Topic 4: The Dividend-Discount Model of Stock Prices

Rational Expectations and Macroeconomics

Almost all economic transactions rely crucially on the fact that the economy is not a "one-period game." In the language of macroeconomists, most economic decisions have an *intertemporal* element to them. Consider some obvious examples:

- We accept cash in return for goods and services because we know that, in the future, this cash can be turned into for goods and services for ourselves.
- You don't empty out your bank account today and go on a big splurge because you're still going to be around tommorrow and will have consumption needs then.
- Conversely, sometimes you spend more than you're earning because you can get a bank loan in anticipation of earning more in the future, and paying the loan off then.
- Similarly, firms will spend money on capital goods like trucks or computers largely in anticipation of the benefits they will bring in the future.

Another key aspect of economic transactions is that generally involve some level of *uncertainty*, so we don't always know what's going to happen in the future. Take two the examples just give. While it is true that one can accept cash in anticipation of turning into goods and services in the future, uncertainty about inflation means that we can't be sure of the exact quantity of these goods and services. Similarly, one can borrow in anticipation of higher income at a later stage, but few people can be completely certain of their future incomes.

For these reasons, people must often make economic decisions based on *expectations* of important future variables. In valuing cash, we must formulate an expectation of future values of inflation; in taking out a bank loan, we must have some expectation of our future income. These expectations will almost certainly turn out to have been incorrect to some degree, but one still has to formulate them before making these decisions.

So, a key issue in macroeconomic theory is how people formulate expectations of economic variables in the presence of uncertainty. Prior to the 1970s, this aspect of macro theory was largely *ad hoc*. Different researchers took different approaches, but generally it was assumed that agents used some simple extrapolative rule whereby the expected future value of a variable was close to some weighted average of its recent past values. However, such models were widely criticised in the 1970s by economists such as Robert Lucas and Thomas Sargent. Lucas and Sargent instead proposed the use of an alternative approach which they called "rational expectations."

Now, the idea that agents expectations are somehow "rational" has various possible interpretations. However, when economists say that agents in a model have rational expectations, they mean a very specific thing: The agents understand the structure of the model economy and base their expectations of variables on this knowledge.[1]

To many economists, this is a natural baseline assumption: We usually assume agents behave in an optimal fashion, so why would we assume that the agents don't understand the structure of the economy, and formulate expectations in some sub-optimal fashion. That said, rational expectations models generally produce quite strong predictions, and these can be tested. Ultimately, any assessment of a rational expectations model must be based upon its ability to fit the relevant macro data.

Stock Prices

The first class of rational expectations models that we will look relate to the determination of stock prices. One reason to start here is that the determination of stock prices is a classic example of the importance of expectations. When one buys a stock today, there is usually no immediate benefit at all: The benefit comes in the future when one receives a flow of dividend payments, and/or sells the stock for a gain. Another reason to study this topic is that understanding the behaviour of asset prices is important for macroeconomists because the movements in wealth caused by asset price fluctuations have important effects on aggregate demand. Finally, as we will see in the rest of the course, the modern theory of the determination of stock prices provides a very useful example of the type of methods used in the so-called rational expectations class of macroeconomic models.

Definitions

A person who purchases a stock today for price P_t and sells it tomorrow for price P_{t+1} generates a rate of return on this investment of

$$r_{t+1} = \frac{D_t + \Delta P_{t+1}}{P_t} \tag{1}$$

[1]This type of expectational assumption is sometimes labelled *model-consistent* expectations.

This rate of return has two components, the first reflects the *dividend payment*, D_t received during the period the stock was held, and the second reflects the *capital gain* (or loss) due to the price of the stock changing from period t to period $t+1$. This can also be written in terms of the so-called *gross return* which is just one plus the rate of return.

$$1 + r_{t+1} = \frac{D_t + P_{t+1}}{P_t} \tag{2}$$

A useful re-arrangement of this equation that we will repeatedly work with is the following:

$$P_t = \frac{D_t}{1 + r_{t+1}} + \frac{P_{t+1}}{1 + r_{t+1}} \tag{3}$$

Stock Prices with Rational Expectations and Constant Expected Returns

We will now consider a *rational expectations* approach to the determination of stock prices. In the context of stock prices, rational expectations means investors understand equation (3) and that all expectations of future variables must be consistent with it. This implies that

$$E_t P_t = E_t \left[\frac{D_t}{1 + r_{t+1}} + \frac{P_{t+1}}{1 + r_{t+1}} \right] \tag{4}$$

where E_t means the expectation of a variable formulated at time t. The stock price at time t is observable to the agent so $E_t P_t = P_t$, implying

$$P_t = E_t \left[\frac{D_t}{1 + r_{t+1}} + \frac{P_{t+1}}{1 + r_{t+1}} \right] \tag{5}$$

A second assumption that we will make for the moment is that the return on stocks is expected to equal some constant value for all future periods:

$$E_t r_{t+k} = r \qquad k = 1, 2, 3, ... \tag{6}$$

This allows equation (5) to be re-written as

$$P_t = \frac{D_t}{1 + r} + \frac{E_t P_{t+1}}{1 + r} \tag{7}$$

The Repeated Substitution Method

Equation (7) is a specific example of what is known as a *first-order stochastic difference equation*.[2] Because such equations are commonly used in macroeconomics, it will be useful

[2] Stochastic means random or incorporating uncertainty. It applies to this equation because agents do not actually know P_{t+1} but instead formulate expectations of it.

to write down the general approach to solving these equations, rather than just focusing only on our current stock-price example. In general, this type of equation can be written as

$$y_t = ax_t + bE_t y_{t+1} \tag{8}$$

Its solution is derived using a technique called *repeated substitution.* This works as follows. Equation (8) holds in all periods, so under the assumption of rational expectations, the agents in the economy understand the equation and formulate their expectation in a way that is consistent with it:

$$E_t y_{t+1} = aE_t x_{t+1} + bE_t y_{t+2} \tag{9}$$

Substituting this into the previous equation, we get

$$y_t = ax_t + abE_t x_{t+1} + b^2 E_t y_{t+2} \tag{10}$$

Repeating this method by substituting in for $E_t y_{t+2}$, and then $E_t y_{t+3}$ and so on, we get a general solution of the form

$$y_t = ax_t + abE_t x_{t+1} + ab^2 E_t x_{t+2} + ... + ab^{N-1} E_t x_{t+N-1} + b^N E_t y_{t+N} \tag{11}$$

which can be written in more compact form as

$$y_t = a \sum_{k=0}^{N-1} b^k E_t x_{t+k} + b^N E_t y_{t+N} \tag{12}$$

The Dividend-Discount Model

Comparing equations (7) and (8), we can see that our stock price equation is a specific case of the first-order stochastic difference equation with

$$y_t = P_t \tag{13}$$

$$x_t = D_t \tag{14}$$

$$a = \frac{1}{1+r} \tag{15}$$

$$b = \frac{1}{1+r} \tag{16}$$

This implies that the stock-price can be expressed as follows

$$P_t = \sum_{k=0}^{N-1} \left(\frac{1}{1+r}\right)^{k+1} E_t D_{t+k} + \left(\frac{1}{1+r}\right)^N E_t P_{t+N} \tag{17}$$

Another assumption usually made is that this final term tends to zero as N gets big:

$$\lim_{N\to\infty}\left(\frac{1}{1+r}\right)^{N} E_t P_{t+N} = 0 \tag{18}$$

What is the logic behind this assumption? One explanation is that if it did not hold then we could set all future values of D_t equal to zero, and the stock price would still be positive. But a stock that never pays out should be inherently worthless, so this condition rules this possibility out. With this imposed, our solution becomes

$$P_t = \sum_{k=0}^{\infty}\left(\frac{1}{1+r}\right)^{k+1} E_t D_{t+k} \tag{19}$$

This equation, which states that stock prices should equal a discounted present-value sum of expected future dividends, is usually known as the *dividend-discount model*.

Constant Expected Dividend Growth: The Gordon Growth Model

A useful special case that is often used as a benchmark for thinking about stock prices is the case in which dividends are expected to grow at a constant rate such that

$$E_t D_{t+k} = (1+g)^k D_t \tag{20}$$

In this case, the dividend-discount model predicts that the stock price should be given by

$$P_t = \frac{D_t}{1+r}\sum_{k=0}^{\infty}\left(\frac{1+g}{1+r}\right)^{k} \tag{21}$$

Now, remember the old multiplier formula, which states that as long as $0 < c < 1$, then

$$1 + c + c^2 + c^3 + ... = \sum_{k=0}^{\infty} c^k = \frac{1}{1-c} \tag{22}$$

This geometric series formula gets used *a lot* in modern macroeconomics, not just in examples involving the multiplier. Here we can use it as long as $\frac{1+g}{1+r} < 1$, i.e. as long as r (the expected return on the stock market) is greater than g (the growth rate of dividends). We will assume this holds. Thus, we have

$$\begin{aligned} P_t &= \frac{D_t}{1+r}\frac{1}{1-\frac{1+g}{1+r}} && (23)\\ &= \frac{D_t}{1+r}\frac{1+r}{1+r-(1+g)} && (24)\\ &= \frac{D_t}{r-g} && (25)\end{aligned}$$

When dividend growth is expected to be constant, prices are a multiple of current dividend payments, where that multiple depends positively on the expected future growth rate of dividends and negatively on the expected future rate of return on stocks. This formula is often called the *Gordon growth model*, after the economist that popularized it.[3]

Allowing for Variations in Dividend Growth

A more flexible way to formulate expectations about future dividends is to assume that dividends fluctuate around a steady-growth trend. An example this is

$$D_t = c(1+g)^t + u_t \tag{26}$$

$$u_t = \rho u_{t-1} + \epsilon_t \tag{27}$$

These equations state that dividends are the sum of two processes: The first grows at rate g each period. The second, u_t, measures a cyclical component of dividends, and this follows what is known as a first-order autoregressive process (AR(1) for short). Here ϵ_t is a zero-mean random "shock" term. Over large samples, we would expect u_t to have an average value of zero, but deviations from zero will be more persistent the higher is the value of the parameter ρ.

We will now derive the dividend-discount model's predictions for stock prices when dividends follow this process. The model predicts that

$$P_t = \sum_{k=0}^{\infty} \left(\frac{1}{1+r}\right)^{k+1} E_t \left(c(1+g)^{t+k} + u_{t+k}\right) \tag{28}$$

Let's split this sum into two. First the trend component,

$$\sum_{k=0}^{\infty} \left(\frac{1}{1+r}\right)^{k+1} E_t \left(c(1+g)^{t+k}\right) = \frac{c(1+g)^t}{1+r} \sum_{k=0}^{\infty} \left(\frac{1+g}{1+r}\right)^k \tag{29}$$

$$= \frac{c(1+g)^t}{1+r} \frac{1}{1-\frac{1+g}{1+r}} \tag{30}$$

$$= \frac{c(1+g)^t}{1+r} \frac{1+r}{1+r-(1+g)} \tag{31}$$

$$= \frac{c(1+g)^t}{r-g} \tag{32}$$

[3] The formula appeared in Myron Gordon's 1962 book *The Investment, Financing and Valuation of the Corporation.*

Second, the cyclical component. Because $E(\epsilon_{t+k}) = 0$, we have

$$E_t u_{t+1} = E_t(\rho u_t + \epsilon_{t+1}) = \rho u_t \tag{33}$$
$$E_t u_{t+2} = E_t(\rho u_{t+1} + \epsilon_{t+2}) = \rho^2 u_t \tag{34}$$
$$E_t u_{t+k} = E_t(\rho u_{t+k-1} + \epsilon_{t+k}) = \rho^k u_t \tag{35}$$

So, this second sum can be written as

$$\sum_{k=0}^{\infty} \left(\frac{1}{1+r}\right)^{k+1} E_t u_{t+k} = \frac{u_t}{1+r} \sum_{k=0}^{\infty} \left(\frac{\rho}{1+r}\right)^k \tag{36}$$
$$= \frac{u_t}{1+r} \frac{1}{1-\frac{\rho}{1+r}} \tag{37}$$
$$= \frac{u_t}{1+r} \frac{1+r}{1+r-\rho} \tag{38}$$
$$= \frac{u_t}{1+r-\rho} \tag{39}$$

Putting these two sums together, the stock price at time t is

$$P_t = \frac{c(1+g)^t}{r-g} + \frac{u_t}{1+r-\rho} \tag{40}$$

In this case, stock prices don't just grow at a constant rate. Instead they depend positively on the cyclical component of dividends, u_t, and the more persistent are these cyclical deviations (the higher ρ is), the larger is their effect on stock prices. To give a concrete example, suppose $r = 0.1$. When $\rho = 0.9$ the coefficient on u_t is

$$\frac{1}{1+r-\rho} = \frac{1}{1.1-0.9} = 5 \tag{41}$$

But if $\rho = 0.6$, then the coefficient falls to

$$\frac{1}{1+r-\rho} = \frac{1}{1.1-0.6} = 2 \tag{42}$$

Note also that when taking averages over long periods of time, the u components of dividends and prices will average to zero. Thus, over longer averages the Gordon growth model would be approximately correct, even though the dividend-price ratio isn't always constant. Instead, prices would tend to be temporarily high relative to dividends during periods when dividends are expected to grow at above-average rates for a while, and would be temporarily low when dividend growth is expected to be below average for a while. This is why the Gordon formula is normally seen as a guide to long-run average valuations rather than a prediction as to what the market should be right now.

Problems for the Model

Despite its widespread popularity as an analytical tool in stock market analysis, the version of the dividend-discount model that we have been analyzing has some problems as an empirical model of stock prices. One important aspect of the data which doesn't match the model is the *volatility* of stock prices. In an important 1981 contribution, Yale economist Robert Shiller argued that stocks were much too volatile to be generated by this model.[4] Shiller pointed out that equation (19) can be re-written as

$$\sum_{k=0}^{\infty}\left(\frac{1}{1+r}\right)^{k+1} D_{t+k} = P_t + \sum_{k=0}^{\infty}\left(\frac{1}{1+r}\right)^{k+1}\left(D_{t+k} - E_t D_{t+k}\right) \tag{43}$$

In other words, the *realized* present value of dividends starting at time t should equal P_t plus a term that reflects the expectational errors made at time t. Shiller pointed out that if expectations were rational then these errors should not be correlated with the information available at time t: Otherwise a better forecast could have been constructed. This means that the two series on the right-hand-side of (43) should be uncorrelated.

This reasoning has implications for the predicted volatility of stock prices. From basic statistical theory, we know that the variance of the sum of two uncorrelated series is just the sum of the two variances. Thus

$$Var\left[\sum_{k=0}^{\infty}\left(\frac{1}{1+r}\right)^{k+1} D_{t+k}\right] = Var(P_t) + Var\left[\sum_{k=0}^{\infty}\left(\frac{1}{1+r}\right)^{k+1}\left(D_{t+k} - E_t D_{t+k}\right)\right] \tag{44}$$

This implies that the variance of stock prices must be less than the variance of the present value of subsequent dividend movements:

$$Var(P_t) < Var\left[\sum_{k=0}^{\infty}\left(\frac{1}{1+r}\right)^{k+1} D_{t+k}\right] \tag{45}$$

A simple check reveals that this inequality does not hold: Stocks are actually much more volatile than suggested by realized movements in dividends.[5]

[4] "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?" *American Economic Review*, June 1981

[5] While technically, the infinite sum of dividends can't be calculated because we don't have data going past the present, Shiller filled in all terms after the end of his sample based on plausible assumptions, and the results are not sensitive to these assumptions.

Two other patterns that are related to Shiller's findings, but that shed extra light on the problems for the dividend-discount model:

- Dividend expectations can't be directly observed. But when economists use regression models to forecast future values of dividends and plug these forecasts in as $E_t D_{t+k}$, the resulting present value sums don't look very like the actual stock prices series. For example, Campbell and Shiller (1988) find that the dividend-price ratio is twice as volatile as the series they generate from plugging in forecasts of dividend growth.[6]

- Shiller pointed out that there appears to be a lot of movements in stock prices that never turn out to be fully justified by later changes dividends. Campbell and Shiller (2001) go farther.[7] They point out that stock prices are essentially no use at all in forecasting future dividends. They note that a high ratio of prices to dividends, instead of forecasting high growth in dividends, tends to forecast lower future returns on the stock market.

- To understand this last finding, look at the chart at the end of these notes. It shows the dividend-price ratio for the S&P 500. Though it goes through long-lasting swings, the ratio has still tended to revert over time to its mean of about 3.5 percent. If the ratio doesn't predict future dividends, then this mean-reversion must imply forecasting power of the ratio for future stock prices. In other words, if dividends are low relative to prices (as they are today) then the future mean-reversion is likely to occur through sluggish price growth, rather than through fast dividend growth.

Time-Varying Expected Returns

This last finding suggests a way to "mend" the dividend-discount model and perhaps explain the extra volatility that affects stock prices: Change the model to allow for variations in expected returns. Again a solution can be derived using repeated substitution. Let

$$R_t = 1 + r_t \tag{46}$$

and start again from the first-order difference equation for stock prices

$$P_t = \frac{D_t}{R_{t+1}} + \frac{P_{t+1}}{R_{t+1}} \tag{47}$$

[6] "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors," *Review of Financial Studies*, Autumn 1988.

[7] NBER Working Paper No. 8221.

Moving the time-subscripts forward one period, this implies

$$P_{t+1} = \frac{D_{t+1}}{R_{t+2}} + \frac{P_{t+2}}{R_{t+2}} \tag{48}$$

Substitute this into the original price equation to get

$$\begin{aligned} P_t &= \frac{D_t}{R_{t+1}} + \frac{1}{R_{t+1}}\left(\frac{D_{t+1}}{R_{t+2}} + \frac{P_{t+2}}{R_{t+2}}\right) \\ &= \frac{D_t}{R_{t+1}} + \frac{D_{t+1}}{R_{t+1}R_{t+2}} + \frac{P_{t+2}}{R_{t+1}R_{t+2}} \end{aligned} \tag{49}$$

Applying the same trick to substitute for P_{t+2} we get

$$P_t = \frac{D_t}{R_{t+1}} + \frac{D_{t+1}}{R_{t+1}R_{t+2}} + \frac{D_{t+2}}{R_{t+1}R_{t+2}R_{t+3}} + \frac{P_{t+3}}{R_{t+1}R_{t+2}R_{t+3}} \tag{50}$$

The general formula is

$$P_t = \sum_{k=0}^{N-1} \frac{D_{t+k}}{\prod\limits_{m=1}^{k+1} R_{t+m}} + \frac{P_{t+N}}{\prod\limits_{m=1}^{N} R_{t+m}} \tag{51}$$

where $\prod\limits_{n=1}^{h} x_i$ means the product of x_1, x_2 .... x_h. Again setting the limit of the $t+N$ term to zero and taking expectations, we get a version of the dividend-discount model augmented to account for variations in the expected rate of return.

$$P_t = \sum_{k=0}^{\infty} E_t \left(\frac{D_{t+k}}{\prod\limits_{m=1}^{k+1} R_{t+m}} \right) \tag{52}$$

This equation gives one potential explanation for the failure of news about dividends to explain stock price fluctuations—perhaps it is news about future stock returns that explains movements in stock prices.

What About Interest Rates?

Changing interest rates on bonds are the most obvious source of changes in expected returns on stocks. Up to now, we haven't discussed what determines the rate of return that investors require to invest in the stock market, but it is usually assumed that there is an arbitrage equation linking stock and bond returns, so that

$$E_t r_{t+1} = E_t i_{t+1} + \pi \tag{53}$$

In other words, next period's expected return on the market needs to equal next period's expected interest rate on bonds, i_{t+1}, plus a risk premium, π, which we will assume is constant.

Are interest rates the culprit accounting for the volatility of stock prices? They are certainly a plausible candidate. Stock market participants spend a lot of time monitoring the Fed and the ECB and news interpreted as implying higher interest rates in the future certainly tends to provoke declines in stock prices. Perhaps surprisingly, then, Campbell and Shiller (1988) have shown that this type of equation still doesn't help that much in explaining stock market fluctuations. Their argument involves plugging in forecasts for future interest rates and dividend growth into the right-hand-side of (52) and checking how close the resulting series is to the actual dividend-price ratio. They conclude that expected fluctuations in interest rates contribute little to explaining the volatility in stock prices. A recent Federal Reserve Board study examing the link between monetary policy and the stock market comes to the same conclusions.[8]

Time-Varying Risk Premia?

So, changes in interest rates do not appear to explain the volatility of stock market fluctuations. The final possible explanation for how the dividend-discount model may be consistent with the data is that changes in expected returns do account for the bulk of stock market movements, but that the principal source of these changes comes, not from interest rates, but from changes in the *risk premium* that determines the excess return that stocks must generate relative to bonds: The π in equation (53) must be changing over time. In an important 1991 paper, John Campbell argued this mechanism—changing expectations about future equity premia—lies behind most of the fluctuations in stock returns.[9]

A problem with this conclusion is that it implies that, most of the time, when stocks are increasing it is because investors are anticipating lower stock returns at a later date. However, the evidence that we have on this seems to point in the other direction. For example, in a recent paper, Northwestern University economist Annette Vissing-Jorgensen found that even at the peak of the most recent bull market, most investors still anticipated

[8] See "What Explains the Stock Market's Reaction to Federal Reserve Policy?", by Ben Bernanke and Kenneth Kuttner. Federal Reserve Board Finance and Economics Discussion Series, No. 2004-16. This can be downloaded at http://www.federalreserve.gov/pubs/feds/2004/200416/200416abs.html

[9] "A Variance Decomposition for Stock Returns," *Economic Journal*, March 1991

high future returns on the market.[10]

Behavioural Finance

If one rejects the idea that, together, news about dividends and news about future returns explain all of the changes in stock prices, then one is forced to reject the rational expectations dividend-discount model as a complete model of the stock market. What is missing from this model? Many believe that the model fails to take into account of various human behavioural traits that lead people to act in a manner inconsistent with pure rational expectations. Indeed, the inability to reconcile aggregate stock price movements with rational expectations is not the only well-known failure of modern financial economics. For instance, there are many studies documenting the failure of optimisation-based models to explain various cross-sectional patterns in asset returns, e.g. why the average return on stocks exceeds that on bonds by so much, or discrepancies in the long-run performance of small- and large-capitalisation stocks.

For many, the answers to these questions lie in abandoning the pure rational expectations, optimising approach. Indeed, the field of *behavioural finance* is booming, with various researchers proposing all sorts of different non-optimising models of what determines asset prices.[11] That said, at present, there is no clear front-runner "alternative" behavioural-finance model of the determination of aggregate stock prices. Also, one should not underestimate the rational expectations model as a benchmark. Given that low dividend-price ratios tend not to be driven by rational expectations of low future returns, the ability of the dividend-price ratio to predict future returns is probably due to a slow tendency of prices to move back towards the "fundamentals".

Does the stock market predictability associated with this return-to-fundamentals finding imply that investors can "beat the market" and get better returns than the market by timing their investments—buying when prices are low relative to dividends, and selling or shorting the market when the reverse holds? Possibly, but it will always be difficult to distinguish stock price fluctuations due to rational responses to news about fundamentals (dividends

[10] "Perspectives on Behavioral Finance: Does Irrationality Disappear with Wealth? Evidence from Expectations and Actions" by Annette Vissing-Jorgensen. This paper can be downloaded at www.kellogg.nwu.edu/faculty/vissing/htm/research1.htm

[11] The papers presented at the bi-annual NBER workshop on behavioural finance give a good flavour of this work. See http://cowles.econ.yale.edu/behfin/. See also Andrei Shleifer's book, *Inefficient Markets* and Robert Shiller's *Irrational Exuberance*

and rates of return) from those that constitute overreaction to fundamentals, or reaction to things that have nothing to do with fundamentals. And the return predictability suggested by the data is of a very long-horizon type. Don't expect to get rich quickly by timing the market!

Figure 1

Dividend-Price Ratio for the S&P 500 (1947-2003)