PY2N20
Material Properties and Phase Diagrams
Lecture 4

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Elastic means reversible!

Elastic Deformation

1. Initial
2. Small load
3. Unload

Linear-elastic
Non-Linear-elastic
Plastic Deformation (Metals)

Plastic means permanent!

1. Initial

2. Small load
   - bonds stretch & planes shear

3. Unload
   - planes still sheared

δ elastic + plastic

F

δ plastic

Linear elastic
Engineering Stress

- **Tensile stress, $\sigma$:**

$$\sigma = \frac{F_t}{A_o} = \text{Pa or} \frac{\text{N}}{\text{m}^2}$$

original area before loading

- **Shear stress, $\tau$:**

$$\tau = \frac{F_s}{A_o}$$

\[ \therefore \text{Stress has units:} \quad \text{N/m}^2 \text{ or Pa} \]
Engineering Strain

- **Tensile strain:**
  \[ \varepsilon = \frac{\delta}{L_0} \]

- **Lateral strain:**
  \[ \varepsilon_L = -\frac{\delta_L}{W_0} \]

- **Shear strain:**
  \[ \gamma = \frac{\Delta x}{y} = \tan \theta \]

Strain is always dimensionless.

Adapted from Fig. 6.1 (a) and (c), *Callister 7e.*
Stress-Strain Testing

• Typical tensile test

Adapted from Fig. 6.3, Callister 7e. (Fig. 6.3 is taken from H.W. Hayden, W.G. Moffatt, and J. Wulff, The Structure and Properties of Materials, Vol. III, Mechanical Behavior, p. 2, John Wiley and Sons, New York, 1965.)

• Typical tensile specimen

Adapted from Fig. 6.2, Callister 7e.
Linear Elastic Properties

- Modulus of Elasticity, $E$: (also known as Young's modulus)

- Hooke's Law:

\[ \sigma = E \varepsilon \]
Poisson's ratio, $\nu$

- **Poisson's ratio, $n$:**

  \[ \nu = -\frac{\varepsilon_L}{\varepsilon} \]

  - **Units:**
    - $E$: [GPa] or [psi]
    - $\nu$: dimensionless

  - **Metals:** $\nu \sim 0.33$
  - **Ceramics:** $\nu \sim 0.25$
  - **Polymers:** $\nu \sim 0.40$

- For $\nu > 0.50$ density increases
- For $\nu < 0.50$ density decreases (voids form)
Mechanical Properties

- Slope of stress-strain plot (which is proportional to the elastic modulus) depends on bond strength in metals

Adapted from Fig. 6.7, *Callister 7e.*
Anisotropy – Crystals and Textures

Tensor

\[
\begin{bmatrix}
\sigma_{11} & \sigma_{12} & \sigma_{13} \\
\sigma_{21} & \sigma_{22} & \sigma_{23} \\
\sigma_{31} & \sigma_{32} & \sigma_{33}
\end{bmatrix}
\]

or

\[
\begin{bmatrix}
\sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\
\sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\
\sigma_{zx} & \sigma_{zy} & \sigma_{zz}
\end{bmatrix}
\]

For an arbitrary direction

\[
\begin{bmatrix}
T_1 \\
T_2 \\
T_3
\end{bmatrix}
= \begin{bmatrix}
\sigma_{11} & \sigma_{12} & \sigma_{13} \\
\sigma_{21} & \sigma_{22} & \sigma_{23} \\
\sigma_{31} & \sigma_{32} & \sigma_{33}
\end{bmatrix}
\begin{bmatrix}
n_1 \\
n_2 \\
n_3
\end{bmatrix}
\]

Not all components are independent – the tensor is symmetric!

\[
\begin{bmatrix}
\sigma_x & \tau_{xy} & \tau_{xz} \\
\tau_{yx} & \sigma_y & \tau_{yz} \\
\tau_{zx} & \tau_{yz} & \sigma_z
\end{bmatrix}
\]

Just for an idea – detailed understanding is not required…
Other Elastic Properties

• Elastic Shear modulus, $G$:

$$
\tau = G \gamma
$$

• Elastic Bulk modulus, $K$:

$$
P = -K \frac{\Delta V}{V_0}
$$

• Special relations for isotropic materials:

$$
G = \frac{E}{2(1 + \nu)} \quad \quad \quad K = \frac{E}{3(1 - 2\nu)}
$$

- Simple torsion test
- Pressure test: Init. vol = $V_0$. Vol chg. = $\Delta V$
Based on data in Table B2, *Callister 7e.*

Composite data based on reinforced epoxy with 60 vol% of aligned carbon (CFRE), aramid (AFRE), or glass (GFRE) fibers.
Useful Linear Elastic Relationships

- **Simple tension:**
  \[ \delta = \frac{FLo}{EAo} \]
  \[ \delta L = -\nu \frac{Fw_o}{EA_o} \]

- **Simple torsion:**
  \[ \alpha = \frac{2MLo}{\pi r_o^4 G} \]

- Material, geometric, and loading parameters all contribute to deflection.
- Larger elastic moduli minimize elastic deflection.
Plastic (Permanent) Deformation

(at lower temperatures, i.e. $T < T_{\text{melt}}/3$)

- Simple tension test:
  - Engineering stress, $\sigma$
  - Engineering strain, $\varepsilon$

Elastic initially

Elastic+Plastic at larger stress

Permanent (plastic) after load is removed

Plastic strain

Adapted from Fig. 6.10 (a), Callister 7e.
Yield Strength, $\sigma_y$

- Stress at which *noticeable* plastic deformation has occurred.

When $\varepsilon_p = 0.002$

$$\sigma_y = \text{yield strength}$$

Note: for 2 inch sample

$$\varepsilon = 0.002 = \frac{\Delta z}{z}$$

$$\therefore \Delta z = 0.004 \text{ in or about } 100 \mu m$$

Adapted from Fig. 6.10 (a), *Callister 7e.*
Based on data in Table B4, *Callister 7e.*

- **a** = annealed
- **hr** = hot rolled
- **ag** = aged
- **cd** = cold drawn
- **cw** = cold worked
- **qt** = quenched & tempered
Tensile Strength, TS

• Maximum stress on engineering stress-strain curve.

Adapted from Fig. 6.11, *Callister 7e.*

\[ F = \text{fracture or ultimate strength} \]

Neck – acts as stress concentrator

• **Metals**: occurs when noticeable necking starts.
• **Polymers**: occurs when polymer backbone chains are aligned and about to break.
**Tensile Strengths - Comparison**

<table>
<thead>
<tr>
<th>Metals/Alloys</th>
<th>Graphite/Ceramics/Semicond</th>
<th>Polymers</th>
<th>Composites/fibers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si crystal &lt;100&gt;</td>
<td>Graphite/Al oxide</td>
<td>Composites/fibers</td>
<td>Wood (fiber)</td>
</tr>
<tr>
<td><strong>Steel (4140) ag</strong></td>
<td><strong>Si nitride</strong></td>
<td>**AFRE(</td>
<td></td>
</tr>
<tr>
<td><strong>W (pure)</strong></td>
<td><strong>Al oxide</strong></td>
<td>**GFRE(</td>
<td></td>
</tr>
<tr>
<td><strong>Ti (5Al-2.5Sn) a</strong></td>
<td><strong>Si crystal</strong></td>
<td>**wood(</td>
<td></td>
</tr>
<tr>
<td><strong>Steel (4140) a</strong></td>
<td><strong>Diamond</strong></td>
<td><strong>GFRE(⊥ fiber)</strong></td>
<td><strong>GFRE(⊥ fiber)</strong></td>
</tr>
<tr>
<td><strong>Cu (71500) cw</strong></td>
<td><strong>Glass-soda</strong></td>
<td><strong>CFRE(⊥ fiber)</strong></td>
<td><strong>CFRE(⊥ fiber)</strong></td>
</tr>
<tr>
<td><strong>Cu (71500) hr</strong></td>
<td><strong>Concrete</strong></td>
<td><strong>wood(⊥ fiber)</strong></td>
<td><strong>wood(⊥ fiber)</strong></td>
</tr>
<tr>
<td><strong>Ti (pure) a</strong></td>
<td><strong>Graphite</strong></td>
<td><strong>HDPE</strong></td>
<td><strong>HDPE</strong></td>
</tr>
<tr>
<td><strong>Al (6061) a</strong></td>
<td><strong>LDPE</strong></td>
<td><strong>PET</strong></td>
<td><strong>PET</strong></td>
</tr>
<tr>
<td><strong>Ti (pure) a</strong></td>
<td><strong>PP</strong></td>
<td><strong>Nylon 6.6</strong></td>
<td><strong>Nylon 6.6</strong></td>
</tr>
<tr>
<td><strong>Ta (pure)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Al (6061) a</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Room Temp. values**

Based on data in Table B4, *Callister 7e*.

- **a** = annealed
- **hr** = hot rolled
- **ag** = aged
- **cd** = cold drawn
- **cw** = cold worked
- **qt** = quenched & tempered
- **AFRE, GFRE, & CFRE** = aramid, glass, & carbon fiber-reinforced epoxy composites, with 60 vol% fibers.
Ductility

- Plastic tensile strain at failure:

\[ \% \text{EL} = \frac{L_f - L_o}{L_o} \times 100 \]

- Another ductility measure:

\[ \% \text{RA} = \frac{A_o - A_f}{A_o} \times 100 \]

Adapted from Fig. 6.13, Callister 7e.
Toughness

- Energy to break a unit volume of material
- Approximate by the area under the stress-strain curve.

Brittle fracture: elastic energy
Ductile fracture: elastic + plastic energy

Adapted from Fig. 6.13, Callister 7e.
Resilience, $U_r$

- Ability of a material to store energy
- Energy stored best in elastic region

If we assume a linear stress-strain curve this simplifies to:

$$U_r = \int_{0}^{\varepsilon_y} \sigma \, d\varepsilon$$

Adapted from Fig. 6.15, *Callister 7e.*
Elastic Strain Recovery

Adapted from Fig. 6.17, *Callister 7e.*
Hardness

- Resistance to permanently indenting the surface.
- Large hardness means:
  - resistance to plastic deformation or cracking in compression.
  - better wear properties.

- Smaller indents mean larger hardness.

E.g., 10 mm sphere apply known force

Measure size of indent after removing load

Increasing hardness:

- Most plastics
- Brasses Al alloys
- Easy to machine steels
- File hard
- Cutting tools
- Nitrided steels
- Diamond
Hardness Measures

- Rockwell
  - No major sample damage
  - Each scale runs to 130 but only useful in range 20-100.
  - Minor load 10 kg
  - Major load 60 (A), 100 (B) & 150 (C) kg
    - A = diamond, B = 1/16 in. ball, C = diamond

- HB = Brinell Hardness
  - \[ TS \text{ (psia)} = 500 \times HB \]
  - \[ TS \text{ (MPa)} = 3.45 \times HB \]
# Hardness Measurements

## Table 6.4 Hardness Testing Techniques

<table>
<thead>
<tr>
<th>Test</th>
<th>Indenter</th>
<th>Shape of Indentation</th>
<th>Load</th>
<th>Formula for Hardness Number</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brinell</td>
<td>10-mm sphere of steel or tungsten carbide</td>
<td><img src="image" alt="Brinell Diagram" /></td>
<td>$P$</td>
<td>$\text{HB} = \frac{2P}{\pi D[D - \sqrt{D^2 - d^2}]}$</td>
<td></td>
</tr>
<tr>
<td>Vickers microhardness</td>
<td>Diamond pyramid</td>
<td><img src="image" alt="Vickers Diagram" /></td>
<td>$P$</td>
<td>$\text{HV} = 1.854P/d_1^3$</td>
<td></td>
</tr>
<tr>
<td>Knoop microhardness</td>
<td>Diamond pyramid</td>
<td><img src="image" alt="Knoop Diagram" /></td>
<td>$P$</td>
<td>$\text{HK} = 14.2P/l^2$</td>
<td>$l/b = 7.11$, $b/t = 4.00$</td>
</tr>
<tr>
<td>Rockwell and Superficial Rockwell</td>
<td>Diamond cone, $\frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{3}{16}$ in. diameter steel spheres</td>
<td><img src="image" alt="Rockwell Diagram" /></td>
<td>$60$ kg, $100$ kg, $150$ kg, $15$ kg, $30$ kg, $45$ kg</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*For the hardness formulas given, $P$ (the applied load) is in kg, while $D$, $d$, $d_1$, and $l$ are all in mm.

Note: S.A. changes when sample stretched

- True stress: $\sigma_T = \frac{F}{A_i}$
- True Strain: $\varepsilon_T = \ln \left( \frac{l_i}{l_o} \right)$

Adapted from Fig. 6.16, *Callister 7e.*
Hardening

- Curve fit to the stress-strain response:
  \[ \sigma_T = K T \varepsilon^n \]

  - "true" stress \((F/A)\)
  - "true" strain: \(\ln(L/L_0)\)

  - An increase in \(\sigma_Y\) due to plastic deformation.

  - Large hardening for \(n = 0.15\) (some steels)
  - Small hardening for \(n = 0.5\) (some coppers)
Design or Safety Factors

- Design uncertainties mean we do not push the limit.
- Factor of safety, $N$

$$\sigma_{working} = \frac{\sigma_y}{N}$$

Often $N$ is between 1.2 and 4

- Example: Calculate a diameter, $d$, to ensure that yield does not occur in the 1045 carbon steel rod below. Use a factor of safety of 5.

1045 plain carbon steel:
- $\sigma_y = 310$ MPa
- $TS = 565$ MPa
- $F = 220,000N$

$$\frac{220,000N}{\pi(d^2 / 4)} = 5$$

$$\frac{\sigma_y}{N} = \frac{310}{5} = 62$$

$d = 0.067 m = 6.7 cm$