Chapter 13: Applications of Hard Magnets

1. Magnetic Circuits
2. Materials
3. Static Applications
4. Dynamic Applications with Mechanical Recoil
5. Dynamic Applications with Active Recoil
6. Microsystems

Comments and corrections please: jcoey@tcd.ie
Further reading

• R. Skomski and J. M. D. Coey, *Permanent Magnetism*, IOP, 1999
  A monograph focussed on the physics of permanent magnetism, with chapters on experimental methods, materials and applications.

• P. Campbell, *Permanent Magnet Materials and their Applications*, CUP, 1994 207 pp
  A short and readable book for engineers.

  A monograph on magnet structures which generate static magnetic fields.
Permanent magnets are ferromagnets with a wide hysteresis loop. Once magnetized, they sit at a working point in the second quadrant of the loop which is determine by the magnet shape and the rest of the magnetic circuit.

Slope of the load line is $B_m/H_m$

$$= -\mu_0(1/N - 1)$$

Permanent magnets generate magnetic flux with no continual expenditure of energy!

The $B$-field may be uniform or nonuniform, static or time-dependent. The magnetic flux density $B_0$ in the airgap is the natural field to consider in permanent magnet applications because flux is conserved in a magnetic circuit, and forces on electric charges and magnetic moments all depend on $B$.

The best permanent magnets are intermetallic compounds of a ferromagnetic 3$d$ element and a 4$f$ element; e.g. SmCo$_5$ or Nd$_2$Fe$_{14}$B. Most common are the cheap hexagonal ferrites BaFe$_{12}$O$_{19}$ and SrFe$_{12}$O$_{19}$. These powders are sometimes bonded in plastic
Examples of permanent magnet applications.

<table>
<thead>
<tr>
<th>Field</th>
<th>Magnetic effect</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>Zeeman splitting</td>
<td>magnetic resonance imaging</td>
</tr>
<tr>
<td></td>
<td>Torque</td>
<td>magnetic powder alignment</td>
</tr>
<tr>
<td></td>
<td>Hall effect,</td>
<td>sensors</td>
</tr>
<tr>
<td></td>
<td>Magnetoresistance</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Force on conductor</td>
<td>motors, actuators, loudspeakers</td>
</tr>
<tr>
<td></td>
<td>Induced emf</td>
<td>generators, microphones</td>
</tr>
<tr>
<td>Nonuniform</td>
<td>Forces on charged particles</td>
<td>beam control, radiation sources</td>
</tr>
<tr>
<td></td>
<td>Force on paramagnet</td>
<td>(microwave, uv, X-ray)</td>
</tr>
<tr>
<td></td>
<td>Force on iron</td>
<td>mineral separation</td>
</tr>
<tr>
<td></td>
<td>Force on magnet</td>
<td>holding magnets</td>
</tr>
<tr>
<td></td>
<td></td>
<td>bearings, couplings, maglev</td>
</tr>
<tr>
<td>Time-varying</td>
<td>Variable field</td>
<td>magnetometry</td>
</tr>
<tr>
<td></td>
<td>Force on iron</td>
<td>switchable clamps</td>
</tr>
<tr>
<td></td>
<td>Eddy currents</td>
<td>brakes, metal separation</td>
</tr>
</tbody>
</table>

Other uses of magnets, in acupuncture, pain control, electrochemistry, supression of wax formation in oil wells or control of limescale deposits in pipes carrying hard water are difficult to classify, but worthy of investigation!
Applications depend on one of the following effects:

- A static uniform field generates torque on a magnet and tends to align pre-existing magnetic moments since $\mathbf{T} = m \times \mathbf{B}$. The compass is an example.

Charged particles moving through a uniform field with velocity $\mathbf{v}$ are deflected by the Lorentz force $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$, which causes them to move in a helix. (c.f Busch’s e/m experiment)

- If the charged particles are electrons confined in a conductor of length $L$ where they constitute a current $I$ flowing perpendicular to the field, and the Lorentz force leads to the familiar expression $\mathbf{F} = \mathbf{B}l\mathbf{I}$. This is the basis of electric motors and other drives.

Conversely, moving a conductor through the uniform field generates an induced electromotive force (emf) given by Faraday's law $\mathcal{E} = -d\Phi/dt$ where $\Phi = BA$ is the flux threading the circuit of which the conductor forms a part. Eddy currents are generated to oppose the motion.

- Spatially nonuniform fields offer another series of useful effects. They exert a force on a magnetic moment given by the energy gradient $\mathbf{F} = -\nabla(m \cdot \mathbf{B})$.

They also exert nonuniform forces on moving charged particles, which can be used to focus ion or electron beams or generate electromagnetic radiation from accelerating electron beams passing through the nonuniform field.

- Time-varying fields can be produced by displacing or rotating the magnets. This may induce an emf in a conductor according to Faraday's law exert forces on the induced eddy currents. Uniform time-varying fields are valuable for magnetic measurements.

Finally, a spatially nonuniform time-varying field will exert a time-dependent force on a magnetic moment or particle beam. Applications include magnetic switches and magnetic measurements such as the Faraday balance.
13.1 Magnetic Circuits

Assuming no flux leakage
\[ B_m A_m = -B_g A_g \]

Assuming ideal soft material \( \mu = \infty \)
Ampere’s law \( \int H \cdot dl = 0 \)
\[ H_m l_m = -H_g l_g \]

Multiplying \( B_m H_m V_m = -B_g^2 V_g / \mu_0 \)

Dividing \( -B_m / H_m = \mu_0 A_g l_m / A_m l_g \)
The permeance coefficient \( P = 1/R \)

**Figure 13.1** A simple magnetic circuit, and its electrical equivalent, with and without losses.
13.1.1 Static and dynamic Applications

Figure 13.2 Hysteresis loops showing working points for a static application (a), a dynamic application with mechanical recoil (b) and a dynamical application with active recoil (c)
13.2 Materials

\[ H = -NM \]
\[ B = \mu_0(H + M) \]
\[ BH = -\mu_0(-NM + M)NM \]
\[ = -\mu_0M^2(N - N^2) \]
\[ d(BH)/dN = 1 - 2N = 0 \]

\[ N = 1/2 \]

New icon for permanent magnets! ⇒
**Figure 13.3** Influence of permanent magnet properties on the design of a dc motor and a loudspeaker
Tonnage production
Ferrite: 1,000,000
Nd-Fe-B  50,000

Permanent Magnet Market

6 B$
Table 13. 3 Properties of commercial oriented magnets.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\mu_0 M_r$ (T)</th>
<th>$J_s$ (T)</th>
<th>$iH_c$ (kA m$^{-1}$)</th>
<th>$BH_{c}$ (kA m$^{-1}$)</th>
<th>$(BH)_{\text{max}}$ (kJ m$^{-3}$)</th>
<th>$\mu_0 M_r^2 / 4$ (kJ m$^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SrFe$<em>{12}$O$</em>{19}$</td>
<td>0.42</td>
<td>0.47</td>
<td>275</td>
<td>265</td>
<td>34</td>
<td>35</td>
</tr>
<tr>
<td>Alnico 5</td>
<td>1.25</td>
<td>1.40</td>
<td>54</td>
<td>52</td>
<td>43</td>
<td>310</td>
</tr>
<tr>
<td>SmCo$_5$</td>
<td>0.88</td>
<td>0.95</td>
<td>1700</td>
<td>660</td>
<td>150</td>
<td>154</td>
</tr>
<tr>
<td>Sm$<em>2$Co$</em>{17}*$</td>
<td>1.08</td>
<td>1.15</td>
<td>800</td>
<td>800</td>
<td>220</td>
<td>232</td>
</tr>
<tr>
<td>Nd$<em>2$Fe$</em>{14}$B</td>
<td>1.28</td>
<td>1.54</td>
<td>1000</td>
<td>900</td>
<td>300</td>
<td>326</td>
</tr>
</tbody>
</table>

* intergrown with 1:5 phase

\[ BH_{\text{max}} < (1/4) \mu_0 M_s^2 \]
Table 13.4. Properties of sintered and bonded ferrite magnets

<table>
<thead>
<tr>
<th></th>
<th>$M_r$ (kA m$^{-1}$)</th>
<th>$H_c$ (kA m$^{-1}$)</th>
<th>$(BH)_{max}$ (kJ m$^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intrinsic SrFe$<em>{12}$O$</em>{19}$</td>
<td>380</td>
<td>-</td>
<td>45$^a$</td>
</tr>
<tr>
<td>Oriented, sintered</td>
<td>330</td>
<td>270</td>
<td>34</td>
</tr>
<tr>
<td>Isotropic, sintered</td>
<td>180</td>
<td>310</td>
<td>9</td>
</tr>
<tr>
<td>Oriented, bonded$^b$</td>
<td>240</td>
<td>245</td>
<td>16</td>
</tr>
<tr>
<td>Isotropic, bonded$^c$</td>
<td>100</td>
<td>180</td>
<td>5</td>
</tr>
<tr>
<td>Material</td>
<td>$d$ (kg m$^{-3}$)</td>
<td>$\alpha$ ($10^{-6}$C$^{-1}$)</td>
<td>$\rho$ ($\mu\Omega$m)</td>
</tr>
<tr>
<td>--------------------------</td>
<td>-------------------</td>
<td>-------------------------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>SrFe$<em>{12}$O$</em>{19}$ sintered</td>
<td>4300</td>
<td>10</td>
<td>$10^8$</td>
</tr>
<tr>
<td>SrFe$<em>{12}$O$</em>{19}$ bonded</td>
<td>3600</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alnico 5 cast</td>
<td>7200</td>
<td>12</td>
<td>0.5</td>
</tr>
<tr>
<td>SmCo$_5$ sintered</td>
<td>8400</td>
<td>11</td>
<td>0.6</td>
</tr>
<tr>
<td>Sm$<em>2$Co$</em>{17}$* sintered</td>
<td>8400</td>
<td>10</td>
<td>0.9</td>
</tr>
<tr>
<td>Nd$<em>2$Fe$</em>{14}$B sintered</td>
<td>7400</td>
<td>-2</td>
<td>1.5</td>
</tr>
<tr>
<td>Nd$<em>2$Fe$</em>{14}$B bonded</td>
<td>6000</td>
<td>200</td>
<td></td>
</tr>
</tbody>
</table>

* intergrown with 1:5 phase. $\alpha$ = thermal expansion, $\rho$ = resistivity
13.3 Static Applications

13.3.1 Uniform fields. The magnetic field produced by a point dipole of moment $m$ Am$^2$ is quite inhomogeneous. In polar coordinates, it is

$$H_r = 2m \cos \theta/4\pi r^3, \quad H_\theta = m \sin \theta/4\pi r^3, \quad H_\phi = 0$$

The field due to an extended line dipole of length $L$ and dipole moment $\lambda$ Am per unit length is significantly different:

$$H_r = \lambda \cos \theta/\pi r^2, \quad H_\theta = \lambda \sin \theta/4 \pi r^2, \quad H_z = 0$$

The magnitude of $\mathbf{H}$, $\sqrt{(H_r^2 + H_\theta^2 + H_\phi^2)}$, is now independent of $\theta$ and its direction makes an angle $2\ell$ with the orientation of the magnet.

Comparison of the magnetic field produced by a) a point dipole $m$ and b) a line dipole $\lambda$. 
Magnetic circuits made of long cylindrical segments may be used to generate uniform fields. An open cylinder or a design with flat cuboid magnets and a soft iron return path is used to for nuclear magnetic resonance (NMR). Permanent magnet flux sources supply fields of order 0.3 T with homogeneity of 1 part in $10^5$ in a whole-body scanner:

Fig 13.5 Designs for magnetic cylinders which produce a uniform transverse field. Figure (c) shows a design where the direction of magnetization of any segment at angular position $\theta$ in the cylinder is at $2\theta$ from the vertical axis. According to the equations for the line dipole, all segments now contribute to create a uniform field across the airgap in a vertical direction. Unlike the structure of Fig (a), the radii $r_1$ and $r_2$ can take any values without creating a stray field outside the cylinder. This ingenious device is known as a Halbach cylinder, The field in the airgap is

$$B_0 = B_r \ln(r_2/r_1)$$

In practice it is convenient to assemble the device from $n$ trapezoidal segments, as illustrated in fig. (d) for $n = 8$. 
13.3.2 Nonuniform fields

**Figure 13.6** Some cylindrical magnet structures which produce inhomogeneous fields: (a) a quadrupole field (b) a hexapole field and (c) a uniform field gradient.
**Figure 13.7** A wiggler magnet used to generate intense electromagnetic radiation from an electron beam.
Nonuniform magnetic fields offer social benefits from the tiphead to the haematology laboratory. The expression for the energy of a pre-existing magnetic moment $m$ in a field $H$ is $-\mu_0 m \cdot H$, leading to

$$F = \mu_0 \nabla (m \cdot H).$$

However when a small moment $m = \chi V H$ is induced by the field in a material of volume $V$ and susceptibility $\chi$, the expression becomes

$$F = (1/2)\mu_0 \chi V \nabla (H^2)$$

This expression is the basis of magnetic separation

Fig 13.8 Magnetic separation a) open gradient separation; b) electromagnetic separation with permanent magnets.
Figure 13.9 Field and force patterns around a cylindrical iron wire in a high-gradient magnetic separator.
Figure 13.10 Permanent magnet variable flux sources: (a) a double Halbach cylinder. (b) four–rod mangle.
Figure 13.11 A MULTIMAG permanent-magnet variable flux source and controller. The magnet head produces a variable field of up to 1.8 T in any transverse direction in the bore.
Figure 13.12 A vector vibrating-sample magnetometer based on a MULTIMAG
**Holding magnets**

Magnets exert forces on each other, and on other ferromagnetic materials such as soft iron. The plastic-bonded ferrite magnets on the ‘fridge are magnetized in a pattern of stripes about 3 mm wide magnetized alternately inwards and outwards from the sheet.

Check this by gently dragging two pieces of plastic magnet past each other!

![Magnetization pattern of plastic magnet sheet.](image)

To work out the maximum force that can be generated at the face of a magnet, consider a toroid that is cut into two C-shaped segments and then separated slightly. If the separation is \(d\) and the cross section area is \(A_g\), the energy appearing in the air gaps is:

\[
\frac{1}{2} \mu_0 H_g^2 A_g d
\]

\[= \frac{B_g^2 A_g d}{\mu_0}
\]

The work done separating the segment is \(2Fd\), hence the force per unit area is:

\[
F/A_g = \frac{B_g^2}{2\mu_0}
\]

Forces of up to 40 N cm\(^{-2}\) can be achieved for \(B_g = 1\ T\).  

![A magnetic toroid cut and separated to produce a field in the airgap.](image)

The flux density at the surface of the plastic magnet is about 50 mT. Estimate the force on a piece the size of a credit card.
Figure 13.14 Two designs for switchable magnetic clamps. (a) is a rotatable magnet design shown in the ‘on’ position, (b) is a design where the magnet array is displaced laterally, shown in the ‘off’ position.
Figure 13.15  (a) A face-type coupling with four axially-magnetized segments, (b) A 2:1 magnetic gear with radially-magnetised segments.
Figure 11.16 Two elementary magnetic bearings made from axially-magnetised rings; (a) a radial bearing and (b) an axial bearing.

Figure 11.17 A linear magnetic bearing.
**Figure 11.18** A Maglev system based on eddy-current repulsion.
Figure 13.19 A magnetically-compensated hinge.
Figure 13.20 Variable-reluctance sensor
Figure 13.21 A flat voice-coil actuator for a personal-computer disk drive.
Figure 13.21 Moving-iron actuators: (a) print hammer and (b) reed switch.
Figure 13.22 DC motor designs: (a) brush motor with magnets on the stator and (b) brushless motor with magnets on the rotor.
Figure 13.23 Motors: (a) A two-pole dc brush motor, (b) a two-pole four-phase brushless dc motor and (c) a variable reluctance motor.
Figure 13.24 Variants of the brushless dc motor: (a) normal design (b) cup-type (c) disk-type. 1 – magnet; 2 – stator; 3 – stator winding; 4 – position sensor.
Figure 13.24 A four pole synchronous motor with a permanent magnet rotor. A 16 bar squirrel cage winding is incorporated so that the machine will operate as an induction motor for startup.
Figure 13.25  A two pole stepping motor used in clocks and watches. In watches the magnet made of bonded Sm$_2$Co$_{17}$ has a mass of a few mg.
Figure 13.26 Miniature hybrid stepping motor
Magnet applications; A 30 B€ market

- Soft Magnets
  - Fe-Si
  - Fe-Si (oriented)
  - Ni-Fe/Fe-Co
  - Amorphous
- Hard Magnets
  - Ni-Fe/Fe-Co (heads)
  - Fe-Si (oriented)
  - Others
- Others

- Others
- Co-Cr (hard discs)
- CrO2 (tapes)
- Co3-yFe2O3 (tapes, floppy discs)
- Iron (tapes)
- Others
- Nd-Fe-B
- Sm-Co
- Alnico
- Others