EC4050 Economics of Securities Markets

Tutorial 1

25th October 2007

Standard practical portfolio decisions involve the computation of a very large number of inputs. The definition of the efficient frontier requires information about expected returns, variances, and covariances or correlations among securities:

$$ER_p = \sum_{i=1}^{N} w_i ER_i$$

$$\sigma_p^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \rho_{ij} \sigma_i \sigma_j$$

It is therefore desirable to find a way to reduce the amount of information required to implement the portfolio decision problem. The single-index model assumes that the comovement (or covariance) between returns arises exclusively from a common factor. In particular, we will assume that returns are affected by two components, one part due to the market, and another part that is independent of the market:

$$R_i = a_i + \beta_i R_m$$

The parameter β_i measures the sensitivity of the return on security i to the return on the market. It is useful to breakdown the component independent of the market between a constant parameter α_i and a random component e_i. Thus,

$$R_i = \alpha_i + \beta_i R_m + e_i$$

We will assume that $E(e_i) = E(e_i e_j) = E(e_i R_m) = 0$. The random component e_i has a zero mean. Moreover, the only reason why stocks vary together is because of a common market source. Finally, the ability of the single-index model to describe returns is independent of the return on the market. The variance of R_m is denoted as σ_m^2, and the variance of e_i is written as $\sigma_{e_i}^2$.

(a) Compute the expected return and the variance of security i, and the covariance between security i and security j, using the single-index model.

(b) Compute the expected return and the variance of a portfolio consisting of N securities.

(c) Show that the standard deviation of this portfolio is equal to $\sigma_m \sum_{i=1}^{N} w_i \beta_i$.