Nash Equilibrium

- **A game consists of**
  - **a set of players**
  - **a set of strategies for each player**
  - **A mapping from set of strategies to a set of payoffs, one for each player**

<u>N.E.:</u> A Set of strategies form a NE if, for player i, the strategy chosen by i maximises i's payoff, given the strategies chosen by all other players

- NE is the set of strategies from which no player has an incentive to unilaterally deviate

- NE is the central concept of non-cooperative game theory I.e. situtations in which binding agreements are not possible

Example

Player 2

		C	D
Player 1	C	(10,10)	(0,20)
	D	(20,0)	(1,1)

This is the game's payoff matrix.

Player A's payoff is shown first.
Player B's payoff is shown second.

NE: (DD) = (1,1)

Another Example….

		Player B	
		L	**R**
Player A	**U**	**(3,9)**	**(1,8)**
	D	**(0,0)**	**(2,1)**

Two Nash equilibria: (U,L) = (3,9)

(D,R) = (2,1)

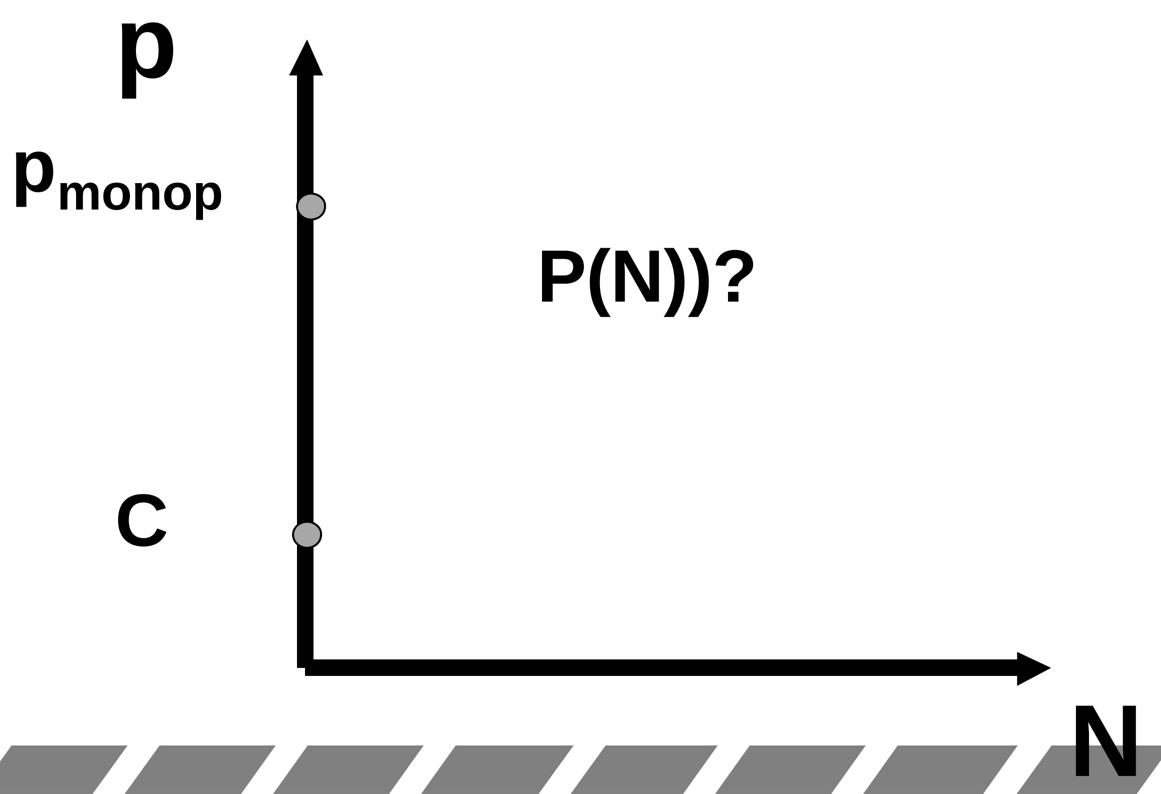
Applying the NE Concept

Modelling Short Run 'Conduct'

Bertrand Competition

Cournot Competition

[Building blocks in modeling the intensity of competition in an industry in the short run]


p
p_{monop}
P(N))?
c
N

Bertrand Price Competition

- **What if firms compete using only price-setting strategies,?**
- **Games in which firms use only price strategies and play simultaneously are Bertrand games.**

Bertrand Games (1883)

1. **2 players, firms i and j**
2. **Bertrand Strategy - All firms simultaneously set their prices.**
3. **Homogenous product**
4. **Perfect Information**
5. **Each firm's marginal production cost is constant at c.**

Bertrand Games

$p_i = 0$ if $p_i > p_j$

$p_i = \frac{1}{2}(p_i - c)Q$ if $p_i = p_j$

$p_i = (p_i - c)Q$ if $p_i < p_j$

- Q: Is there a Nash equilibrium?
- A: Yes. Exactly one.

All firms set their prices equal to the marginal cost c. Why?

Bertrand Games

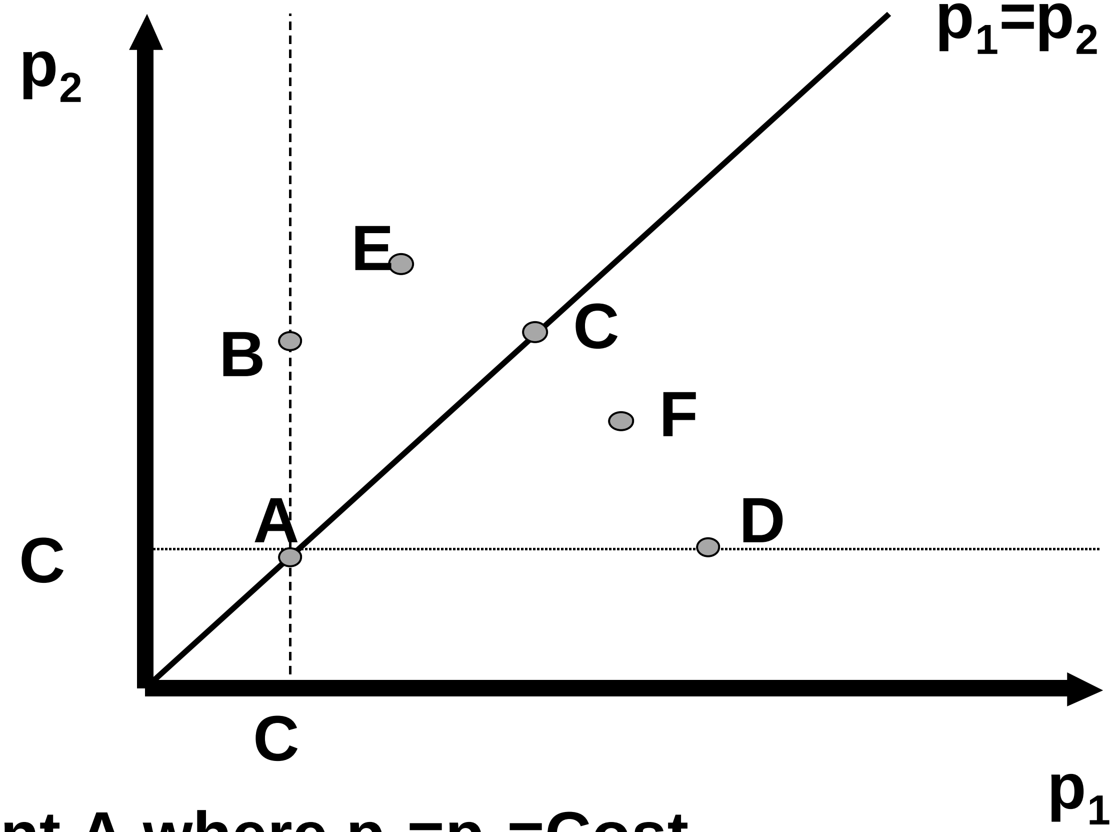
Proof by Contradiction

- **Suppose one firm sets its price higher than another firm's price.**
- **Then the higher-priced firm would have no customers.**
- **Hence, at an equilibrium, all firms must set the same price.**

Bertrand Games

- Suppose the common price set by all firm is higher than marginal cost c.
- Then one firm can just slightly lower its price and sell to all the buyers, thereby increasing its profit.
- The only common price which prevents undercutting is c. Hence this is the only Nash equilibrium.

Illustration



NE: at point A where $p_1 = p_2 = \text{Cost}$

Bertrand Paradox

- For n>=2 with firms simultaneously setting prices, prices = marginal cost and profits are zero……. Perfectly competitive outcome is replicated
- Intuitive assumption …..surprising result!

- This result holds where firms have identical costs
- If firms have different costs, then there may or may not be a pure strategy equilibrium

If firms are capacity constrained, then a mixed strategy equilibrium results

- **Edgeworth (1897) - Capacity Constraints**

Neither firm can meet the entire market demand, but can meet half market demand.

Constant MC to a point, then decreasing returns

Under these conditions, Edgeworth cycle: prices fluctuate between high and low

Kreps & Scheinkman (1983)

- If there is a two stage game,
- in which firms set capacity in stage 1
- And in stage 2, given their capacity, set price
- Then the Cournot result is observed

Differentiated Products resolve the Bertrand Paradox

- **Differentiated Products allow price competing oligopolists to mark up**

Cournot Competition (1838)

1. **2 Players (identical)**
2. **Cournot strategy - All firms simultaneously set their output**
3. **Homogenous product**
4. **Perfect Information**
5. **Linear demand**
6. **Constant MC**

2 identical firms, linear demand, constant marginal cost

$$P = a - Q \; = a - q_1 - q_2 \qquad TC_i = c(q_i)$$

$$\Pi_1 = (a - q_1 - q_2).q_1 - c(q_1)$$

Choose q_1 to max $\mathbf{p}_1$, given q_2

$$\frac{\partial \Pi_1}{\partial q_1} = a - 2q_1 - q_2 - c = 0$$

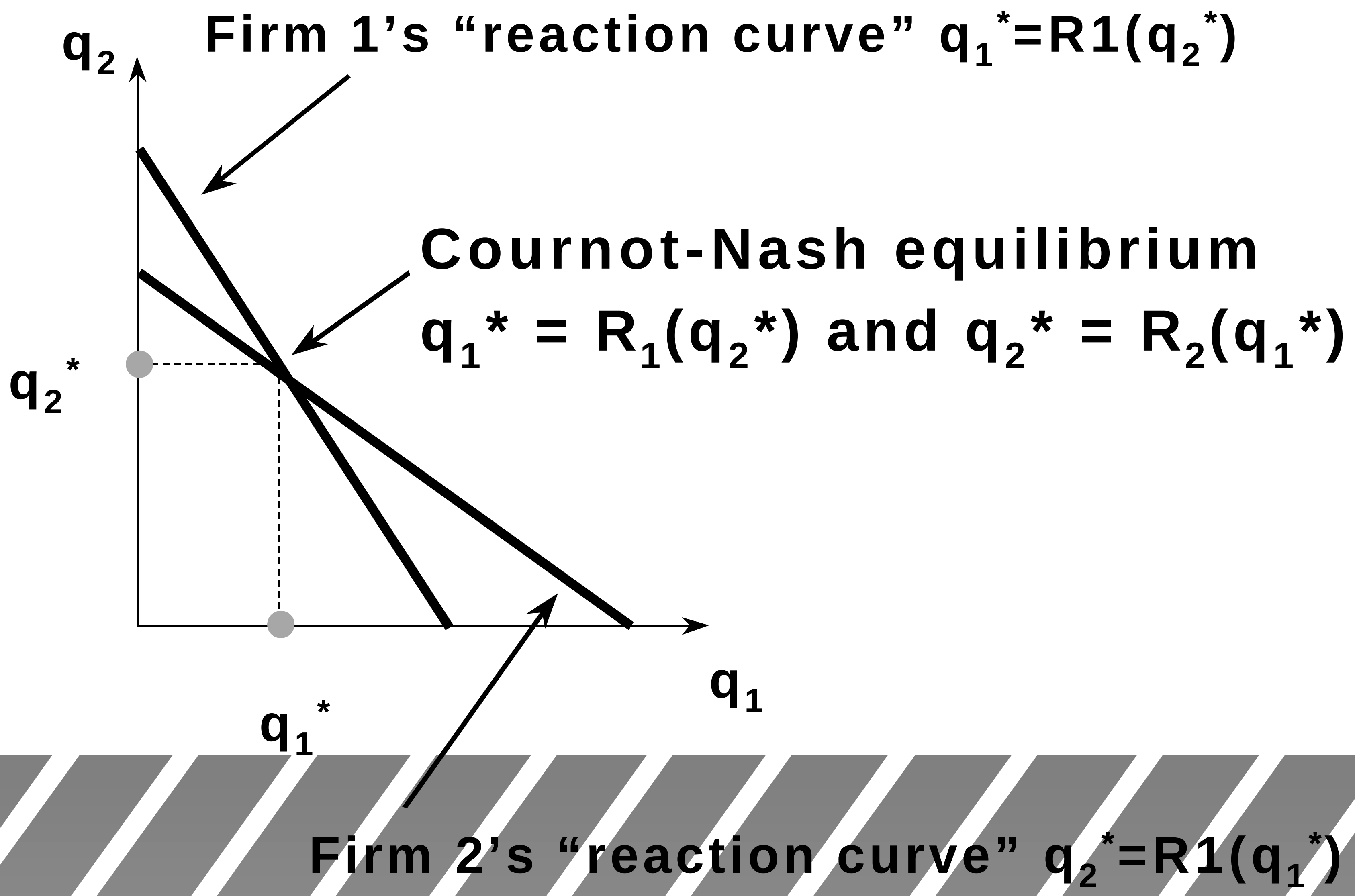
Higher q_2, lower level q_1 to max $\mathbf{p}_1$

$$q_1 = \frac{a - q_2 - c}{2} \qquad q_1 = R(q_2)$$

$$Similarily, identical\ \ firms \Rightarrow$$

$$q_2 = \frac{a - q_1 - c}{2} \qquad q_2 = R(q_1)$$

- An equilibrium is when each firm's output level is a best response to the other firm's output level - then neither wants to deviate from its output level.
- A pair of output levels (q_1^*, q_2^*) is a Cournot-Nash equilibrium if $q_1^* = R1(q_2^*)$ and $q_2^* = R1(q_1^*)$


Firm 1's "reaction curve" $q_1^*=R1(q_2^*)$
q_2
Cournot-Nash equilibrium
$q_1^* = R_1(q_2^*)$ and $q_2^* = R_2(q_1^*)$
q_2^*
q_1
q_1^*
Firm 2's "reaction curve" $q_2^*=R1(q_1^*)$

Solve reaction curves to find cournot

$$R_1(q_2) = R_2(q_1) =$$

$$q_1 = \frac{a - q_2 - c}{2} \Rightarrow q_1 = \frac{a - (\frac{a - q_1 - c}{2}) - c}{2}$$

$$solving \quad : \quad q_1^* = \frac{a - c}{3}$$

$$identical \quad firms \quad : in \ \ equilibriu \quad m$$

$$q_1^* = q_2^* = \frac{a - c}{3} \ cournot \quad nash \quad equil$$

$$Total \quad Cournot \quad quantity \quad : \frac{2(a - c)}{3}$$

$$Solve \quad for \quad price \quad : p = a - Q = \frac{(a + 2c)}{3}$$

- Q: Are the Cournot-Nash equilibrium profits the largest that the firms can earn in total?
- A: Firms could earn higher profits if both agreed to set half the monopoly output (and thus earn half monopoly profit each)

HOWEVER

- Collusive\Joint profit max output levels $q_{m1}q_{m2}$ not sustainable – incentives to unilaterally deviate – not a NE
- if firm 1 continues to produce q_1^{m}, firm 2's profit-maximizing response is $q_2 = R_2(q_1^{m})$

$$P_{monop} > P_{cournot} > P_{perfcomp=bertrand}$$

$$Q_{monop} < Q_{cournot} < Q_{perfcomp=bertrand}$$

$$p_{monop} > p_{cournot} > p_{perfcomp=bertrand}$$

Example:

P = 140 – Q; $C_i = 60(q_i)$;

2 firms play Cournot. What are equilibrium outcomes?

$$P = 140 - q_1 - q_2$$

$$\Pi_1 = (140 - q_1 - q_2).q_1 - 60q_1$$

$$\frac{\partial \Pi_1}{\partial q_1} = 140 - 2q_1 - q_2 - 60 = 0$$

$$q_1 = 40 - \tfrac{1}{2}q_2$$

and solving profit max q_2, *given* q_1

$$q_2 = 40 - \tfrac{1}{2}q_1$$

$$\textit{solve reaction functions}:$$

$$q_1 = 40 - \tfrac{1}{2} q_2 \quad \textit{and} \quad q_2 = 40 - \tfrac{1}{2} q_1$$

$$q_1 = 40 - \tfrac{1}{2}(40 - \tfrac{1}{2} q_1) \Rightarrow q_1^* = 80/3 = 26\ 2/3$$

$$\textit{identical firms}: q_1^* = q_2^* = 26\ 2/3$$

$$\textit{Total Cournot quantity}: 160/3 = 53\ 1/3$$

$$\textit{Solve for price}: p = 140 - Q = \frac{260}{3} = 86\ 2/3$$

$$\textit{Solveforpr ofit}: \pi_1 = \pi_2 =$$

Monopoly....1 firm

$$P = 140 - q_m$$

$$\Pi_m = (140 - q_m).q_m - 60q_m$$

$$\frac{\partial \Pi_m}{\partial q_m} = 140 - 2q_m - 60 = 0$$

$$q_m = 40$$

$$p_m = 140 - Q = 140 - 40 = 100$$

Perfect Competition (& Bertrand)

$P = MC = 60$

$P = a - Q$ so $Q = a - c = 140 - 60 = 80$

Profit = $p.q - c.q = 0$ since $p = c$

Shows $Q^m < Q^c < Q^{pc}$

And $P^m > P^c > P^{pc}$

Thus, Bertrand ® for N> or = 2, get perfectly competitive outcomes

Can show that as - N, cournot outcome ® perfectly competitive

N player Cournot

1. **N Players (identical)**
2. **Cournot strategy - All firms simultaneously set their output**
3. **Homogenous product**
4. **Perfect Information**
5. **Linear demand**
6. **Zero Cost**

P = a - bQ ; $Q = \sum_{i=1}^{N} q_i$

since N firms are identical $Q = Nq_i$

$TC_i = 0+0q_1$ so c = 0

$Firm_i$: $\Pi_i = (a - bQ).q_i$

$$\frac{\partial \Pi_i}{\partial q_i} = a - bQ - bq_i = 0$$

$$a - b(Nq_i) - bq_i = 0$$

$$a - b(1 + N)q_i = 0$$

$$q_i^* = \frac{a}{b(N+1)}$$

$$p_i^* = a - b.N.q_i = \frac{a}{(N+1)}$$

Note: if $y = bQ.q_i$ &

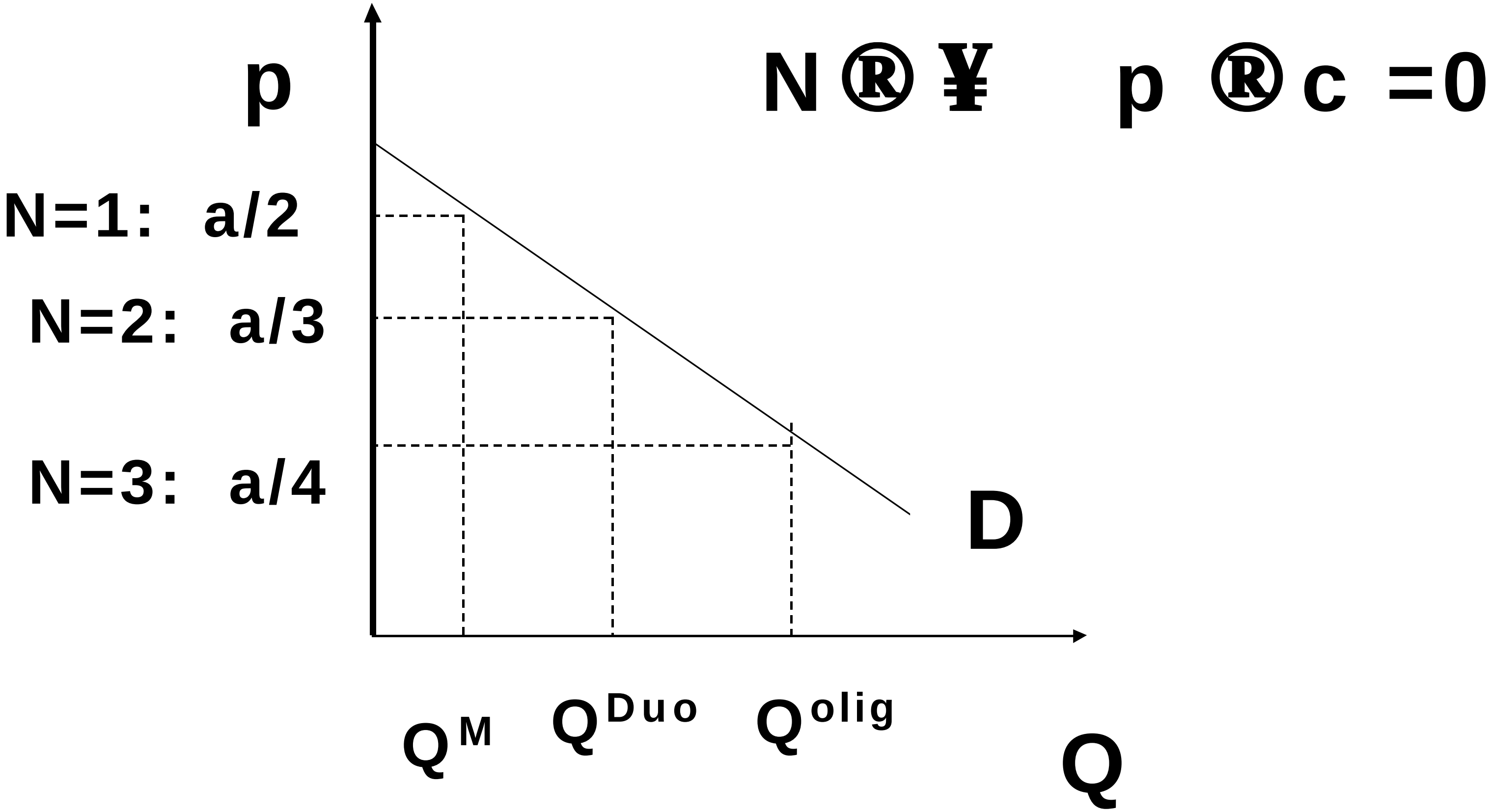
$Q = q_i + q_j + \ldots + q_n$

(so $dQ/dq_i = 1$)

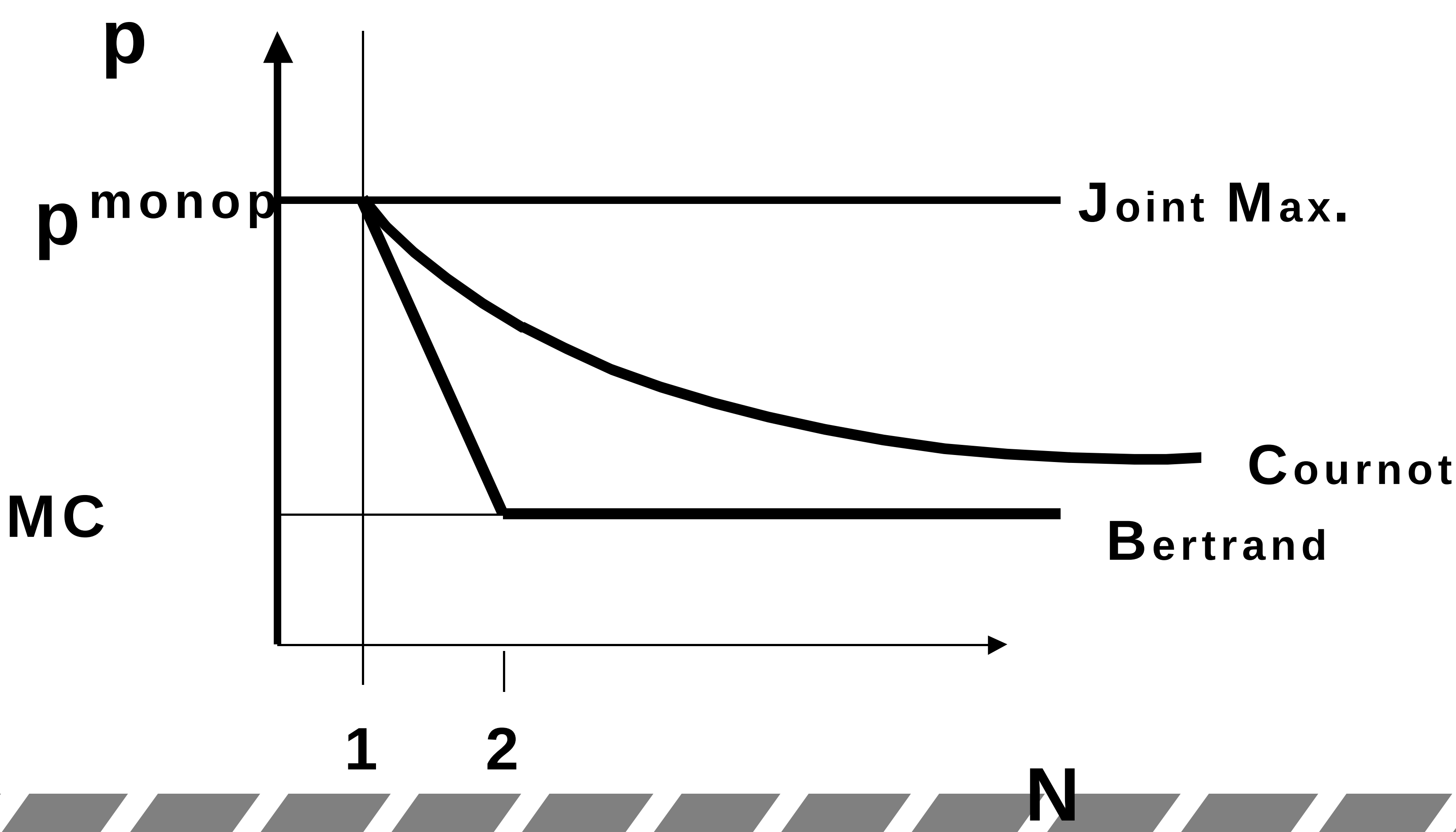
$dy/dq_i = bQ.1 + bq_i.dQ/dq_i$

$= bQ + bq_i$

$$P = a / N+1$$

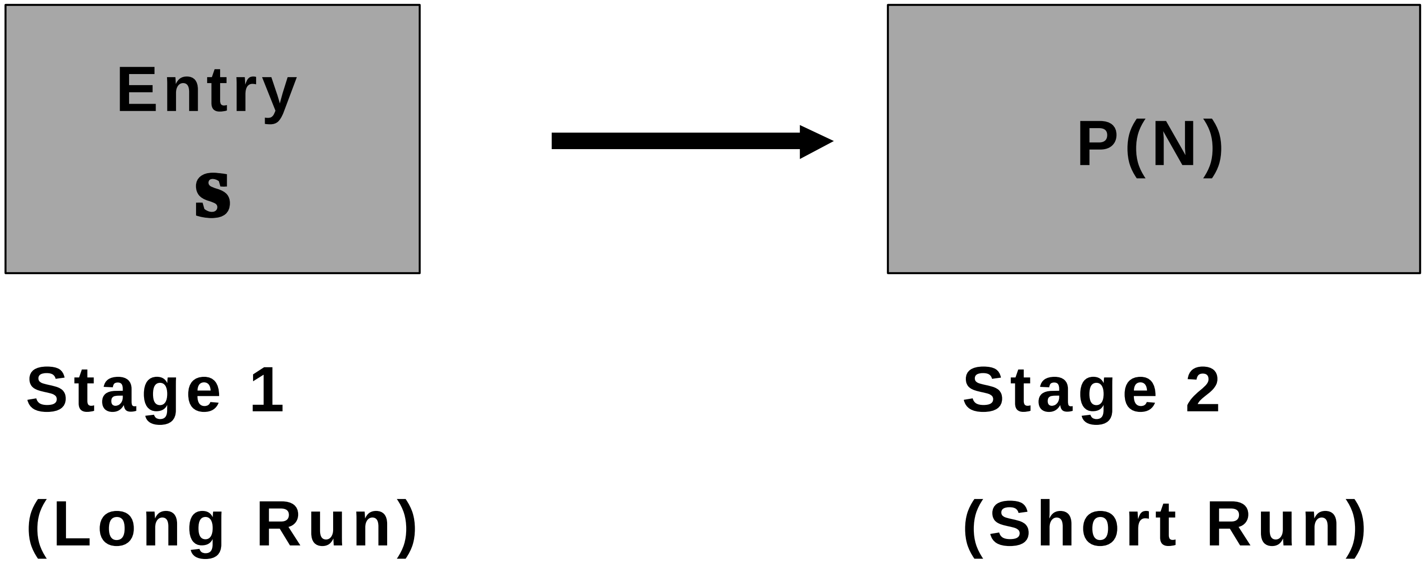

p
$N \rightarrow \infty \quad p \rightarrow c = 0$
N=1: a/2
N=2: a/3
N=3: a/4
D
Q^M
Q^{Duo}
Q^{olig}
Q

P(N) function links price cost margins to a given N



P for any given N depends on the 'intensity of competition' (Bertrand: Most intense)

Entry?



The entry decision is a backward induction procedure

We return to modelling entry, where N is endogenous and depends on P(N) in the latter part of the course……