5. Models of the atom

5.1 Thomson’s cake

Atoms electrically neutral: negative charged electrons like raisins in positively charged dough?

displacement of electrons from equilibrium positions: simple harmonic motion → thus emission of radiation.
gives wrong frequencies! (not in line with measured atomic spectra)
5.2 Rutherford’s scattering experiments

- Thomson, Millikan: almost all the mass of an atom is associated with the positive charge, not the electrons.
- size of atom approximately $10^{-10}$ m.
- distribution of positive charge in atom?
- Rutherford: fire alpha-particles ($\text{He}^{2+}$ ions, section 6.1), of radioactive source against gold-foil target.
- $\alpha$-particles pass through thin sheets of metal foil, through the interior of atoms! Mass: 7300 times that of the electron, hardly scattering due to electrons.
• Observe **backscattering** of some $\alpha$-particles (nearly 180 degrees)!

• Computations: only possible if positive mass concentrated within range of $10^{-14}$ m of the atom.

(left) Scattering due to nucleus of radius $7.0 \times 10^{-15}$ m (actual size) and (right) radius 10 times the actual value. (YF38.17)
5.3 Bohr’s model of the atom (1913)

**Ingredients**

- atom essentially empty, nucleus occupies only about $10^{-12}$ of total volume of atom, but all the positive charge and 99.95% of the total mass of the atom.

- discrete energy levels
mini-solar system?

• standard electromagnetic theory, known at the end of 19\textsuperscript{th} century (after Maxwell), shows that accelerating charges radiate energy (not a trivial calculation)

• Thus orbiting electrons loose energy, atoms would emit continuous spectra and would not be stable!

PROBLEM!
Bohr postulates

- electrons move in **stable circular orbits** without emission of radiation.
- definite energy associated with each stable orbit.
- **radiation** only for transition from one orbit to another.

Radiation involves one photon: \( \Delta E = hf = E_i - E_f \)

- stable orbits characterized by **quantized** angular momentum.
- angular momentum: \( \vec{L} = mr \times \vec{v} \) Circular motion: \( \vec{r} \perp \vec{v} \)
- quantisation: \( L_n = mr_n v_n = n\hbar \)

- \( n \) is **principal quantum number**, \( n = 1, 2, 3, \ldots \)

\[ \hbar = \frac{\hbar}{2\pi} \]
Resulting model of Hydrogen atom: circular motion of one electron around positive nucleus

- Coulomb force:

\[ F_c = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r_n^2} \]

- Newton’s 2nd law for circular motion (m: mass of electron)

\[ F_n = ma = m \frac{v_n^2}{r_n} \]

- Set \( F_c = F_n \)

- Insert \( r_n = \frac{n\hbar}{mv_n} \)

- compute the velocities in the orbits as

\[ v_n = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{n\hbar} \]

- Insert \( v_n = \frac{n\hbar}{m r_n} \)

- compute the radii of the orbits as

\[ r_n = 4\pi\varepsilon_0 \frac{n^2\hbar^2}{e^2 m} \]
• **Minimum radius** for $n=1$:

$$r_1 = a_0 = 4\pi\varepsilon_0 \frac{\hbar^2}{e^2 m} = 5.29 \times 10^{-11} \text{ m}$$

• find diameter $d \approx 1 \times 10^{-10} \text{ m}$, consistent with previous estimates!

• $r_n = n^2 a_0$

• Orbital speed $v_1 \approx 2.19 \times 10^6 \text{ m/s} \ll c = 3.0 \times 10^8 \text{ m/s}$, no relativistic treatment required.

**Computation of energy levels:**

- **Kinetic energy:**

$$E_{kin} = \frac{1}{2} m v_n^2 = \frac{1}{\varepsilon_0^2} \frac{me^4}{8n^2 \hbar^2}$$

- **Potential energy (zero at $r = \infty$):**

$$E_{pot} = -\frac{1}{4\pi\varepsilon_0} \frac{e^2}{r_n} = -\frac{1}{\varepsilon_0^2} \frac{me^4}{4n^2 \hbar^2}$$

- **total energy:**

$$E_{tot} = E_{kin} + E_{pot} = -\frac{1}{\varepsilon_0^2} \frac{me^4}{8n^2 \hbar^2} < 0$$
• Change in energy from level \( n=n_i \) to level \( n=n_f \):

\[
\Delta E = hf = E_{ni} - E_{nf} = \frac{1}{\varepsilon_0^2} \frac{me^4}{8\hbar^2} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)
\]

• agrees with Rydberg equation by setting

\[
R := \frac{me^4}{8\varepsilon_0^2\hbar^3 c} = 1.097 \times 10^7 \text{ m}^{-1}
\]

• excellent agreement with the value of \( R \) determined from the spectrum of Hydrogen

• ionisation energy: \( n_i=1, \ n_f = \text{infinity} \)

\[
E = \frac{me^4}{8\varepsilon_0^2\hbar^2} = 13.606 \text{ eV}
\]

goes with experimental data.  eV: electron-volt
Comment on Bohr’s semi-classical model

• successful for computation of spectral lines, ionisation energy, size (for one electron system!)

BUT

• In proper quantum mechanical treatment there are no orbits (uncertainty relation) and energy levels are eigenvalues of the Schrödinger equation

• Electrons are described by wave-functions whose squared amplitude gives probability of finding particle at certain moment in time
5.4 The Franck-Hertz experiment

- shows existence of **energy levels** in Mercury atoms, in agreement with Bohr

[Diagram of the Franck-Hertz experiment after Krane]

figure: [http://hyperphysics.phy-astr.gsu.edu/hbase/FrHz.html](http://hyperphysics.phy-astr.gsu.edu/hbase/FrHz.html)
Observation:

- distance between two maxima: 4.9V
- mercury vapour emits UV light of wavelength 0.25μm when electrons have kinetic energy higher than 4.9 eV.
Explanation: for \( E > 4.9 \text{eV} \), collisions excite Mercury atoms, electrons lose 4.9eV in collision, remaining energy not sufficient to overcome retarding electric field to collector.

- Further increase in accelerating voltage, electrons gain energy again, possibility of further excitation.

- **Excitation** of one “orbiting” electron of Mercury to higher orbit, followed by drop and emission of photon:

\[
\Delta E = hf = \frac{hc}{\lambda}.
\]

- thus \( \lambda = 0.25\mu\text{m} \), in agreement with observation!
5.5 Comment on energy unit electron-volt, \( 1 \text{eV} = 1.602 \times 10^{-19} \text{J} \)

charge \( q \) released into capacitor, applied voltage \( U \), distance between plates \( d \)

- Strength of electric field: \( E = \frac{U}{d} \)
- Force \( F \) on charge: \( F = qE \), resulting in acceleration
- Energy: force \( \times \) distance = \( Fd = qEd = qU \)
- **One electron-volt**: energy that an electron acquires when accelerated by one Volt:
  \( 1 \text{eV} = e \times 1 \text{V} = 1.6 \times 10^{-19} \text{C V} = 1.6 \times 10^{-19} \text{J} \)
- Electron-volt: standard energy unit in atomic and (sub-) nuclear physics.
Express mass $m$ of a particle using Einstein’s relation $E=mc^2$

*Example:* Rest mass of electron,

$$m = 9.109 \times 10^{-31} \text{ kg}$$

$$E = 9.109 \times 10^{-31} \text{ kg} \times (2.998 \times 10^8 \text{ m/s})^2 = 8.187 \times 10^{-14} \text{ J};$$

$$E = 0.511 \times 10^6 \text{ eV} = 0.511 \text{ MeV}; \text{ (Mega electron volt)}$$

Write rest mass as $m = E/c^2 = 0.511 \text{ MeV} / c^2$

The $c^2$ factor is omitted in some text-books.