Topic 3: Endogenous Technology & Cross-Country Evidence

In this handout, we examine an alternative model of endogenous growth, due to Paul Romer ("Endogenous Technological Change," *Journal of Political Economy*, 1990). Then, we finish our coverage of growth topics with a discussion of some evidence relating to cross-country patterns in output per worker.

The Romer Model

So what is this technology term A anyway? The Romer model takes a specific concrete view on this issue. Romer describes the aggregate production function as

$$Y = L_Y^{1-\alpha}\left(x_1^{\alpha} + x_2^{\alpha} + ... + x_A^{\alpha}\right) = L_Y^{1-\alpha}\sum_{i=1}^{A} x_i^{\alpha} \tag{1}$$

where L_Y is the number of workers producing output and the x_i's are different types of capital goods. The crucial feature of this production function is that diminishing marginal returns applies to each of the individual capital goods (because $0 < \alpha < 1$) so that if A was fixed, then growth would eventually taper off to zero.

However, in the Romer model, A is not fixed. Instead, L_A workers are engaged in R&D and this leads to the invention of new capital goods. We will describe this using a "production function" for the change in the number of capital goods:

$$\dot{A} = \gamma L_A^{\lambda} A^{\phi} \tag{2}$$

The change in the number of capital goods depends positively on the number of researchers (λ is an index of how slowly diminishing marginal productivity sets in for researchers) and also on the prevailing value of A itself. This latter effect stems from the "giants shoulders" effect.[1] For instance, the invention of a new piece of software will have relied on the previous invention of the relevant computer hardware, which itself relied on the previous invention of semiconductor chips, and so on. Note that Romer's original paper only examined the case $\lambda = 1, \phi = 1$, whereas we will not restrict λ and will generally consider the case $\phi < 1$.

As described in Romer's paper and in the Jones textbook, the model contains a full description of what determines the fraction of workers that work in the research section. In keeping with the spirit of the Solow model, I'm going to just treat this as an exogenous

[1] Stemming from Isaac Newton's observation "If I have seen farther than others, it is because I was standing on the shoulders of giants."

parameter (but will discuss later some of the factors that should determine this share). So, we have

$$L = L_A + L_Y \quad (3)$$

$$L_A = s_A L \quad (4)$$

And again we assume that the total number of workers grows at an exogenous rate n:

$$\frac{\dot{L}}{L} = n \quad (5)$$

One can define the aggregate capital stock as

$$K = \sum_{i=1}^{A} x_i \quad (6)$$

Again, I'll treat the savings rate as exogenous and assume

$$\dot{K} = s_K Y \quad (7)$$

One observation that simplifies the analysis of the model is the fact that all of the capital goods play an identical role in the production process. For this reason, the demand for each is the same, implying that

$$x_i = \bar{x} \qquad i = 1, 2, ...A \quad (8)$$

This means that the production function can be written as

$$Y = A L_Y^{1-\alpha} \bar{x}^{\alpha} \quad (9)$$

Note now that

$$K = A\bar{x} \Rightarrow \bar{x} = \frac{K}{A} \quad (10)$$

so output can be re-expressed as

$$Y = A L_Y^{1-\alpha} \left(\frac{K}{A}\right)^{\alpha} = (A L_Y)^{1-\alpha} K^{\alpha} \quad (11)$$

This looks just like the Solow model's production function. The TFP term is written as $A^{1-\alpha}$ as opposed to just A as it was in our first handout, but this makes no difference to the substance of the model.

Steady-State Growth in The Romer Model

One can use the same arguments as before to show that this economy converges to a steady-state growth path in which capital and output grow at the same rate. So, one can derive the steady-state growth rate as follows. Re-write the production function as

$$Y = (As_Y L)^{1-\alpha} K^{\alpha} \tag{12}$$

where

$$s_Y = 1 - s_A \tag{13}$$

Take logs and derivatives to get

$$\frac{\dot{Y}}{Y} = (1-\alpha)\left(\frac{\dot{A}}{A} + \frac{\dot{s_Y}}{s_Y} + \frac{\dot{L}}{L}\right) + \alpha\frac{\dot{K}}{K} \tag{14}$$

Now use the fact that the steady-state growth rates of capital and output are the same to derive that this steady-state growth rate is given by

$$\left(\frac{\dot{Y}}{Y}\right)^* = (1-\alpha)\left(\frac{\dot{A}}{A} + \frac{\dot{s_Y}}{s_Y} + \frac{\dot{L}}{L}\right) + \alpha\left(\frac{\dot{Y}}{Y}\right)^* \tag{15}$$

Finally, because the share of labour allocated to the non-research sector cannot be changing along the steady-state path (otherwise the fraction of researchers would eventually go to zero or become greater than one, which would not be feasible) we have

$$\left(\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L}\right)^* = \frac{\dot{A}}{A} \tag{16}$$

The steady-state growth rate of output per worker equals the steady-state growth rate of A. The only difference from the Solow model is that writing the TFP term as $A^{1-\alpha}$ makes this growth rate $\frac{\dot{A}}{A}$ as opposed to $\frac{1}{1-\alpha}\frac{\dot{A}}{A}$.

The big difference relative to the Solow model is that the A term is determined within the model as opposed to evolving at some fixed rate unrelated to the actions of the agents in the model economy. To derive the steady-state growth rate in this model, note that the growth rate of the number of capital goods is

$$\frac{\dot{A}}{A} = \gamma\left(s_A L\right)^{\lambda} A^{\phi-1} \tag{17}$$

The steady-state of this economy features A growing at a constant rate. This can only be the case if the growth rate of the right-hand-side of (17) is zero. Taking logs and derivatives, this implies

$$\lambda \left(\frac{\dot{s}_A}{s_A} + \frac{\dot{L}}{L} \right) - (1 - \phi) \frac{\dot{A}}{A} = 0 \tag{18}$$

Again, in steady-state, the growth rate of the fraction of researchers must be zero. So, along the model's steady-state growth path, the growth rate of the number of capital goods (and hence output per worker) is

$$\left(\frac{\dot{A}}{A} \right)^* = \frac{\lambda n}{1 - \phi} \tag{19}$$

The long-run growth rate of output per worker in this model depends on positively on three factors:

- The growth rate of the number of workers n. The higher this, the faster the economy adds researchers. This may seem like a somewhat unusual prediction, but it is worth noting that both economic growth and population growth have accelerated over the past 200 years. There is also a well-known paper by Harvard's Michael Kremer, who surveys human history back to one million years BC and concludes that those civilisations with higher populations tended to have higher living standards.[2]
- The parameter λ, which describes the extent to which diminishing marginal productivity sets in as we add researchers.
- The strength of the "standing on shoulders" effect, ϕ. The more past inventions help to boost the rate of current inventions, the faster the growth rate will be.

A Special Case

An important exception to these results is the original Romer model in which $\phi = 1$. In this case, the growth rate of the number of capital goods is

$$\frac{\dot{A}}{A} = \gamma L_A^{\lambda} \tag{20}$$

so there is no steady-state growth path. The growth rate increases over time in line with increases in the number of researchers. Charles Jones's textbook argues fairly convincingly

[2] Michael Kremer, "Population Growth and Technological Change: One Million B.C. to 1990", *Quarterly Journal of Economics*, Vol. 108, No. 3. (Aug., 1993), pp. 681-716.

that this is not a very plausible model. For instance, Jones points out that the number of researchers has grown considerably over the post-War period, so growth rates should have accelerated accordingly if the model was correct. But this has not happened.

The Steady-State Level of Output Per Worker

One can use the same arguments as in our first handout to decompose output per worker into a capital-output ratio component and a TFP component. In other words, one can re-arrange equation (11) to get

$$\frac{Y}{L_Y} = \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} A \tag{21}$$

and use the fact that $L_Y = (1 - s_A)L$ to get

$$\frac{Y}{L} = (1 - s_A)\left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} A \tag{22}$$

Note that the s_A term reflects the reduction in the production of goods and services due to a fraction of the labour force being employed as researchers. One can also use the same arguments to show that, along the steady-state growth path the capital-output ratio is

$$\left(\frac{K}{Y}\right)^* = \frac{s_K}{n + \frac{\lambda n}{1-\phi} + \delta} \tag{23}$$

(The $\frac{\lambda n}{1-\phi}$ here takes the place of the $\frac{g}{1-\alpha}$ in the first handout's expression for the steady-state capital-output ratio because this is the new formula for the growth rate of output per worker). Finally, we can also figure out the level of A along the steady-state growth path as follows. Along the steady-state path, we have

$$\frac{\dot{A}}{A} = \gamma \left(s_A L\right)^{\lambda} A^{\phi-1} = \frac{\lambda n}{1-\phi} \tag{24}$$

This latter equality can be re-arranged as

$$A^* = \left(\frac{\gamma\left(1-\phi\right)}{\lambda n}\right)^{\frac{1}{1-\phi}} \left(s_A L\right)^{\frac{\lambda}{1-\phi}} \tag{25}$$

So, along the steady-state growth path, output per worker is

$$\left(\frac{Y}{L}\right)^* = (1 - s_A)\left(\frac{s_K}{n + \frac{\lambda n}{1-\phi} + \delta}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\gamma\left(1-\phi\right)}{\lambda n}\right)^{\frac{1}{1-\phi}} \left(s_A L\right)^{\frac{\lambda}{1-\phi}} \tag{26}$$

Convergence Dynamics for A

We noted already that the arguments showing that the capital-output ratio tends to converge towards its steady-state are the same here as in the Solow model. What about the

A term? How do we know, for instance, that A always reverts back eventually to the path given by equation (25)? To see that this is the case, let

$$g_A = \frac{\dot{A}}{A} = \gamma \left(s_A L \right)^{\lambda} A^{\phi - 1} \tag{27}$$

Taking logs and derivatives of this we get

$$\frac{\dot{g}_A}{g_A} = \lambda \left(\frac{\dot{s}_A}{s_A} + n \right) - (1 - \phi) \, g_A \tag{28}$$

One can use this equation to show that g_A will be falling whenever

$$g_A > \frac{\lambda n}{1 - \phi} + \frac{\lambda}{1 - \phi} \frac{\dot{s}_A}{s_A} \tag{29}$$

So, apart from periods when the share of researchers is changing, the growth rate of A will be declining whenever it is greater than its steady-state value of $\frac{\lambda n}{1-\phi}$. The same argument works in reverse when g_A is below its steady-state value. Thus, the growth rate of A displays convergent dynamics, always tending back towards its steady-state value. And equation (25) tells us exactly what the *level* of A has to be if the growth rate of A is at its steady-state value.

Optimal R&D?

We haven't discussed the various factors that may determine the share of the labour force allocated to the research sectors, s_A. However, in equation (26) we have diagnosed two separate effects two offsetting effects that s_A has on output: A negative one caused by the fact the researchers don't actually produce output, and a positive one due to the positive effect of the share of researchers on the level of technology.

It is a relatively simple calculus problem to figure out the level of s_A that maximises the level of output per worker along the steady-state growth path. In other words, one can can differentiate equation (26) with respect to s_A, set equal to zero, and solve to obtain that this optimizing share of researchers is

$$s_A^{**} = \frac{\frac{\lambda}{1-\phi}}{1 + \frac{\lambda}{1-\phi}} = \frac{\lambda}{1 - \phi + \lambda} \tag{30}$$

When one fills in the model to determine s_A endogenously, does the economy generally arrive at this optimal level? No. The reason for this is that research activity generates *externalities* that affect the level of output per worker, but which are not taken into account by private individuals or firms when they make the choice of whether or not to conduct research. Looking at the "ideas" production function, equation (2), one can see both positive and negative externalities:

- A positive externality due to the "giants shoulders" effect. Researchers don't take into account the effect their inventions have in boosting the future productivity of other researchers. The higher is θ, the more likely it is that the R&D share will be too low.
- A negative externality due to the fact that $\lambda < 1$, so diminishing marginal productivity applies to the number of researchers.

Whether there is too little or too much research in the economy relative to the optimal level depends on the strength of these various externalities. However, using empirical estimates of the parameters of equation (2), Charles Jones and John Williams have calculated that it is far more likely that the private sector will do too little research relative to the social optimum.[3]

To give some insight into this result, note that the steady-state growth rate in this model is $\frac{\lambda n}{1-\phi}$, so $\frac{\lambda}{1-\phi}$ is the ratio of the growth rate of output per worker to the growth rate of population. Suppose this equals one, so growth in output per worker equals growth in population—a not unreasonable assumption. In this case $\frac{\lambda}{1-\phi} = 1$ and the optimal share of researchers is one-half. Indeed, for any reasonable steady-state growth rate, the optimal share of researchers is very high, so it is hardly surprising that the economy does not automatically generate this share.

This result points to the potential for policy interventions to boost the rate of economic growth by raising the number of researchers. For instance, laws to strength patent protection may raise the incentives to conduct R&D. This points to a potential conflict between macroeconomic policies aimed at raising growth and microeconomic policies aimed at reducing the inefficiencies due to monopoly power: Some amount of monopoly power for patent-holders may be necessary if we want to induce a high level of R&D and thus a high level of output.

[3] Charles I. Jones and John C. Williams, "Too Much of a Good Thing? The Economics of Investment in R&D", *Journal of Economic Growth*, March 2000, Vol. 5, No. 1, pp. 65-85.

Interpreting the Romer Model

One final set of observations on the Romer model are in order. The model should not be thought of as a model of growth in any one particular country. No country uses only technologies that were invented in that country; rather, products invented in one country end up being used all around the world. Thus, the model is best thought of as a very long-run model of the world economy. How then should long-run growth rates be determined for individual countries? By itself, the model has no clear answer, but it suggests that the ability of a country's citizens to learn about the usage of new technologies will plays a key role in determining its level of output per worker. We will not discuss models describing this process here, but Chapter 6 of the Jones textbook describes such a model.

Hall and Jones (1999): Levels Accounting and Social Infrastructure

We will conclude our study of economic growth with an examination of the determinants of the differences in output per worker across countries. Specifically, we will consider some results from a paper by Robert Hall and Charles Jones (*QJE*, February 1999). The basis of the study is a "levels accounting" exercise that starts from the following production function

$$Y_i = K_i^{\alpha} \left(h_i A_i L_i\right)^{1-\alpha} \tag{31}$$

where h_i is the average level of human capital per worker. This can be re-expressed using our usual formulation as

$$\frac{Y_i}{L_i} = \left(\frac{K_i}{Y_i}\right)^{\frac{\alpha}{1-\alpha}} h_i A_i \tag{32}$$

Hall and Jones constructed a measure h_i using evidence on levels of educational attainment and they set $\alpha = 1/3$ in line with the usual value. This allowed them to use (32) to express all cross-country differences in output per worker in terms of three multiplicative terms: capital intensity, human capital per worker, and technology or total factor productivity. They found that output per worker in the richest five countries was 31.7 times that in the poorest five countries. This was explained as follows:

- Differences in capital intensity contributed a factor of 1.8.
- Differences in human capital contributed a factor of 2.2
- The remaining difference—a factor of 8.3—was due to differences in TFP.

What determines these huge differences in total factor productivity across countries? Though interesting, the Romer model is perhaps too mechanistic in its view of what generates cross-country differences in TFP. According to the model, these differences must be due to variations in the extent to which countries have adopted the latest technologies. While this is almost certainly a factor in differences in TFP, it leaves open the question of what drives the pace of technology adoption in poorer countries.

Also, TFP is really just a measure of the efficiency with which an economy makes use of its resources and one can imagine a whole range of other factors that can affect this. For example:

- Crime: Time spent on crime does not produce output. Neither do resources devoted to protecting inviduals and firms from crime.
- Bureaucratic Inefficiency and Corruption: Satisfaction of bureaucratic requirements and bribing of officials can be important diversions of resources in poor economies.
- Restrictions on Market Mechanisms: Protectionism, price controls, and central planning can all lead to resources being allocated in an inefficient manner.

Hall and Jones coined the term *social infrastructure* to describe the social institutions that affect incentives to produce and invest. Using a simple empirical proxy for social infrastructure, they concluded that these institutions are crucial determinants of TFP. In Chapter 7 his book, Jones also describes how social infrastructure is a key determinant of the accumulation of factors.

The Hall-Jones findings indicate the limits of the type of mechanical growth theory that we have studied. Countries can certainly get richer by devoting more resources to factor accumulation and by combining their factors in a more efficient manner. But what can be done if a country's institutions are curbing investment and leading to an inefficient allocation of resources? Growth theory has little to say about how institutions affect accumulation or productive efficiency. That said, the findings are part of a growing field that emphasises the importance of understanding how institutions affect economic outcomes, and this is likely to be an interesting field of research in the future. (See for example, Djankov et al, The New Comparative Economics, NBER Working Paper 9608. `www.nber.org/papers/w9608`).