Lecture 1 Review

• Basic Concepts
  — Charge, Current, Voltage, Energy, and Power
  — Passive Convention
  — Ground
  — Circuit Network: Branch/Node/Loop

• Basic Laws:
  — Ohm’s Law
  — Kirchhoff’ laws
  — Series, parallel, series-parallel circuits and voltage/current dividers

• Safety
Lecture 2  DC Circuits
Analysis Methods
Lecture Objectives

• Methods of Analysis
  — Nodal analysis
    • Without Voltage Sources
    • With Voltage Sources
  — Mesh analysis
    • Without Current Sources
    • With Voltage Sources
  — Analysis by Inspection

General procedure!
Example: Apply Ohm’s and Kirchhoff's laws

Calculate the voltage across the elements and the current flowing through each element.

- Identify nodes
- Set reference point
- Reference:
  - Current direction
- Assign Node voltages
- Calculate the branch currents: Ohm’s law
- KCL law
Nodal analysis: Node Voltages

• Identify nodes: N.
• Select a node as the reference node \( v_r = 0 \).
• Assign current direction for each branch
• Assign voltages to \((N-1)\) nodes: \( v_1, v_2, \ldots, v_{N-1} \).
• Use Ohm’s law to express the branch current.
• Apply KCL to each of the \((N-1)\) nodes.
• Solve the resulting simultaneous equations.

Use KCL to obtain equations for `node voltage`
A **supernode** is formed by enclosing a (dependent or independent) voltage source connected between two non-reference nodes and any elements connected in parallel with it.
A mesh is a loop which does not contain any other loops within it.
Example: Apply Ohm’s law and Kirchhoff's Laws

Calculate the voltage across the elements and the current flowing through each element.

• Identify loops and meshes
• Assign mesh currents
• KVL + Ohm’s law:
• Solve the equations

Mesh current = the current flowing through the element
Mesh analysis

• Identify loops and find $N$ meshes.
• Assign mesh currents: $i_1$, $i_2$, ... $i_N$.
• Express voltages in terms of the mesh currents: Ohm’s law.
• Apply KVL to get $N$ simultaneous equations
• Solve the equations to get the mesh current

Use KVL to set equations for `mesh current`
Mesh analysis with current sources

We apply KVL and need to know voltages. But, for an ideal current source?

**Case 1:** a current source exists only in one mesh.

\[ i_2 = \text{the current of the current source} \]

**Case 2:** a current source exists between two meshes.

A **supermesh** results when two meshes have a current source in common: KVL and KCL
Mesh analysis: limits

- Not applicable to nonplanar circuits
Nodal Analysis by Inspection

\[
\begin{bmatrix}
0.3 & -0.2 & 0 & 0 \\
-0.2 & 1.325 & -0.125 & -1 \\
0 & -0.125 & 0.5 & -0.125 \\
0 & -1 & -0.125 & 1.625 \\
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2 \\
v_3 \\
v_4 \\
\end{bmatrix}
=
\begin{bmatrix}
3 \\
-3 \\
0 \\
6 \\
\end{bmatrix}
\]
Nodal Analysis by Inspection

\[
\begin{bmatrix}
G_{11} & G_{12} & \cdots & G_{1N} \\
G_{21} & G_{22} & \cdots & G_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
G_{N1} & G_{N2} & \cdots & G_{NN}
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2 \\
\vdots \\
v_N
\end{bmatrix}
= 
\begin{bmatrix}
i_1 \\
i_2 \\
\vdots \\
i_N
\end{bmatrix}
\]

or simply

\[Gv = i\]

where

\[G_{kk} = \text{Sum of the conductances connected to node } k\]
\[G_{kj} = G_{jk} = \text{Negative of the sum of the conductances directly connecting nodes } k \text{ and } j, k \neq j\]
\[v_k = \text{Unknown voltage at node } k\]
\[i_k = \text{Sum of all independent current sources directly connected to node } k, \text{ with currents entering the node treated as positive}\]

\(G\) is called the *conductance matrix*
Mesh Analysis with Inspection

Clockwise, rise: positive

\[
\begin{bmatrix}
9 & -2 & -2 & 0 & 0 \\
-2 & 10 & -4 & -1 & -1 \\
-2 & -4 & 9 & 0 & 0 \\
0 & -1 & 0 & 8 & -3 \\
0 & -1 & 0 & -3 & 4
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2 \\
i_3 \\
i_4 \\
i_5
\end{bmatrix} =
\begin{bmatrix}
4 \\
6 \\
-6 \\
0 \\
-6
\end{bmatrix}
\]
Mesh Analysis by Inspection

\[
\begin{bmatrix}
R_{11} & R_{12} & \cdots & R_{1N} \\
R_{21} & R_{22} & \cdots & R_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
R_{N1} & R_{N2} & \cdots & R_{NN}
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2 \\
\vdots \\
i_N
\end{bmatrix}
=
\begin{bmatrix}
v_1 \\
v_2 \\
\vdots \\
v_N
\end{bmatrix}
\]

or simply

\[R_i = v\]

where

- \(R_{kk}\) = Sum of the resistances in mesh \(k\)
- \(R_{kj} = R_{jk}\) = Negative of the sum of the resistances in common with meshes \(k\) and \(j\), \(k \neq j\)
- \(i_k\) = Unknown mesh current for mesh \(k\) in the clockwise direction
- \(v_k\) = Sum taken clockwise of all independent voltage sources in mesh \(k\), with voltage rise treated as positive

\(R\) is called the resistance matrix.
Mesh vs. Nodal

• To select the method that results in the smaller number of equations. For example:

1. Choose nodal analysis for circuit with fewer nodes than meshes.
   — *Choose mesh analysis for circuit with fewer meshes than nodes.
   — *Networks that contain many series connected elements, voltage sources, or supermeshes are more suitable for mesh analysis.
   — *Networks with parallel-connected elements, current sources, or supernodes are more suitable for nodal analysis.

2. If node voltages are required, it may be expedient to apply nodal analysis. If branch or mesh currents are required, it may be better to use mesh analysis.
Appendix
Cramer’s Rules

\[
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\
\vdots \\
a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n
\]

\[
\begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn}
\end{bmatrix}
\begin{bmatrix}
x_1 \\ x_2 \\ \vdots \\ x_n
\end{bmatrix}
=
\begin{bmatrix}
b_1 \\ b_2 \\ \vdots \\ b_n
\end{bmatrix}
\]

\[
\Delta = \begin{vmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn}
\end{vmatrix}
\]

\[
M_{ij} = \begin{vmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn}
\end{vmatrix}_{i,j}
\]

\[
\Delta_j = a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13} + \cdots + (-1)^{i+n}a_{1n}M_{1n}
\]

\[
\Delta_1 = \begin{vmatrix}
b_1 \\ b_2 \\ \vdots \\ b_n
\end{vmatrix}
\]

\[
\Delta_2 = \begin{vmatrix}
a_{11} & b_1 & \cdots & a_{1n} \\ a_{21} & b_2 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & b_n & \cdots & a_{nn}
\end{vmatrix}
\]

\[
\Delta_n = \begin{vmatrix}
a_{11} & a_{12} & \cdots & b_1 \\ a_{21} & a_{22} & \cdots & b_2 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & b_n
\end{vmatrix}
\]

\[
x_1 = \frac{\Delta_1}{\Delta} \\
x_2 = \frac{\Delta_2}{\Delta} \\
\vdots \\
x_n = \frac{\Delta_n}{\Delta}
\]
Scilab

- http://www.scilab.org/
\[
\begin{align*}
x_1 + 2x_2 - x_3 & = 1 \\
-2x_1 - 6x_2 + 4x_3 & = -2 \\
x_1 - 3x_2 + 3x_3 & = 1
\end{align*}
\]

\[
A = \begin{bmatrix} 1 & 2 & -1 \\ -2 & -6 & 4 \\ -1 & -3 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}.
\]

\[
Ax = b.
\]

\[
A = \begin{bmatrix} 1 & 2 & -1 \\ -2 & -6 & 4 \\ -1 & -3 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}
\]

// Set up a system
// of equations.

x = A\backslash b

// Find x with A x = b.

\[
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}.
\]
Circuit Simulation: Pspice /Cadence

Lecture 3  DC Circuits
Circuit Theorems