MSc Macroeconomics

Problem set 3

Deadline: Friday 2nd November 2007, 10:00

Each student must hand in an answer sheet. Answer sheets should be written legibly and unreadable scribblings will be ignored. Answer sheets returned after the deadline will be awarded a zero grade.

Problem 1 (compulsory)

In the model of optimal consumption a representative consumer maximises the present value of his lifetime utility subject to a lifetime budget constraint. The optimisation problem is given by

$$\max_{c_1,\ldots,c_T} U = \sum_{t=1}^{T} \frac{1}{(1+\rho)^t} u(C_t) \tag{1}$$

subject to

$$\sum_{t=1}^{T} \frac{1}{(1+r)^t} C_t = A_0 + \sum_{t=1}^{T} \frac{1}{(1+r)^t} Y_t \tag{2}$$

Suppose that the period utility function is given by

$$u(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma} \tag{3}$$

(a) Write down the Lagrangian function for this optimisation problem.

(b) Compute the first-order conditions corresponding to time t and time $t+1$, and show that

$$\frac{C_{t+1}}{C_t} = \left(\frac{1+r}{1+\rho}\right)^{\frac{1}{\sigma}}$$

(c) Suppose that $\rho = r = 0$. Use the lifetime budget constraint to compute a solution for C_t, and use this solution to discuss intuitively the effects of permanent vs. temporary changes in taxes.

Problem 2 (compulsory)

This exercise focuses on a discrete-time version of the investment model with adjustment costs. Suppose that a firm facing a market interest rate $1 + r$ has a production function given by $Y_t = A_t F(K_t)$, where A_t is a productivity parameter. Labour supply is constant. The firm's objective is to maximise the present discounted value of its lifetime profits. However, the firm faces adjustment costs to changing its capital stock. Specifically, we will assume that it must pay $\chi I^2/2$ in adjustment costs in any period where it invests at rate I. The optimisation problem is given by

$$\max \sum_{s=t}^{\infty} (1+r)^{-(s-t)} \left[A_s F(K_s) - I_s - \frac{\chi I_s^2}{2} \right] \tag{4}$$

subject to

$$K_{t+1} = K_t + I_t \tag{5}$$

(a) Write down the current-value Lagrangian function.
(b) Show that the first-order conditions imply the following system in q and K:

$$K_{t+1} - K_t = \frac{q_t - 1}{\chi}$$

$$q_{t+1} - q_t = r q_t - A_{t+1} F' \left(K_t + \frac{q_t - 1}{\chi} \right)$$