Statistical Inference

Continuous Probability Distributions

Chelydra serpentina (Snapping turtle)

This creature has no natural enemies once it reaches its adulthood. The maximum theoretical longevity has been estimated to be 170 years!

The relationship between the carapace size and the age is roughly (ie, some randomness exists) known.

Then we can find the probability that a 30cm turtle is, say, 40 or older.

You can read more about this creature at http://www.tortoisetrust.org/articles/snappers.htm

A continuous probability distribution

… describes the likelihood that a continuous random variable

with an infinite number of possible values

fall within a certain interval.

NB: We focus on an interval because those possible values are continuous, ie, not separable.

A uniform probability distribution

… is defined by a specified interval.

The situation

All values over the interval are equally likely.

Let X be a random variable.

Let x be a particular value that X can take.

Let x_{min} be the minimum value that X can take.

Let x_{max} be the maximum value that X can take.

So the interval is

$$x_{min} \leq x \leq x_{max}$$

So the length of the interval is

$$X_{max} - X_{min}$$

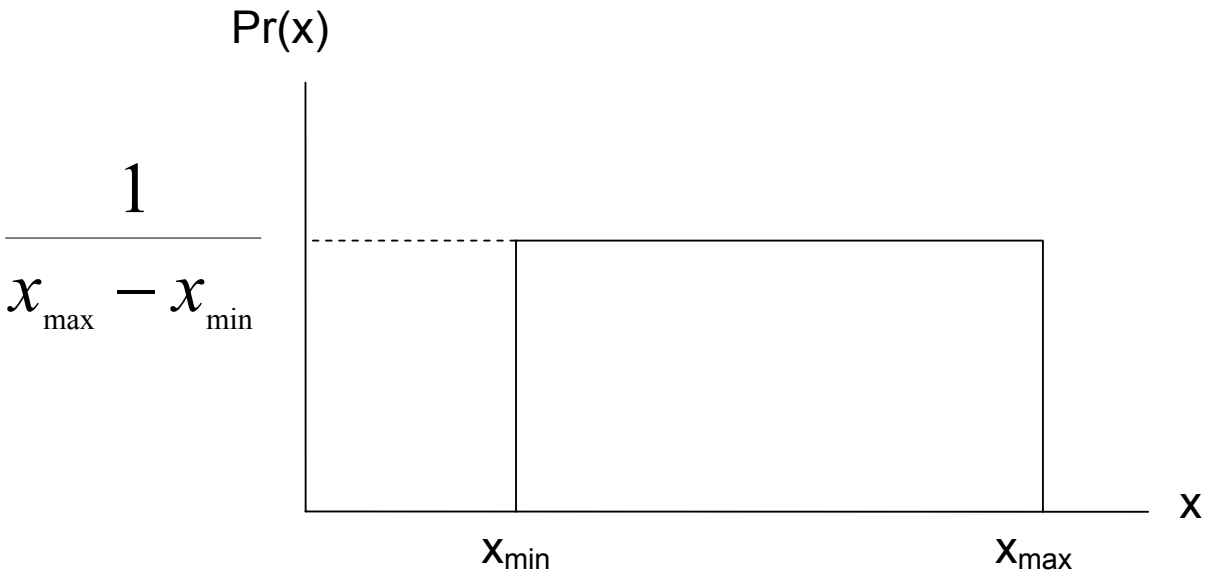
Probabilities sum to 1, and we have equally likely values.

So by dividing 1 by the interval length, we get

the probability that a particular value over the interval is observed.

So, for any range of values over the interval,

we can easily find the probability that those values are observed.



- **The height of the distribution is equal over the interval.**
- **The total area of the rectangle is 1.**

The mean is the centre of the interval, ie,

$$\mu = \frac{x_{\min} + x_{\max}}{2}$$

The variance is given by

$$\sigma^2 = \left(x_{\max} - x_{\min}\right)^2 / 12$$

The standard deviation is the square root of the variance, ie, σ.

Back to the snapping turtle example

Suppose it is known that the age of a 30cm snapping turtle is uniformly distributed between 35 and 47 years inclusive.

What's the probability that a 30cm snapping turtle is 40 or older?

What are the mean age and the standard deviation?

The length of the interval is

$$47 - 35 = 12$$

The range for "40 or older" is

$$47 - 40 = 7$$

So

$$\Pr(x \geq 40) = 7/12 \approx .583$$

The mean is $(35 + 47)/2 = 41$

The s.d. is $(47 - 35)/\sqrt{12} = 12/\sqrt{12} \approx 3.464$

Question

Suppose that, at the Bank of ΩΘΔ counters,

the time required for the credit card application process follows

a uniform distribution ranging from 4 minutes to 10 minutes.
(fairly efficient!)

What's the likelihood an application would take at most 6 minutes?

$$(6 - 4)/(10 - 4) = 2/6 = 1/3$$

Exercises to be prepared for the week-7 meeting

Exercises 2, 4, 6
and 10 to 52 (even numbered ones only)
in Chapter 7

Your teacher may go through some or all of the following ones at the meeting:

6, 12, 14, 20, 26, 44