Nash Equilibrium

- A game consists of
  - a set of players
  - a set of strategies for each player
  - A mapping from set of strategies to a set of payoffs, one for each player

N.E.: A Set of strategies form a NE if, for player i, the strategy chosen by i maximises i’s payoff, given the strategies chosen by all other players
NE is the set of strategies from which no player has an incentive to unilaterally deviate.

NE is the central concept of non-cooperative game theory i.e. situations in which binding agreements are not possible.
Example

Player 2
C  D

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>(10,10)</td>
<td>(0,20)</td>
</tr>
<tr>
<td>D</td>
<td>(20,0)</td>
<td>(1,1)</td>
</tr>
</tbody>
</table>

This is the game’s payoff matrix.

Player A’s payoff is shown first.
Player B’s payoff is shown second.

NE: (DD) = (1,1)
Another Example....

Two Nash equilibria: (U,L) = (3,9)  
(D,R) = (2,1)
Applying the NE Concept

Modelling Short Run ‘Conduct’

Bertrand Competition
Cournot Competition

[Building blocks in modeling the intensity of competition in an industry in the short run]
Bertrand Price Competition

- What if firms compete using only price-setting strategies?
- Games in which firms use only price strategies and play simultaneously are Bertrand games.
Bertrand Games (1883)

1. 2 players, firms i and j
2. Bertrand Strategy - All firms simultaneously set their prices.
3. Homogenous product
4. Perfect Information
5. Each firm’s marginal production cost is constant at c.
Bertrand Games

\[ \pi_i = 0 \quad \text{if } p_i > p_j \]
\[ \pi_i = \frac{1}{2} (p_i - c)Q \quad \text{if } p_i = p_j \]
\[ \pi_i = (p_i - c)Q \quad \text{if } p_i < p_j \]

Q: Is there a Nash equilibrium?
A: Yes. Exactly one.

All firms set their prices equal to the marginal cost c. Why?
Proof by Contradiction

- Suppose one firm sets its price higher than another firm’s price.
- Then the higher-priced firm would have no customers.
- Hence, at an equilibrium, all firms must set the same price.
Suppose the common price set by all firm is higher than marginal cost $c$.
Then one firm can just slightly lower its price and sell to all the buyers, thereby increasing its profit.
The only common price which prevents undercutting is $c$. Hence this is the only Nash equilibrium.
Illustration

NE: at point A where $p_1=p_2=\text{Cost}$
Bertrand Paradox

- For $n \geq 2$ with firms simultaneously setting prices, prices = marginal cost and profits are zero……. Perfectly competitive outcome is replicated
- Intuitive assumption .....surprising result!
- This result holds where firms have identical costs.
- If firms have different costs, then there may or may not be a pure strategy equilibrium.
If firms are capacity constrained, then a mixed strategy equilibrium results

- **Edgeworth (1897) - Capacity Constraints**

Neither firm can meet the entire market demand, but can meet half market demand.

Constant MC to a point, then decreasing returns

Under these conditions, Edgeworth cycle: prices fluctuate between high and low
Kreps & Scheinkman (1983)

- If there is a two stage game,
- in which firms set capacity in stage 1
- And in stage 2, given their capacity, set price
- Then the Cournot result is observed
Differentiated Products resolve the Bertrand Paradox

- Differentiated Products allow price competing oligopolists to mark up
Cournot Competition (1838)

1. 2 Players (identical)
2. Cournot strategy - All firms simultaneously set their output
3. Homogenous product
4. Perfect Information
5. Linear demand
6. Constant MC
2 identical firms, linear demand, constant marginal cost

\[ P = a - Q = a - q_1 - q_2 \]
\[ TC_i = c(q_i) \]
\[ \Pi_1 = (a - q_1 - q_2)q_1 - c(q_1) \]
\[ \frac{\partial \Pi_1}{\partial q_1} = a - 2q_1 - q_2 - c = 0 \]

Choose \( q_1 \) to max \( \pi_1 \), given \( q_2 \)

\[ q_1 = \frac{a - q_2 - c}{2} \]

Higher \( q_2 \), lower level \( q_1 \) to max \( \pi_1 \)

\[ q_1 = R(q_2) \]

Similarly, identical firms \( \Rightarrow \)

\[ q_2 = \frac{a - q_1 - c}{2} \]

\[ q_2 = R(q_1) \]
An equilibrium is when each firm’s output level is a best response to the other firm’s output level - then neither wants to deviate from its output level.

A pair of output levels \((q_1^*, q_2^*)\) is a Cournot-Nash equilibrium if 
\[ q_1^* = R_1(q_2^*) \text{ and } q_2^* = R_1(q_1^*) \]
Firm 1’s “reaction curve” $q_1^* = R_1(q_2^*)$

Cournot-Nash equilibrium $q_1^* = R_1(q_2^*)$ and $q_2^* = R_2(q_1^*)$

Firm 2’s “reaction curve” $q_2^* = R_2(q_1^*)$
Solve reaction curves to find cournot...

\[ R_1(q_2) = R_2(q_1) = \]

\[ q_1 = \frac{a - q_2 - c}{2} \Rightarrow q_1 = \frac{a - (\frac{a - q_1 - c}{2}) - c}{2} \]

solving : \( q_1^* = \frac{a - c}{3} \)

identical firms : in equilibrium \( m \)

\( q_1^* = q_2^* = \frac{a - c}{3} \) cournot nash equil

Total Cournot quantity : \( \frac{2(a - c)}{3} \)

Solve for price : \( p = a - Q = \frac{(a + 2c)}{3} \)
Q: Are the Cournot-Nash equilibrium profits the largest that the firms can earn in total?

A: Firms could earn higher profits if both agreed to set half the monopoly output (and thus earn half monopoly profit each)
HOWEVER

Collusive\Joint profit max output levels $q_{m1}q_{m2}$ not sustainable – incentives to unilaterally deviate – not a NE

if firm 1 continues to produce $q_1^m$, firm 2’s profit-maximizing response is $q_2 = R_2(q_1^m)$
$P_{\text{monop}} > P_{\text{cournot}} > P_{\text{perfcomp=bertrand}}$

$Q_{\text{monop}} < Q_{\text{cournot}} < Q_{\text{perfcomp=bertrand}}$

$\pi_{\text{monop}} > \pi_{\text{cournot}} > \pi_{\text{perfcomp=bertrand}}$
Example:

\[ P = 140 - Q; \quad C_i = 60(q_i); \]

2 firms play Cournot. What are equilibrium outcomes?

\[
P = 140 - q_1 - q_2 \\
\Pi_1 = (140 - q_1 - q_2) \cdot q_1 - 60 q_1 \\
\frac{\partial \Pi_1}{\partial q_1} = 140 - 2q_1 - q_2 - 60 = 0 \\
q_1 = 40 - \frac{1}{2}q_2 \\
\text{and solving profit max } q_2, \text{ given } q_1 \\
q_2 = 40 - \frac{1}{2}q_1\]
Solve reaction functions:

\[ q_1 = 40 - \frac{1}{2} q_2 \quad \text{and} \quad q_2 = 40 - \frac{1}{2} q_1 \]

\[ q_1 = 40 - \frac{1}{2} \left(40 - \frac{1}{2} q_1\right) \Rightarrow q_1^* = \frac{80}{3} = 26 \frac{2}{3} \]

Identical firms: \( q_1^* = q_2^* = 26 \frac{2}{3} \)

Total Cournot quantity: \( \frac{160}{3} = 53 \frac{1}{3} \)

Solve for price: \( p = 140 - Q = \frac{260}{3} = 86 \frac{2}{3} \)

Solve for profits: \( \pi_1 = \pi_2 = \)
Monopoly....1 firm

\[ P = 140 - q_m \]

\[ \Pi_m = (140 - q_m) \cdot q_m - 60 q_m \]

\[ \frac{\partial \Pi_m}{\partial q_m} = 140 - 2q_m - 60 = 0 \]

\[ q_m = 40 \]

\[ p_m = 140 - Q = 140 - 40 = 100 \]
Perfect Competition (& Bertrand)

\[ P = MC = 60 \]
\[ P = a - Q \text{ so } Q = a - c = 140 - 60 = 80 \]
\[ \text{Profit} = p.q - c.q = 0 \text{ since } p = c \]

Shows \( Q^m < Q^c < Q^{pc} \)

And \( P^m > P^c > P^{pc} \)
Thus, Bertrand $\rightarrow$ for $N \geq 2$, get perfectly competitive outcomes

Can show that as $\uparrow N$, Cournot outcome $\rightarrow$ perfectly competitive
1. **N Players (identical)**
2. **Cournot strategy - All firms simultaneously set their output**
3. **Homogenous product**
4. **Perfect Information**
5. **Linear demand**
6. **Zero Cost**
\[ P = a - bQ \quad ; \]

since \( N \) firms are identical

\[ TC_i = 0 + 0q_1 \quad \text{so} \quad c = 0 \]

Firm \(_i\):

\[ \Pi_i = (a - bQ)q_i \]

\[ \frac{\partial \Pi_i}{\partial q_i} = a - bQ - bq_i = 0 \]

\[ a - b(Nq_i) - bq_i = 0 \]

\[ a - b(1 + N)q_i = 0 \]

\[ q_i^* = \frac{a}{b(N + 1)} \]

\[ p_i^* = a - b.N.q_i = \frac{a}{(N + 1)} \]

Note: if \( y = bQ.q_i \) & \( Q = q_i + q_j + \ldots + q_n \)

(so \( dQ/dq_i = 1 \))

\[ dy/dq_i = bQ.1 + bq_i.dQ/dq_i \]

\[ = bQ + bq_i \]
\[ P = \frac{a}{N+1} \]

For \( N = 1 \):
\[ a/2 \]

For \( N = 2 \):
\[ a/3 \]

For \( N = 3 \):
\[ a/4 \]

As \( N \to \infty \), \( p \to c = 0 \).
$P(N)$ function links price cost margins to a given $N$

$P$ for any given $N$ depends on the ‘intensity of competition’ (Bertrand: Most intense)
The entry decision is a backward induction procedure.
We return to modelling entry, where $N$ is endogenous and depends on $P(N)$ in the latter part of the course......