Topic 7: The New-Keynesian Phillips Curve

The Phillips curve has been a central topic in macroeconomis since the 1950s and its successes and failures have been a major element in the evolution over time of the discipline. We will now discuss how a popular modern version of the Phillips curve, known as the "New Keynesian" Phillips curve, that is consistent with rational expectations. We will start, however, with a brief review of the history of the Phillips curve relationship. It is strongly recommended that, in addition to these notes, you take a look at "Inflation Dynamics: A Structural Econometric Analysis" by Jordi Galí and Mark Gertler. This paper can be downloaded at `www.nyu.edu/econ/user/gertlerm/jme99.pdf`

The Phillips Curve

The idea that there should be some sort of positive relationship between inflation and output has been around almost as long as economics itself, but the modern incarnation of this relationship is usually traced to a late 1950s study by the LSE's A.W. Phillips, which documented a statistical relationship between wage inflation and unemployment in the UK. This "Phillips Curve" relationship was then also found to work well for price inflation and for other economies, and it became a key part of the standard Keynesian textbook model of the 1960s. As Keynesian economists saw it, the Phillips curve provided a menu of tradeoffs for policy-makers: They could use demand management policies to increase output and decrease unemployment, but this could only be done at the expense of higher inflation.

In his 1968 presidential address to the American Economic Association, Milton Friedman presented a sharp critique of the Keynesian Phillips curve. In particular, he criticized its treatment of expectations.[1] The Keynesian model implicitly relied on the idea that low unemployment could be sustained by allowing high inflation to erode real wages and thus boost labour demand. Friedman pointed out that if policy tried to keep output above its "potential" or "equilibrium" level, then wage-bargainers would get used to the higher level of inflation and adjust their nominal wage demands upwards. The result would be higher inflation without the sustainable low unemployment. Empirical evidence seemed to subsequently back up Friedman's argument, as the 1970s saw the "stagflation" combination of high inflation and high unemployment that the Phillips curve relationship seemed to rule out.

This "demise" of the traditional Phillips curve, and the sense that it was due to inade-

[1] Milton Friedman (1968). "The Role of Monetary Policy." *American Economic Review*, 58, 1-17.

quate modelling of expectations, was a major impetus for the rational expectations school of thought in the 1970s, led by Robert Lucas and Thomas Sargent. And, in addition to being more precise about expectations formation, this school of economists relied more heavily on neoclassical "microfoundations" for macroeconomic models. Often, as well as rejecting the Phillips curve, these economists also questioned the whole basis for Keynesian economics, i.e. the assumption that monetary policy could systematically affect output even in the short-run.

The principal response of Keynesian economists to these theoretical critiques has been to attempt to build models that incorporate rational expectations and that provide a microeconomic justification for monetary policy having at least short-run effects. The principal microeconomic rationale has been *sticky prices*. Without some type of price rigidity, it is difficult to rationalise the idea that there can be periods during which factors of production, such as labour, are under-utilized, with aggregate output being below its so-called potential level. Once we assume that at least some prices are rigid, then not all markets are clearing instantaneously and aggregate output may sometimes be below what would obtain when all prices move flexibly. Also, with sticky prices, an increase in the money stock can produce a short-run increase in real spending power and thus can boost real output.

This modern approach featuring rational expectations and some form of microfoundations is known as *New Keynesian* macroeconomics. We will now describe one of the key New-Keynesian models, and explore its implications for the behaviour of inflation and output.

Pricing à la Calvo

There are lots of different ways of formulating the idea that prices may be sticky. Some of the best known formulations were those introduced in papers in the late seventies by John Taylor and Stanley Fischer.[2] These papers essentially invented New Keynesian economics. Here, however, we will use a formulation known as *Calvo pricing*, after the economist who first introduced it.[3] Though not the most realistic formulation of sticky prices, it turns out to provide analytically convenient expressions, and has implications that are very similar

[2] See, for instance, Stanley Fischer (1977), "Long-Term Contracts, Rational Expectations, and the Optimal Money Supply Rule," *Journal of Political Economy*, 85, 191-205, and John Taylor (1979), "Staggered Wage Setting in a Macro Model," *American Economic Review*, Papers and Proceedings, Vol. 69, 108-113.

[3] Guillermo Calvo, "Staggered Contracts in a Utility-Maximizing Framework" *Journal of Monetary Economics*, September 1983.

to those of more realistic (but more complicated) formulations.

The form of price rigidity faced by the Calvo firm is as follows. Each period, only a random fraction $(1-\theta)$ of firms are able to reset their price; all other firms keep their prices unchanged. When firms do get to reset their price, they must take into account that the price may be fixed for many periods. We assume they do this by choosing a log-price, z_t, that minimizes the "loss function"

$$L(z_t) = \sum_{k=0}^{\infty} (\theta\beta)^k E_t \left(z_t - p^*_{t+k}\right)^2 \tag{1}$$

where β is between zero and one, and p^*_{t+k} is the log of the optimal price that the firm would set in period $t+k$ if there were no price rigidity.

This expression probably looks a bit intimidating, so it's worth discussing it a bit to explain what it means. The loss function has a number of different elements:

- The term $E_t\left(z_t - p^*_{t+k}\right)^2$ describes the expected loss in profits for the firm at time $t+k$ due to the fact that it will not be able to set a frictionless optimal price that period. This quadratic function is intended just as an approximation to some more general profit function. What is important here is to note that because the firm may be stuck with the price z_t for some time, it will lose profits relative to what it would have been able to obtain if there were no price rigidities.
- The summation $\sum_{k=0}^{\infty}$ shows that the firm considers the implications of the price set today for all possible future periods.
- However, the fact that $\beta < 1$ implies that the firm places less weight on future losses than on today's losses. A dollar today is worth more than a dollar tomorrow because it can be re-invested. By the same argument, a dollar lost today is more important than a dollar lost tomorrow.
- Future losses are actually discounted at rate $(\theta\beta)^k$, not just β^k. This is because the firm only considers the *expected* future losses from the price being fixed at z_t. The chance that the price will be fixed until $t+k$ is θ^k. So the period $t+k$ loss is weighted by this probability. There is no point in the firm worrying too much about losses that might occur from having the wrong price far off in the future, when it is unlikely that the price will remained fixed for that long.

The Optimal Reset Price

After all that, the actual solution for the optimal value of z_t, (i.e. the price chosen by the firms who get to reset) is quite simple. Each of the terms featuring the choice variable z_t—that is, each of the $\left(z_t - p^*_{t+k}\right)^2$ terms—need to be differentiated with respect to z_t and then the sum of these derivatives is set equal to zero. This means

$$L'(z_t) = 2\sum_{k=0}^{\infty} (\theta\beta)^k E_t \left(z_t - p^*_{t+k}\right) = 0 \tag{2}$$

Separating out the z_t terms from the p^*_{t+k} terms, this implies

$$\left[\sum_{k=0}^{\infty} (\theta\beta)^k\right] z_t = \sum_{k=0}^{\infty} (\theta\beta)^k E_t p^*_{t+k} \tag{3}$$

Now, we can use our old pal the geometric sum formula to simplify the left side of this equation. In other words, we use the fact that

$$\sum_{k=0}^{\infty} (\theta\beta)^k = \frac{1}{1-\theta\beta} \tag{4}$$

to re-write the equation as

$$\frac{z_t}{1-\theta\beta} = \sum_{k=0}^{\infty} (\theta\beta)^k E_t p^*_{t+k} \tag{5}$$

implying a solution of the form

$$z_t = (1-\theta\beta)\sum_{k=0}^{\infty} (\theta\beta)^k E_t p^*_{t+k} \tag{6}$$

Stated in English, all this equation says is that the optimal solution is for the firm to set its price equal to a weighted average of the prices that it would have expected to set in the future if there weren't any price rigidities. Unable to change price each period, the firm chooses to try to keep close "on average" to the right price.

And what is this "frictionless optimal" price, p^*_t? We will assume that the firm's optimal pricing strategy without frictions would involve setting prices as a fixed markup over marginal cost:

$$p^*_t = \mu + mc_t \tag{7}$$

Thus, the optimal reset price can be written as

$$z_t = (1-\theta\beta)\sum_{k=0}^{\infty} (\theta\beta)^k E_t \left(\mu + mc_{t+k}\right) \tag{8}$$

The New-Keynesian Phillips Curve

Now, we can show how to derive the behaviour of aggregate inflation in the Calvo economy. The following derivation is a bit subtle, and you will not be asked to repeat it in the exam.

The aggregate price level in the Calvo economy is just a weighted average of last period's aggregate price level and the new reset price, where the weight is determined by θ:

$$p_t = \theta p_{t-1} + (1-\theta) z_t, \tag{9}$$

This can be re-arranged to express the reset price as a function of the current and past aggregate price levels

$$z_t = \frac{1}{1-\theta}(p_t - \theta p_{t-1}) \tag{10}$$

Now, let's examine equation (8) for the optimal reset price again. We have shown that the first-order stochastic difference equation

$$y_t = a x_t + b E_t y_{t+1} \tag{11}$$

can be solved to give

$$y_t = a \sum_{k=0}^{\infty} b^k E_t x_{t+k} \tag{12}$$

Examining equation (8), we can see that z_t must obey a first-order stochastic difference equation with

$$\begin{aligned} y_t &= z_t & (13)\\ x_t &= \mu + mc_t & (14)\\ a &= 1-\theta\beta & (15)\\ b &= \theta\beta & (16) \end{aligned}$$

In other words, we can write the reset price as

$$z_t = \theta\beta E_t z_{t+1} + (1-\theta\beta)(\mu + mc_t) \tag{17}$$

Substituting in the expression for z_t in equation (10) we get

$$\frac{1}{1-\theta}(p_t - \theta p_{t-1}) = \frac{\theta\beta}{1-\theta}(E_t p_{t+1} - \theta p_t) + (1-\theta\beta)(\mu + mc_t) \tag{18}$$

After a bunch of re-arrangements, this equation can be shown to imply

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta}(\mu + mc_t - p_t) \tag{19}$$

where $\pi_t = p_t - p_{t-1}$ is the inflation rate.

This equation is known as the *New-Keynesian Phillips Curve.* It states that inflation is a function of two factors:

- Next period's expected inflation rate, $E_t \pi_{t+1}$.
- The gap between the frictionless optimal price level $\mu + mc_t$ and the current price level p_t. Another way to state this is that inflation depends positively on *real marginal cost*, $mc_t - p_t$.

Why is real marginal cost a driving variable for inflation? Firms in the Calvo model would like to keep their price as a fixed markup over marginal cost. If the ratio of marginal cost to price is getting high (i.e. if $mc_t - p_t$ is high) then this will spark inflationary pressures because those firms that are re-setting prices will, on average, be raising them.

Real Marginal Cost and Output

For simplicity, we will denote the deviation of real marginal cost from its frictionless level of $-\mu$ as

$$\hat{mc}_t^r = \mu + mc_t - p_t \tag{20}$$

so we can write the NKPC as

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta} \hat{mc}_t^r \tag{21}$$

One problem with attempting to implement this model empirically, is that we don't actually observe data on real marginal cost. National accounts data contain information on the factors that affect *average* costs such as wages, but do not tell us about the cost of producing an additional unit of output. That said, it seems very likely that marginal costs are procyclical, and more so than prices. When production levels are high relative to potential output, there is more competition for the available factors of production, and this leads to increases in real costs, i.e. increases in the costs of the factors over and above increases in prices. Some examples of the procyclicality of real marginal costs are fairly obvious. For example, the existence of overtime wage premia generally means a substantial jump in the marginal cost of labour once output levels are high enough to require more than the standard workweek.

For these reasons, many researchers implement the NKPC using a measure of the *output gap* (the deviation of output from its potential level) as a proxy for real marginal cost. In other words, they assume a relationship such as

$$\hat{mc}_t^r = \lambda y_t \tag{22}$$

where y_t is the output gap. This implies a New-Keynesian Phillips curve of the form

$$\pi_t = \beta E_t \pi_{t+1} + \gamma y_t \tag{23}$$

where

$$\gamma = \frac{\lambda (1-\theta)(1-\theta\beta)}{\theta} \tag{24}$$

And this approach can be implemented empirically.

The "Asset-Price-Like" Behaviour of NKPC Inflation

The New-Keynesian approach assumes that firms have rational expectations. Thus, we can apply the repeated substitution method to equation (23) to arrive at

$$\pi_t = \gamma \sum_{k=0}^{\infty} \beta^k E_t y_{t+k} \tag{25}$$

Inflation today depends on the whole sequence of expected future output gaps. Thus, the NKPC sees inflation as behaving according to the classic "asset-price" logic that we saw with the dividend-discount stock price model.

The NKPC, Monetary Policy and the Lucas Critique

Essentially all practising macroeconomists now accept Friedman's critique of the original Phillips curve. Thus, it is widely accepted that inflation expectations will move upwards over time if output remains above its potential level, and that there is little or scope for policy-makers to choose a tradeoff between inflation and output. However, many macroeconomists do believe that there is a relationship of the form

$$\pi_t = \pi_{t-1} + \alpha - \beta u_t \tag{26}$$

So there is a relationship between the *change* in inflation and the level of unemployment. In this formulation, the lagged inflation term reflects how last period's level of inflation changes people's expectations and so feeds into today's inflation. This so-called *accelerationist*

Phillips curve fits the data quite well (or, more precisely, empirical approaches based on a weighted average of past inflation rates, not just last period's, fit the data well) and comes with its own well-known terminology. Specifically, economists often speak of the so-called NAIRU—the non-accelerating inflation rate of unemployment. This is the inflation rate consistent with constant inflation and it is defined implicitly by

$$\alpha - \beta u^* = 0 \Rightarrow u^* = \frac{\alpha}{\beta} \tag{27}$$

Empirical estimates of the NAIRU are often invoked in real-world policy discussions, with the policy recommendations made on the basis of whether unemployment is above or below this NAIRU level.[4]

The fact that inflation depends on its own lagged values in this formulation also has important implications for monetary policy. Consider, for instance, a central bank that wants to reduce inflation from a high level. If this Phillips curve is correct, then it will be very difficult to reduce inflation quickly without a significant increase in unemployment. So, this Phillips curve suggests that gradualist policies are the best way to reduce inflation.

The NKPC model turns much of this standard reasoning on its head. While advocates of the NKPC will concede that the accelerationist model, equation (26), fits the data reasonably well, they view this as a so-called reduced-form relationship, not a structural relationship. In other words, if the true model is

$$\pi_t = \beta E_t \pi_{t+1} + \gamma y_t \tag{28}$$

then equation (26) might have a good statistical fit because π_{t-1} is likely to be correlated with $E_t \pi_{t+1}$. However, they would warn policy-makers not to rely on this relationship, because changes in policy may produce a break the correlation between $E_t \pi_{t+1}$ and π_{t-1} and at this point the statistical accelerationst Phillips curve will break down.[5]

But the implications for monetary policy are completely different. There may be a statistical relationship between current and lagged inflation but the NKPC says that there

[4] Note though the NAIRU terminology is actually a misnomer. If unemployment is below u^*, then inflation will be increasing, but not accelerating. The price level is what will be accelerating. Perhaps the NAIRU should be changed to the NAPLRU, but this isn't so catchy so the "slipped derivative" is probably here to stay.

[5] There is currently an ongoing debate about whether or not the coefficients of econometric Phillips curves are stable over time. A contribution to this debate by your lecturer can be found in Gerard O'Reilly and Karl Whelan (2004), "Has Euro-Area Inflation Persistence Changed Over Time?" ECB Working Paper No. 335.

is no structural relationship at all. Thus, there is no need for gradualist policies to reduce inflation. According to the NKPC, low inflation can be achieved immediately by the central bank announcing (and the public believing) that it is committing itself to elminating positive output gaps in the future: This can be seen from equation (25).

Of course, whether these are good policy recommendations depends on whether or not the NKPC is a good model of the inflation-output relationship. We will discuss this issue in our next (final) handout.