EC4050 Economics of Securities Markets

Tutorial 2

1st November 2007

The determination of optimal weights on risky assets in the mean-variance model relies on constrained optimisation. We will derive these optimal weights mathematically and extend the discussion to unlimited borrowing and lending. This serves as an introduction to the mathematics of the standard CAPM.

Suppose initially that the investor has access to three risky securities with expected return $ER_i, i = 1,2,3$ and risk $\sigma_i, i = 1,2,3$. To find a point on the efficient frontier, the investor will minimise risk for a given portfolio expected return. The constrained optimisation problem is given by

$$\min_{w_i} \frac{1}{2}\sigma_p^2 = \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3} w_i w_j \sigma_{ij} \tag{1}$$

subject to

$$\sum_{i=1}^{3} w_i ER_i = ER_p \tag{2}$$

$$\sum_{i=1}^{3} w_i = 1 \tag{3}$$

(a) Write down the Lagrangian function for this problem.

(b) Compute the first-order conditions for this problem.

(c) Write these first-order conditions in a generalised manner to show that there are five equations with five unknowns, so that the optimal weights can be obtained as a solution.