The Effects of Third-Degree Price Discrimination in Oligopoly

By THOMAS J. HOLMES*

Retail firms and restaurants commonly offer senior citizen discounts. I suspect this practice is not motivated by altruism. Rather, the behavior can be better explained as profit-maximizing price discrimination; these firms are setting a low price in a market which probably has high elasticity of demand. But why should the demand of senior citizens be more elastic than average? One possibility is that elderly people have below-average income and they would not eat out if restaurant prices were high. Perhaps a more plausible explanation is that retired people simply have more time on their hands to spend commuting across town to take advantage of discounts. The purpose of these discounts may then be to lure senior citizens away from other restaurants rather than to get them out of the house.

The analysis of third-degree price discrimination has a long history (A. C. Pigou, 1920; Joan Robinson, 1933), and two articles on the subject have recently appeared in this *Review* (Richard Schmalensee, 1981; Hal Varian, 1985). The industry structure in these analyses is monopoly. But an explicit model of some form of competition is obviously needed to study discriminatory discounts meant to attract customers from rival firms. And even discounts meant to get people out of the house may have an oligopoly analysis which is qualitatively different from their monopoly analysis.

In this paper I present a simple duopoly model of a differentiated-products industry. I show that a firm's price elasticity of demand in a market can be expressed as the sum of two parts: the industry-demand elasticity and the cross-price elasticity. The first measures the tendency of consumers in a market to stay home when the price goes up; the second the tendency to switch suppliers. In the monopoly case (in which the cross-price elasticity is zero) and the collusive case (in which the cross-price elasticity is irrelevant), discrimination between markets is due to differences in industry-demand elasticity. In the noncooperative oligopoly case, equilibrium price differentials can be accounted for both by differences in cross-price elasticity as well as differences in industry-demand elasticity.

Total industry output when the firms can discriminate between two markets is compared to what it would be if discrimination were impossible. Which regime has larger output depends on the sum of the "adjusted-concavity condition" and the "elasticity-ratio condition." The first predominates when the rival products are poor substitutes and the firms have approximate monopoly power, while the second predominates at the other extreme where the products are close substitutes and the firms have almost no market power. The adjusted-concavity condition compares the relative curvature of demands in the two markets and was originally discovered by Robinson (1933) in her analysis of monopoly price discrimination. The elasticity-ratio condition compares the relative difference in industry-demand elasticity between the two markets with the relative difference in cross-price elasticity. According to the elasticity-ratio condition, a discriminatory discount due to high industry elasticity of demand would be "more likely" to increase total output than one due to high cross-price elasticity.

The effect of discrimination on profit is also discussed. The distinction made here

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between industry-demand elasticity and cross-price elasticity proves useful in finding an interesting situation in which discrimination can reduce equilibrium profit levels.

As mentioned above, the analysis of discrimination has historically been confined to monopoly environments. Recently, several papers have appeared which consider the practice in industries with rival firms. In particular, this paper is closely related to a paper by Severin Borenstein (1985).[1] His paper and mine share the same focus; he distinguishes between discrimination based on the tendency to drop out of the market from that based on the tendency to switch suppliers. But our approaches are significantly different, as he relies on simulations while I employ analytical methods.

I. The Model

There is an industry composed of two firms, A and B, with the product of A differentiated from the product of B. Each firm produces at constant average cost equal to c. The set of potential consumers of these products is exogenously partitioned into two groups. Using Robinson's terminology, call one group the "weak" market and the other group the "strong" market.

The firms can practice third-degree price discrimination which means it is feasible for each firm to set one price in the strong market and another in the weak market. Let $x_i^j(p_i^A, p_i^B)$ be the quantity demanded from firm j by buyers in market i when p_i^A and p_i^B are the prices set by A and B, i equal to w and s denoting the weak and strong markets. Let $\pi_i^j(p_i^A, p_i^B) \equiv (p_i^j - c)x_i^j(p_i^A, p_i^B)$ be firm j's profit in market i. Equilibrium is defined by the standard Bertrand concept in which each firm chooses its price in market i to maximize profit, taking its rival's price as fixed.

Make the symmetry assumption that A's market i demand when it sets price p and B sets q is the same as B's demand when the prices are reversed.

$$(1)\qquad x_i^A(p,q) = x_i^B(q,p), \quad i = w,s.$$

Assume further that for each market i there exists a unique equilibrium to the price game in which both firms set the same price p_i^*.

The analysis makes use of three notions of demand elasticity. Given the symmetry assumption, these elasticities need only be defined for prices that are equal across firms and such a price will be called an industry price. Let $y_i\ (p) \equiv x_i^A(p,p)$ denote the demand of each firm in market i when the industry price is p in market i. The combined output of both firms, the *industry demand*, is $2y_i(p)$. The *industry-demand* elasticity $e_i^I(p)$ measures the responsiveness of industry demand to increases in the industry price,

$$(2)\qquad e_i^I(p) \equiv -p\cdot\frac{2\cdot dy_i(p)/dp}{2\cdot y_i(p)}$$

$$= -p\cdot\frac{dy_i(p)/dp}{y(p)}.$$

Note the sign convention that the elasticity be positive. The other two elasticities measure how the demands of both firms are affected when A unilaterally departs from the industry price p. The *firm-level* elasticity of demand $e_i^F(p)$ measures the effect on A's demand while the *cross-price* elasticity $e_i^C(p)$ measures the impact on B's demand. Assume the cross-price elasticity is positive so the products of the two firms are substitutes.

Borenstein (1985) noted that the decline in A's sales when it unilaterally increases price can conceptually be broken down into two parts. First, some former customers no longer buy the product of the industry because of the price hike. Second, some former customers substitute the now relatively cheaper product of firm B. I formalize this conceptual distinction by demonstrating that the firm-level elasticity of demand can be decomposed into industry elasticity and cross-price elasticity components. Note that under the symmetry assumption of equation (1), the cross-partial derivatives of the demand

[1] Other papers in the area include Michael L. Katz, 1984, and Phillip J. Lederer and Arthur P. Hurter, Jr., 1986.

functions are symmetric at identical prices,

$$\frac{\partial x_i^A(p,p)}{\partial p^B} = \frac{\partial x_i^B(p,p)}{\partial p^A}. \tag{3}$$

The slope of A's demand with respect to changes in its own price, holding B's price fixed, can then be written as the sum of two negative terms,

$$\begin{aligned} \frac{\partial x_i^A(p,p)}{\partial p^A} &= \left[\frac{\partial x_i^A(p,p)}{\partial p^A} + \frac{\partial x_i^A(p,p)}{\partial p^B}\right] - \frac{\partial x_i^B(p,p)}{\partial p^A} \\ &= \frac{dy_i(p)}{dp} - \frac{\partial x_i^B(p,p)}{\partial p^A}. \end{aligned} \tag{4}$$

The first term accounts for the buyers who stay home and the second those who switch to B. In elasticity terms,

$$\begin{aligned} e_i^F(p) &\equiv -p \cdot \frac{\partial x_i^A(p,p)/\partial p^A}{y_i(p)} \\ &= -\frac{p}{y_i(p)}\frac{dy_i(p)}{dp} - \frac{\partial x_i^B(p,p)}{\partial p^A} \\ &= e_i^I(p) + e_i^C(p). \end{aligned} \tag{5}$$

The equilibrium price in market i depends on the firm-level elasticity of demand according to the familiar price-cost markup formula,

$$\frac{p_i^* - c}{p_i^*} = \frac{1}{e_i^F(p_i^*)} = \frac{1}{e_i^I(p_i^*) + e_i^C(p_i^*)}. \tag{6}$$

This equation succinctly illustrates that in symmetric oligopoly discrimination is based on differences in industry-demand elasticity and/or cross-price elasticity.

II. The Effects of Price Discrimination

In the *uniform-price* regime each firm is constrained to set the same price in the weak market as it does in the strong market. This section compares prices, output, and profit levels in the uniform-price regime with their levels in the discriminatory regime.[2]

Let $M_i(p)$ be the marginal increase in A's market i profit when it unilaterally increases price above the industry level p,

$$M_i(p) \equiv \frac{\partial \pi_i^A(p,p)}{\partial p^A} = y_i(p) + (p-c)\frac{\partial x_i^A(p,p)}{\partial p^A}. \tag{7}$$

It has slope

$$\frac{dM_i(p)}{dp} = \frac{dy_i(p)}{dp} + \frac{\partial x_i^A(p,p)}{\partial p^A} + (p-c)\frac{d}{dp}\left[\frac{\partial x_i^A(p,p)}{\partial p^A}\right]. \tag{8}$$

Assume $M_i(p)$ is strictly decreasing in p, that is, the gain from unilaterally increasing price falls as the industry price rises. Since the first two terms of (8) are negative, this assumes they outweigh any possible offsetting effect of the third term.

In the discriminatory case, the equilibrium price p_i^* must solve the symmetric first-order condition $M_i(p_i^*) = 0$. Assume the equilib-

[2] In the third-degree price discrimination literature, it is traditional to evaluate the effect of the practice on the total of producers' and consumers' surplus. Output is not distributed efficiently in the discriminatory allocation. Extending the earlier analyses of monopoly, Hal Varian, 1985, showed that for total surplus to increase it is necessary (but not sufficient) that total output increase, *regardless* of how prices are determined. In particular, his result holds for this oligopoly analysis.

rium prices differ and name the market with the lower equilibrium price the weak market. In the uniform-price regime, A sets the uniform price which maximizes profits combined over the two markets. The equilibrium uniform price p_u^* solves the symmetric first-order condition $M_w(p_u^*)+M_s(p_u^*)=0$. Since $M_w(p)$ and $M_s(p)$ both decrease in price, p_u^* must lie between p_w^* and p_s^*.

A. *The Effect of Discrimination on Output*

Compared with the uniform-price regime, discrimination raises the price in the strong market and lowers the price in the weak market. Since industry demand is elastic in each market, output falls in the strong market and rises in the weak market. Part A determines the direction of the net effect on total industry output.

As it is difficult to directly compare equilibrium output levels in the two regimes, a continuum of regimes is considered in which the uniform-price and discriminatory regimes are polar cases. Let the parameter r represent the maximum price differential between markets such that in regime r the constraint $|p_s - p_w| \le r$ must be satisfied. The parameter r can be interpreted as the cost to strong-market buyers of pretending to be weak-market buyers. Let $p_i(r)$ be the equilibrium price in market i in regime r. Let $\bar{r}$ be the equilibrium price differential in the unconstrained case, $\bar{r} = p_s^* - p_w^*$. For r greater than $\bar{r}$ the constraint is not binding so price in market i is simply p_i^*, while r equal to zero corresponds to the uniform price regime. For r between 0 and $\bar{r}$, the constraint is binding, and $p_w(r)$ solves the symmetric first-order condition $M_w(p_w)+M_s(p_w+r)=0$. The assumption that $M_i(p)$ decreases in price for both i implies the strong-market price rises with r while the weak-market price falls.

Denote the total quantity of output in regime r as $Q(r) \equiv y_w(p_w(r))+y_s(p_s(r))$. The slope of this function determines the effect on total output of a local increase in the ability to price discriminate. If it can be shown that $Q(r)$ is monotonic in r, increasing or decreasing, then the direction of the effect of moving from uniform pricing ($r=0$) to unconstrained discriminatory pricing (r large enough) is determined.[3] Straightforward calculations show the slope $Q'(r)$ is positive if and only if the following sum of two terms is positive,

$$(9)\quad \left[\frac{(p_s-c)}{y_s'(p_s)}\cdot\frac{d}{dp_s}\left(\frac{\partial x_s^A(p_s,p_s)}{\partial p^A}\right) - \frac{(p_w-c)}{y_w'(p_w)}\cdot\frac{d}{dp_w}\left(\frac{\partial x_w^A(p_w,p_w)}{\partial p^A}\right)\right] + \left[\frac{e_s^C(p_s)}{e_s^I(p_s)} - \frac{e_w^C(p_w)}{e_w^I(p_w)}\right],$$

where the functional dependence of p_w and p_s on r is implicit.

Call the first term the adjusted-concavity condition, using Robinson's terminology. This term compares the relative curvature of demand in the two markets. In the special case of zero cross-price elasticity in both markets, $e_w^C(p)=0, e_s^C(p)=0$, the second term of (9) is zero and the first term reduces to the condition Robinson obtained in her analysis of the monopoly case, with a slight discrepancy due to an error in her analysis.[4] According to this condition, if industry demands in the weak and strong markets are respectively convex and concave then $Q'(r)$ is positive for all r so the discriminatory output $Q(\bar{r})$ is above the uniform price output $Q(0)$. This claim of Robinson (1933) was later formalized by Schmalensee (1981) by use of a technique somewhat related to the one used here.

[3]Of course $Q(\bar{r})$ may still be greater than $Q(0)$ even if $Q(r)$ decreases in r over some range.

[4]Robinson essentially assumes that the demand elasticities in each market are approximately equal at the uniform price, which accounts for the discrepancy. M. L. Greenhut and H. Ohta, 1976, present a numerical example in which Robinson's condition predicts that output decreases when, in fact, it increases. Both markets have constant elasticity of demand in their example. Condition (9) can be used to show that monopoly discrimination always increases output for such demands.

Call the second term of (9) the elasticity-ratio condition. It is helpful to rewrite the condition in another form. The second term of (9) is positive if and only if

$$(10) \qquad \frac{e_w^I(p_w)}{e_s^I(p_s)} > \frac{e_w^C(p_w)}{e_s^C(p_s)}.$$

Suppose the adjusted-concavity condition is zero. Then condition (10) says discrimination increases output if and only if the ratio of industry-demand elasticities between the weak and strong markets is greater than the corresponding ratio of cross-price elasticities. This condition accounts for Borenstein's simulation finding that discrimination based on differences in tendency to drop out of the market had a greater positive effect on output than discrimination based on readiness to switch suppliers. But note that the elasticity-ratio condition is more general, since it also applies when markets differ in both dimensions.

When demand is linear (as given below in equation (12)) the adjusted-concavity condition is zero and the direction of the effect on output depends solely on the elasticity-ratio condition. In the general nonlinear case the direction will depend on the sum of the two conditions. However, if the cross-price elasticities in both markets are high enough, the elasticity-ratio condition will predominate in determining the net effect. To see this, note that when the rival products are close substitutes, equilibrium prices are close to marginal cost. Since $(p_w - c)$ and $(p_s - c)$ enter multiplicatively into the adjusted-concavity condition, high cross-price elasticities tend to make the first term of (9) small. On the other hand, when the cross-price elasticities are increased (keeping their ratio constant) the second term of (9) becomes large in absolute value.[5]

[5]This idea is formalized in my dissertation where I explicitly parameterize demand by cross-price elasticity. I show that the effect on output is determined solely by condition (10) near the limit where the products are perfect substitutes.

B. *The Explanation Behind the Elasticity-Ratio Condition*

The net effect of discrimination on total output depends in part on how the practice affects prices. In particular, it depends on how significant the decline in price in the weak market is relative to the rise in price in the strong market. There is a sense in which discrimination increases "average" price; the increase in price in the strong market above the uniform price is "large" relative to the decrease in the weak-market price. This can be illustrated with a simple example. Assume firm-level demand elasticity is constant in each market equal to e_w^F and e_s^F.[6] Simplify further by assuming that industry demand is perfectly inelastic in each market so each market's share of total output t_i (where $t_w + t_s = 1$) is independent of regime.[7] The firm-level elasticity of the combined markets is $t_w e_w^F + t_s e_s^F$, the mean of the elasticities in the individual markets. The equilibrium uniform price is determined by the inverse-elasticity rule, and by substituting the discriminatory markups for the constant elasticities in each market, and by straightforward application of Jensen's inequality, we have the inequality

$$(11) \qquad \frac{p_u^* - c}{p_u^*} = \frac{1}{t_w e_w^F + t_s e_s^F} < t_w \cdot \frac{p_w^* - c}{p_w^*} + t_s \cdot \frac{p_s^* - c}{p_s^*}.$$

The uniform-price markup is strictly less than the arithmetic mean of the discriminatory markups (and instead equals the harmonic mean). Equation (11) and straightforward arguments yield $p_u^* < t_w p_w^* + t_s p_s^*$. The

[6]The demand function $x_i^A(p,q) = kp^{-a}q^b$ has constant firm-level, industry-demand, and cross-price elasticity. Industry demand is perfectly inelastic when $a = b$.

[7]Of course this is not an interesting example for examining the effect of discrimination on total output. But I am focusing here on the effect on prices and the lesson learned is of somewhat greater generality.

weak market then wields a disproportionate effect in the determination of the uniform price.

The elasticity-ratio condition says that, *ceteris paribus*, a rise in the weak-market cross-price elasticity makes the net effect of discrimination on output more likely to be negative. This is a consequence of the more than proportionate effect of such a change in reducing the uniform price. The fall in the uniform price is greater than the fall in the "average" discriminatory price.

It remains to explain why a rise in the weak-market industry-demand elasticity makes the net effect more likely to be positive. By reasoning parallel to the above, such a change would lower the uniform price relative to the "average" discriminatory price which in itself would make the net effect more likely to be *negative*. But it also makes weak-market output more responsive to any *given* price decrease and this positive effect on total output outweighs the negative effect.

C. *The Effect on Profit*

A monopolist would always prefer the discriminatory regime to the uniform price regime since it could always choose not to discriminate. But the logic of individual decision theory does not extend to noncooperative games; the firms in an oligopoly may be worse off with a larger choice set. Part C illustrates one circumstance in which this can happen.

Consider first the case where the firms collude and set price to maximize industry profit. The optimal uniform price represents a compromise between a lower price that would maximize profit in the weak market and a higher price that would maximize profit on the strong market. The ability to discriminate would enable the collusive oligopoly to raise profit in both markets.

Consider now the noncooperative oligopoly case. If cross-price elasticities are high enough, p_u^* will be below the optimal (i.e., collusive) discriminatory weak-market price. If discrimination is permitted, the price in the weak market falls to p_w^* which is even further below the optimal discriminatory weak-market price. This means that discrimination reduces industry profit in the weak market. This is offset by higher profit in the strong market. The direction of the net effect on total profit is in general ambiguous as can be shown by example.

Assume that demand is linear,

$$(12)\quad x_i^A(p_i^A, p_i^B) = f_i \cdot \left[1 - a_i p_i^A - b_i(p_i^A - p_i^B)\right].$$

The industry demand and cross-price elasticities in market i are measured by a_i and b_i.[8] Assume $2a_w + b_w > 2a_s + b_s$ so the weak market has higher firm-level elasticity of demand at any price. It can be shown that if the elasticity-ratio condition holds, (which in this case is $a_w/a_s > b_w/b_s$), then total profit always increases with discrimination, that is, profit rises if output rises. But for the case in which a_w is less than a_s while b_w is greater than b_s, I was able to find examples in which discrimination reduces total profit (though it should be noted that in these examples profit never declines by more than a few percentage points).[9]

In these examples in which discrimination decreases profit, the weak market has higher cross-price elasticity but lower industry elasticity of demand than the strong market. The outcome has an intuitive explanation. Since industry elasticity of demand is lower in the weak market, a given increase in price would

[8]The monopoly case of $b_w = b_s = 0$ often appears in textbooks where it is shown that the net effect on output is zero, implying the net effect on total surplus is negative (see fn. 2). Suppose the cross-price effects are identical, $b_w = b_s = b$. The elasticity-ratio condition implies the output effect is strictly positive for b above zero. And it can be shown that the net effect on total surplus is also positive, for b above a_w. (This assumes $c = 0$. The point at which the direction of the welfare effect reverses is some b between a_s and a_w.)

[9]For example, when the parameter values are $f_w = f_s = .5$, $a_w = .1$, $a_s = 1$, $b_w = 4$, and $b_s = 1$, discrimination reduces profit by 3.5 percent. This numerical example satisfies all the assumptions of the model including existence of a uniform-price equilibrium (see fn. 10).

be relatively more effective in increasing profit in the weak market than the strong market. However, discrimination decreases the price set in the weak market because its higher cross-price elasticity outweighs its lower industry elasticity of demand. Discrimination results in a lower price in the "wrong" market from the perspective of the firms and total profit falls.

III. Concluding Remarks

I conclude by reviewing some of the assumptions of the paper. Many restrictions were made on endogenous phenomena (for example, existence of equilibrium and global concavity properties). It is beyond the scope of this paper to determine the generality of the demands which satisfy these restrictions. I can say there exist demand specifications consistent with all of the assumptions.[10] Having two firms in the industry rather than n firms simplifies notation but is otherwise not essential. But the assumption of symmetry is important at least to the extent that both firms rank markets by degree of firm-level elasticity in the same way. This holds in many examples of interest (for example, senior-citizen discounts and airline discrimination between business and tourist traffic). Yet in other cases the weak market of one firm is the strong market of another. This results in qualitatively different form of price discrimination which requires a separate analysis.[11]

[10]The example of linear demand given by equation (12) satisfies all the assumptions of the paper as long as certain additional restrictions are placed on the parameters so that a uniform-price equilibrium exists.

[11]Spatial discrimination (for example, Lederer and Hurter, 1986) is an example of this type of discrimination.

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