Written Assignment for EC4010 (Macroeconomics)

This assignment is due in class at the end of the lecture on April 25. It will count for 10% of your total grade for the course. Note that it will usually be possible to proceed and make an attempt at each part of the questions even if you have had difficulty answering some of the preceding parts.

Question 1: Nonrenewable Resources in the Solow Model

This question relates to an extension of the basic Solow model to allow for the use in production of nonrenewable energy resources such as oil. The production function for the model is

$$Y_t = A_t K_t^{\alpha} E_t^{\beta} L_t^{1-\alpha-\beta}$$

where K_t is capital, E_t is energy usage, L_t is the number of workers and A_t is a measure of technological efficiency. Energy usage is assumed to be a constant fraction, s_E of total energy resources, R_t:

$$E_t = s_E R_t$$

while the rate of depletion of energy resources is determined by energy usage:

$$\dot{R}_t = -E_t$$

The other assumptions are standard. Capital accumulates according to

$$\dot{K}_t = s_K Y_t - \delta K_t \tag{1}$$

while A_t is assumed to grow at a constant rate g and L_t is assumed to grow at a constant rate n.

(a) Show that the growth rate of energy resources, R_t, is -s_E

(b) Show that the growth rate of E_t equals the growth rate of R_t.

(c) Derive an equation for the growth rate of output.

(d) Derive an equation for the growth rate of capital.

(e) Show that the growth rate of the capital-output ratio depends negatively on the level of the ratio. This means that the capital-output ratio tends to converge towards a specific equilibrium value. What is this value?

(f) What is the rate at which the capital-output ratio converges towards its equilibrium value? (This is a bit tricky and doesn't affect the rest of the problem.)

(g) Show that output per worker is given by

$$\frac{Y_t}{L_t} = A_t^{\frac{1}{1-\alpha}} \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{E_t}{L_t}\right)^{\frac{\beta}{1-\alpha}}$$

(h) Use the expression derived in (g) to derive the steady-state growth rate of output per worker.

(i) What does this model imply for the effect of population growth and energy efficiency on economic growth?

(j) What do you think of this as a model of the interaction between energy usage and economic growth?

Question 2: Interest Rates in the Solow Model

Suppose firms in the Solow model have to rent, rather than buy, their capital stocks. Thus, firms choose quantities of labour and capital input based on maximizing

$$\pi_t = A_t K_t^{\alpha} L_t^{1-\alpha} - w_t L_t - c_t K_t$$

where w_t is the wage rate and c_t is the rental rate of a unit of capital. (We are assuming there is no inflation in this economy and have normalised the price level to one).

(a) What is the first-order condition determining firm demand for capital?

(b) Assume now and for the rest of the question that the rental rate for capital is given by $c_t = r_t + \delta$, where r_t is the economy's interest rate. What is the intuition behind this condition?

(c) Assume that the Solow model economy is on its steady-state growth path. What is happening to the marginal productivity of capital? (Hint: For a Cobb-Douglas production function $\frac{\partial Y}{\partial K} = \alpha \frac{Y}{K}$.)

(d) Based on the condition determining the demand for capital, can you derive a formula for the real interest rate along the steady-state growth path?

(e) How would an increase in the savings rate affect interest rates along the steady-state growth path? What about an increase in the depreciation rate?

Question 3: Expectations and The Phillips Curve

Consider an economy in which inflation is determined by an expectational Phillips curve of the form

$$\pi_t = E_t \pi_{t+1} + \gamma y_t$$

where y_t is the gap between output and its potential level. Assume also that the output gap evolves according to an $AR(1)$ process

$$y_t = \rho y_{t-1} + \epsilon_t$$

where ϵ_t is a zero-mean shock term. Finally, assume the agents in this economy have rational expectations.

(a) Suppose this economy is on a steady-state inflationary path in which inflation is expected to be constant. What value does the output gap take along this path?

(b) Use the repeated substitution method to obtain an expression for inflation as a function of current and expected future output gaps.

(c) Use the fact that the output gap follows an AR(1) process, and the geometric sum formula, to derive a relationship between inflation and the current value of the output gap.

(d) Suppose an econometrician is doing a statistical study of the relationship between inflation and output, based on the equation derived in (c). Could this relationship be used to assess the effects on inflation of moving the economy from an average output gap of zero (as implied by the process assumed up to now) to an average output gap of one percent?

(e) Suppose an econometrician estimate a regression of inflation on its own lagged value? What will the results look like? Could these findings be used to figure out how "sticky" inflation is? In other words, could they be used to find out whether the lagged value of inflation has an influence on the current value of inflation?