 460$$

$$1.5Y = 1560$$

$$0.5Y = 520$$

$$Y = 1040$$

We now use this new value for Y to get the new interest rate. We sub the new Y value into the new LM relation:

$$i = Y/4000 - 23/100$$

$$i = 1040/4000 - 23/100$$

$$i = 26/100 - 23/100$$

$$i = 3/100$$

$$i = 3\%$$

We can use these new values of i and Y to find the new values of C and I :

$$C = 200 + 0.25(Y_D)$$

$$C = 200 + 0.25(Y - T)$$

We know sub in our new value for Y and our value for T given in the question:

$$C = 200 + 0.25(1040 - 200)$$

$$C = 200 + 0.25(840)$$

$$C = 200 + 210$$

$$\mathbf{C = 410}$$

Investment is given as:

$$I = 150 + 0.25Y - 1000i$$

We sub in our new values for i and Y :

$$I = 150 + 0.25(1040) - 1000(0.03)$$

$$I = 150 + 260 - 30$$

$$\mathbf{I = 380}$$

Again it makes sense to verify that these figures for I and C are right by using the identity:

$$\mathbf{Y = C + I + G}$$

We sub in the values we've found for C and I and also the value for G given in the question:

$$Y = 410 + 380 + 250$$

$$\mathbf{Y = 1040}$$

This is the same value of Y as we got at the start of the part (d) so our answers make sense.

Summary: The monetary expansion takes the form of an increase in the money supply. This has the effect of shifting the LM Curve. The new equilibrium gives us a lower interest rate (3%, compared to 5% in part (b)) and a higher level of output (1040,

compared to 1000 in part (b)). Consumption rises from 400 in (c) to 410 and Investment rises from 350 in (c) to 380.

(e) Here we are asked to look at the effect that a Fiscal Expansion will have on the equilibrium values of Y, i, C and I.

The Fiscal Expansion takes the form of an increase in Government spending – G rises to 400. The Money Supply is back at its original level of 1600, so the LM relation is the same as in part (a):

$$i = Y/4000 - 0.2$$

The IS relation is different as $G = 400$ now:

$$Y = C + I + G$$

$$Y = 200 + 0.25Y_D + 150 + 0.25Y - 1000i + 400$$

$$Y = 200 + 0.25(Y - T) + 150 + 0.25Y - 1000i + 400$$

$$Y = 200 + 0.25Y - 0.25T + 150 + 0.25Y - 1000i + 400$$

$$Y = 200 + 0.25Y - 0.25(200) + 150 + 0.25Y - 1000i + 400$$

$$Y = 200 + 0.25Y - 50 + 150 + 0.25Y - 1000i + 400$$

$$Y = 0.5Y + 700 - 1000i$$

$$0.5Y = 700 - 1000i$$

$$Y = 1400 - 2000i : \text{which is the new IS relation.}$$

We now sub in for i from the LM relation:

$$Y = 1400 - 2000(Y/4000 - 0.2)$$

$$Y = 1400 - 0.5Y + 400$$

$$Y = 1800 - 0.5Y$$

$$1.5Y = 1800$$

$$Y = 1200$$

We use this value of Y to find the corresponding interest rate:

$$i = Y/4000 - 0.2$$

$$i = 1200/4000 - 0.2$$

$$i = 3/10 - 0.2$$

$$i = 0.3 - 0.2$$

$$i = 0.1$$

$$\mathbf{i = 10\%}$$

Using these values of i and Y we find the new values for I and C :

$$C = 200 + 0.25(Y_D)$$

$$C = 200 + 0.25(Y - T)$$

We now sub in our new value for Y and our value for T given in the question:

$$C = 200 + 0.25(1200 - 200)$$

$$C = 200 + 0.25(1000)$$

$$C = 200 + 250$$

$$\mathbf{C = 450}$$

Investment is given as:

$$I = 150 + 0.25Y - 1000i$$

We sub in our new values for i and Y :

$$I = 150 + 0.25(1200) - 1000(0.1)$$

$$I = 150 + 300 - 100$$

$$\mathbf{I = 350}$$

Again it makes sense to verify that these figures for I and C are right by using:

$$\mathbf{Y = C + I + G}$$

We sub in the values we've found for C and I and also the value for G given in the question:

$$Y = 450 + 350 + 4000$$

$$Y = 1200$$

This is the same value of Y as we got at the start of the part (e) so our answers make sense.

Summary: The fiscal expansion takes the form of an increase in government spending. This has the effect of shifting the IS Curve. The new equilibrium gives us a higher level of output (1200, compared to 1000 in part (b)) and a higher interest rate (10%, compared to 5% in part (b)). Consumption rises from 400 in (c) to 450.

Question 3

This question deals with bond prices and interest rates.

P_b = price of the bond today in \$

i = interest rate on a bond

A bond pays out \$100 in one year:

$$P_b + P_b \cdot i = \$100 \Rightarrow P_b(1 + i) = \$100 \quad (1)$$

(a) What is the interest rate on the bond if its price is \$75?

We rearrange the formula (1) to find i :

$$P_b + P_b \cdot i = \$100$$

$$P_b \cdot i = \$100 - P_b$$

$$i = \frac{\$100 - P_b}{P_b}$$

Here, \$100 is the payout in one year's time. Plugging in our value for P_b , i.e. \$75:

$$i = \frac{\$100 - \$75}{\$75} = \frac{\$25}{\$75} = \frac{1}{3} = 33\%$$

The interest rate that corresponds to this bond price is **33%**.

(b) Rearranging the above formula from part (a):

$$i = \frac{\$100 - P_b}{P_b}$$

$$i = \frac{\$100}{P_b} - \frac{P_b}{P_b}$$

$$i = \frac{\$100}{P_b} - 1$$

We can differentiate i with respect to P_b :

$$\frac{di}{dP_b} = -\frac{100}{P_b^2} < 0$$

It is now clear that when the price of the bond increases, the interest rate falls. So there is an inverse relationship between bond prices and interest rates.

(c) What is the price of the bond today if the interest rate is 8%?

We use the formula (1) to find P_b :

$$P_b(1 + i) = \$100$$

$$P_b = \frac{\$100}{1 + i}$$

We sub in our value for the interest rate: $i = 8\%$:

$$P_b = \frac{\$100}{1 + 0.08}$$

$$P_b = \frac{\$100}{1.08}$$

$$\mathbf{P_b = \$92.59}$$

The price of the bond today is \$92.59.