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Abstract

The literature contains many measurements of the fatigue properties of compact bone, but these experimental results have been difficult to interpret and use due to a large amount of apparent scatter: variation in the number of cycles to failure for a given cyclic stress or strain range. Recently Taylor (1998a, Journal of Orthopaedic Research, 16, 163–169) showed that much of this scatter could be explained using a statistical model which took into account specimen size, or more specifically stressed volume. The present paper describes an attempt to test this model by using it to predict some new data, for bovine bone tested in compressive loading at room temperature at physiological loading rates. Twenty specimens were tested at the same applied load range (100 MPa). The theory was able to predict the mean behaviour of the specimens very well, with an accuracy (expressed in terms of stress) of 2%. It was also able to predict the degree of scatter (i.e. the variation of \( N_f \)), which was shown to be similar to that measured by other workers.

1. Introduction

It is well known that bone, like many other materials, experiences the phenomenon of fatigue: the gradual deterioration and eventual failure of a material when subjected to a periodic or cyclic stress. The phenomenon is of importance because fatigue failures occur in vivo, as a result of excessive use of a limb; they are known clinically as “stress fractures”. These fractures occur in about 70% of all thoroughbred racehorses (Nunamaker et al., 1990) and are common amongst athletes and military personnel (Milgrom et al., 1989). In addition it has been suggested that fatigue damage (microcracking) plays a role in complex physiological phenomena such as bone remodelling and adaptation (Martin and Burr, 1982; Prendergast and Taylor, 1994). For these reasons it is clearly important to have precise information on the tendency of bone to fail by fatigue, so that the above phenomena can be analysed and predicted. This requires a body of experimental data and a mathematical model to predict the number of cycles to failure as a function of stress or strain range, also accounting for any other relevant factors such as the frequency of loading, temperature, type of bone, etc.

Over the last 30 years, quite a lot of experimental data has been generated. Taylor (1998a) found 12 separate studies which recorded the number of cycles to failure \( N_f \) as a function of applied stress range \( (\Delta \sigma) \) or strain range \( (\Delta \varepsilon) \), for the relatively high \( N_f \) values of \( 10^5 \)–\( 10^6 \) cycles, which are of clinical interest. When data from these studies are plotted on common axes (Fig. 1) it is clear that there is a huge amount of scatter, with \( N_f \) varying by several orders of magnitude for the same stress. The data were analysed with a view to isolating significant parameters, which were initially listed as: temperature; loading frequency; load type (tension, compression, etc) and; bone type (bovine, human, etc). It proved impossible to understand the data, however, until a further parameter was included, which was the size of the test specimen. Fatigue behaviour in many materials is known to be sensitive to the amount of material under stress. This is because fatigue failure is initiated from...
weak regions — often from pre-existing microcracks — and a large stressed volume will, on average, contain weaker regions than a small volume. This also affects the life of specimens with non-uniform loading, such as those loaded in bending, which effectively have a smaller stressed volume. Traditionally this effect is analysed using a Weibull model, which defines the probability of failure ($P_f$) — in this case the probability that a specimen tested at a given stress range will fail before a given number of cycles has elapsed. The following expression is used:

$$P_f = 1 - \exp[-(V_s/V_{so})(\Delta\sigma_{so}/\Delta\sigma_*)^m]$$

(1)

Here $V_s$ is the volume of material under stress, $V_{so}$ is the stressed volume of a set of standard test specimens, $\Delta\sigma_{so}$ is the applied stress range, and $\Delta\sigma_*$ and $m$ are constants which define the shape of the probability distribution. Eq. (1), written in this form, refers to a specific number of cycles to failure (though it can be rewritten with $N_f$ as a variable). Thus, provided test data exist for one specimen size, $V_{so}$, the values of the two constants can be found. One can then find the probability that any chosen volume of material, subjected to any chosen stress level, will fail within a given number of cycles. $P_f$ varies from 0 (no chance of failing) to 1.0 (certain to fail), so setting $P_f = 0.5$ gives the median of the distribution, which is referred to as the fatigue strength, $\Delta\sigma_{so}$, for a given $N_f$ and specimen size.

Taylor (1998a) showed that the existing data on bone fatigue conformed to this Weibull analysis: fatigue limits at $10^5$ cycles could be successfully predicted when stressed volume was accounted for. When considered in this way, the data also revealed consistent and separable effects of frequency, bone type and loading type. The effect of temperature had already been established by Carter and Hayes (1976). The magnitude of these various effects is summarised in Table 1, which shows the multiplying factor to be applied to the fatigue strength in each case. The constants in Eq. (1), for human bone at physiological temperature and frequency in zero-tension loading, were found to be as follows: $\Delta\sigma_{so} = 59.75 \text{ MPa}; \ m = 8, V_{so} = 64 \text{ mm}^3$. From this, and using the factors in Table 1, $P_f$ for $10^5$ cycles can be found for any set of conditions in human or bovine bone.

As the table shows, the largest effects are those due to variation in frequency and in the source of bone used, bovine bone being stronger than human bone. However, recent results by Zioupos et al. (1996) have shown that, for human bone, fatigue strength decreases significantly with age: their results for bone from a 27 year old person were almost identical to previous bovine data. This suggests that the real reason for the difference may be age rather than species, since most tests on human bone inevitably use relatively old material.

The aim of the present study was to test the predictive capacity of this model by applying it to a set of data which had not previously been considered when formulating the theory. It was noticed that no results existed for bovine bone tested at physiological frequencies in zero-compression loading. The only previous data on bovine bone in compression is that due to Gray and Korbacher (1974), who used a relatively high frequency of 30 Hz. Caler and Carter (1989) tested in zero-compression cycling at physiological frequency but their experiments used

### Table 1

<table>
<thead>
<tr>
<th>Parameter change</th>
<th>Effect on fatigue strength (multiplying factor)</th>
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<tbody>
<tr>
<td><strong>Frequency</strong></td>
<td></td>
</tr>
<tr>
<td>From physiological frequency (0.5–3 Hz) to high frequency (30–125 Hz)</td>
<td>$\times 1.33$</td>
</tr>
<tr>
<td><strong>Bone type</strong></td>
<td></td>
</tr>
<tr>
<td>From human bone to bovine bone</td>
<td>$\times 1.44$</td>
</tr>
<tr>
<td><strong>Temperature</strong></td>
<td></td>
</tr>
<tr>
<td>From physiological temperature to room temperature</td>
<td>$\times 1.16$</td>
</tr>
<tr>
<td><strong>Loading type</strong></td>
<td></td>
</tr>
<tr>
<td>From zero-tension to zero-compression</td>
<td>$\times 1.08$</td>
</tr>
<tr>
<td>From zero-tension to tension-compression</td>
<td>$\times 1.12$</td>
</tr>
<tr>
<td>From zero-compression to tension-compression</td>
<td>$\times 1.04$</td>
</tr>
</tbody>
</table>
human bone. Accordingly it was decided to conduct a series of tests under these conditions, and simultaneously to predict the results using the existing model. A secondary aim was to predict the amount of scatter in the data: the variation in $N_t$ for a given stress range, because this can also be predicted using the same Weibull model.

2. Methods and materials

Specimens were taken from 15 fresh bovine tibiae, each tibia being obtained from a different animal. Specimens were of the typical, waisted, “dog-bone” type (Fig. 2), circular in cross-section. The bone was stored in a freezer at $-20^\circ$C and kept wet during machining and testing, which was conducted at room temperature (approx. $20^\circ$C). Coupons of approximately rectangular shape were cut from the mid-shaft of each bone using a bandsaw. Cylindrical sections were cut from these, oriented parallel to the bone’s long axis, using a high-speed-steel core-milling cutter of internal diameter 7 mm at a rotation speed of 350 rpm and a cutting rate of 10 mm/min, performed in 0.5 mm increments. Firm gripping without damage was achieved by embedding the coupons in plaster held within aluminium box-section moulds. These cylinders were then waisted using a CNC lathe to give a gauge diameter of 5.25 mm over a 7 mm gauge length, using a spindle speed of 900 rpm, a feed rate of 0.07 mm/rev and a cutting depth of 0.5 mm. Continuous liquid cooling was used throughout to prevent overheating of the material. Visual inspection of the final specimens showed good surface finish in the gauge length, with no obvious machining defects. This was confirmed in more detailed examination after testing: a penetrant dye was applied and sections were examined microscopically. Only small amounts of surface damage were found and these did not initiate fatigue damage. Brass end caps were attached to the specimens before testing to facilitate loading without contact damage.

An INSTRON 8501 servo-hydraulic testing machine was used in load control to apply an axial compressive force to the specimens. The machine was fitted with parallel aluminium plates into which depressions had been machined to accommodate the specimens’ end caps. This method was used to avoid errors in the axial alignment of specimens, which is a potential problem in tests of this kind owing to the creation of bending stresses. Alignment was checked periodically during the test programme. All specimens were tested at a frequency of 3 Hz, cycling between 11 and 111 MPa to give a $\Delta \sigma$ value of 100 MPa and a stress ratio of 0.1. A constant stress range was chosen so as to obtain as much information as possible about the scatter in $N_t$. The number of cycles to failure was recorded: failure was defined in terms of an increase of more than 10% in the cyclic deformation range (i.e. 10% loss of stiffness). In practice this always coincided with the presence of at least one large crack, typically oriented at an angle of $45^\circ$ to the specimen axis. A total of 20 usable test results were obtained, 5 specimens having been used to establish the test protocol.

3. Results

Fig. 3 shows the data displayed on the usual logarithmic stress/life curve as in Fig. 1. The mean life obtained from this plot was $N_{t,\text{mean}} = 15\,060$ cycles. The scatter in $N_t$ values can be expressed in terms of a probability value, similar to $P_t$ in Eq. (1), but in this case it is logical to define $P$ as the probability that $N_t$ will be larger than a given value. Fig. 4 shows this probability as a function of $N_t$, normalised with respect to $N_{t,\text{mean}}$ so that it can be compared with scatter from other datasets in the literature.

4. Predictions

Our first aim was to predict the fatigue strength of bone tested according to the above protocol. Since the previous analysis (Taylor, 1998a) defined the fatigue strength at $10^6$ cycles, which is different from the experimental value of 15,060, an assumption has to be made regarding the effect of $\Delta \sigma$ on $N_t$. It is well...

Fig. 2. Geometry of the test specimens.

Fig. 3. Test results: all tests were conducted at a stress range of 100MPa, there was considerable variation in $N_t$. The solid line shows the predicted scatter band for $P$ limits of 20 and 80%.
established that this relationship has the form of an inverse power law:

$$N_t = A/\Delta \sigma^m.$$  \hspace{1cm} (2)

The critical feature here is the exponent $m$, which defines the slope of the line in the logarithmic stress/life plot. Experimental tests have shown a range of different $m$ values: we chose to use a value of 11.88 from the results of Caler and Carter (1989) since these experiments were most similar to the present ones. In fact the choice of $m$ is not too critical in the present analysis because the difference between $N_{t,\text{mean}}$ and $10^5$ cycles is relatively small, so we will move only a small distance on the stress axis, corresponding to a factor of 1.17.

To obtain a prediction from the theory described above, the Weibull equation was used with constants previously obtained by Taylor (1998a) and combined with the appropriate factors from Table 1. Fig. 5 is a plot of fatigue strength as a function of stressed volume, showing the prediction for the present testing protocol (physiological frequency and room temperature), and also showing the experimental point. Agreement between prediction and data for the present test series was very good, with an error of only 2% in the fatigue strength, which is within experimental error for tests of this kind.

Scatter in test data can also be predicted from the model. A scatter band can be defined by two levels of $P_t$: we chose 20 and 80%. Using Eqs. (1) and (2), $N_t$ values corresponding to these upper and lower limits were found to be 3508 and 64 635. As Fig. 3 shows, this is an accurate prediction of these probability levels, since 8 data points (40% of the total) lie outside these limits.

5. Discussion

The primary aim of this paper was to carry out an independent test of the model which had been previously developed to predict fatigue strength in compact bone. The results have been very successful: the model was able to combine the effects of specimen size, temperature, loading frequency and number of cycles in such a way as to predict the strength of bovine bone in compression with high accuracy.

It should be emphasised that this predictive model is not a mechanistically based theory. It is derived essentially from a statistical analysis of previous data, the intention being to show that data from various sources, tested in a variety of different ways, is in fact telling a consistent story which can be interpreted accurately enough to enable new predictions to be made. However, there is a certain mechanistic element to the model because, as explained above, it is based on a “weakest-link” concept: it is assumed that failure will occur from the worst point in the specimen, which in this case can be interpreted as the largest or most dangerous microcrack. If so, the Weibull probability distribution will be a reflection of the distribution of microcrack sizes — an assertion which can be tested by direct observation and, if shown to be correct, paves the way for a more truly mechanistic theory of fatigue in bone.

Scatter in test data can be predicted using the same model because it too is assumed to arise from the distribution of microcracks. In this respect it is encouraging to note that data from three different studies showed the same degree of scatter (Fig. 4). This implies that scatter is not due to inaccuracies in the testing system, but is
indeed inherent in the material. However, there are other possible reasons for inherent scatter, such as variations in porosity, lamellar orientation, mineralisation, etc., so it is clearly too simplistic to assume that all variability is due to microcrack distribution. A mechanistic model will have to include these other sources of variability, either explicitly or else implicitly through, for example, variation in material strength parameters such as the constants \( A \) and \( m \) in Eq. (2). An initial attempt at such a model, based on a population of microcracks growing in a material of variable strength, has been able to capture to some extent the observed scatter in \( N_f \) (Taylor, 1998b).

The present model can be used directly in the analysis of clinical phenomena such as stress fractures. A detailed discussion of this matter is beyond the scope of this paper, but it is worth noting one interesting result of the analysis. If this model is applied to a large bone such as the limb bone of a human being or large animal, then the stressed volume will be much larger than that of typical test specimens, and consequently the predicted fatigue strength of real bones in vivo will be much lower than any of the values measured using test specimens.

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References


