Topic 2: Linear Economic Models

(i) Market Equilibrium
(ii) Market Equilibrium + Excise Tax

Lecture Notes:
sections 2.1 and 2.2

Jacques Text Book (edition 2):
section 1.2 – Algebraic Solution of Simultaneous Linear Equations

section 1.3 – Demand and Supply Analysis
ASIDE:
Solving Simultaneous Equations

⇒ Plot on a graph and then solve to find common co-ordinates…….

OR

⇒ Solve Algebraically

Eg.

\[
\begin{align*}
4x + 3y &= 11 & \text{eq.1} \\
2x + y &= 5 & \text{eq.2}
\end{align*}
\]

1. Express both eq. in terms of the same value of x (or y)

\[
\begin{align*}
4x &= 11 - 3y & \text{eq.1} \\
4x &= 10 - 2y & \text{eq.2'}
\end{align*}
\]
2. Substitute value of eq.1 into eq.2’
   \[ 11 - 3y = 10 - 2y \]

3. collect terms
   \[ 11 - 10 = -2y + 3y \]
   \[ 1 = y \]

4. now compute x
   \[ 4x = 10 - 2y \]
   \[ 4x = 10 - 2 = 8 \]
   \[ x = 2 \]

5. check the solution
   \[ \text{in both equations, when } x = 2, y = 1 \]
   \[ \text{the two lines intersect at } (2,1) \]
Note that if the two functions do not intersect, then cannot solve equations simultaneously…..

\[ x - 2y = 1 \quad \text{eq.1} \]
\[ 2x - 4y = -3 \quad \text{eq.2} \]

\textbf{Step 1}
\[ 2x = 2 + 4y \quad \text{eq.1'} \]
\[ 2x = -3 + 4y \quad \text{eq.2} \]

\textbf{Step 2}
\[ 2 + 4y = -3 + 4y \quad \text{BUT} \Rightarrow \]
\[ 2 + 3 = 0 \ldots \ldots \]

No Solution to the System of Equations
Solving Linear Economic Models

Market Equilibrium
Quantity Demanded = Quantity Supplied

Finding the equilibrium price and quantity levels.....

In general,

Demand Function: \( Q_D = a + bP \)
Supply Function: \( Q_S = c + dP \)

- Set \( Q_D = Q_S \) and solve simultaneously for \( P^e = (a - c)/(d - b) \)
- Knowing \( P^e \), find \( Q^e \) given the demand/supply functions
- \( Q^e = (ad - bc)/(d - b) \)
Eg.1

\[ Q_D = 50 - P \]  \hspace{1cm} (i)
\[ Q_S = 20 + 2P \]  \hspace{1cm} (ii)

\( \Rightarrow \) Set \( Q_D = Q_S \)

\[ 50 - P = 20 + 2P \]
\[ 3P = 30 \]
\[ P = 10 \]

\( \Rightarrow \) Knowing \( P \), find \( Q \)

\[ Q = 50 - P \]
\[ = 50 - 10 = 40 \]

\( \Rightarrow \) Check the solution

i) 40 = 50 − 10 and (ii) 40 = 20 + (2*10)

In both equations if \( P=10 \) then \( Q=40 \)
Changes in Demand or Supply…

Shift the curves and results in a new equilibrium price and quantity

Section 2.2 Notes: Market Equilibrium + Excise Tax

Impose a tax t on suppliers per unit sold…….
Shifts the supply curve to the left

\[ Q_D = a + bP \]
\[ Q_S = d + eP \text{ with no tax} \]
\[ Q_S = d + e(P-t) \text{ with tax } t \text{ on suppliers} \]

\[ Q_D = 50 - P, \text{ and } Q_S = 20 + 2P \text{ becomes } \]
\[ Q_S = 20 + 2(P-t) \]
Write Equilibrium $P$ and $Q$ as functions of $t$

$\Rightarrow$ Set $Q_D = Q_S$

\[50 - P = 20 + 2(P-t)\]
\[30 = 3P - 2t\]
\[3P = 30 + 2t\]
\[P = 10 + \frac{2}{3}t\]

Knowing $P$, find $Q$

\[Q = 50 - P\]
\[= 50 - (10 + \frac{2}{3}t)\]
\[= 40 - \frac{2}{3}t\]
Comparative Statics: effect on $P$ and $Q$ of $\uparrow t$

(i) As $\uparrow t$, then $\uparrow P$ paid by consumers by $\frac{2}{3}t$

$\Rightarrow$ remaining tax ($\frac{1}{3}$) is paid by suppliers

$$\text{total tax } t = \frac{2}{3}t + \frac{1}{3}t$$

Consumers pay $\quad$ Suppliers pay

**Price consumers pay** $-$ **price suppliers receive** $= \text{total tax } t$

e.g. $t = £3$

*Consumer $P$: £12 (pre-tax eq. $p + \frac{2}{3}t$)*

*Supplier $P$: £9 (pre-tax eq. $p - \frac{1}{3}t$)*

(ii) and $\downarrow Q$ by $\frac{2}{3}t$, reflecting a shift to the left of the supply curve
Another Tax Problem….

\[ Q_D = 132 - 8P \]
\[ Q_S = 6 + 4P \]

(i) Find the equilibrium \( P \) and \( Q \).
(ii) How does a per unit tax \( t \) affect outcomes?
(iii) What is the equilibrium \( P \) and \( Q \) if unit tax \( t = 4.5 \)?
**Solution…..**

(i) Equilibrium values

⇒ Set \( Q_D = Q_S \)

\[
132 - 8P = 6 + 4P
\]

\[
12P = 126
\]

\[
P = 10.5
\]

⇒ Knowing \( P \), find \( Q \)

\[
Q = 6 + 4P
\]

\[
= 6 + 4(10.5) = 48
\]

Equilibrium values: \( P = 10.5 \) and \( Q = 48 \)
(ii) The comparative Statics of adding a tax……

\[
Q_D = 132 - 8P \\
Q_S = 6 + 4(P - t)
\]

\[\Rightarrow \quad \text{Set } Q_D = Q_S \\
132 - 8P = 6 + 4(P - t) \\
12P = 126 + 4t \\
P = 10.5 + \frac{1}{3} t
\]

\[\Rightarrow \quad \text{Knowing } P, \text{ find } Q \\
Q = 6 + 4[P - t] \\
= 6 + 4[(10.5 + \frac{1}{3} t) - t] \\
= 48 - \frac{8}{3} t
\]

*Imposing* \( t \) \( \Rightarrow \)  \( \uparrow \) consumer \( P \) by \( \frac{1}{3} t \), supplier pays \( \frac{2}{3} t \), and \( \downarrow Q \) by \( \frac{8}{3} t \)
(iii) If per unit \( t = 4.5 \)

\[
P = 10.5 + \frac{1}{3}(4.5) = 12
\]

*Consumer P:* £12 (pre-tax eq. \( p + \frac{1}{3}t \))

*Supplier P:* £7.5 (pre-tax eq. \( p - \frac{2}{3}t \))

\[
Q = 48 - \frac{8}{3}(4.5) = 36
\]
Market Equilibrium and Income

Increase in Income Y => Shift Out of Demand Curve => ↑ Q_D and ↑ P

\[ Q_D = a + bP + cY \]
\[ Q_S = d + eP \]

Let,

\[ Q_D = 200 - 2P + \frac{1}{2}Y \]
\[ Q_S = 3P - 100 \]

Given the above Demand and Supply functions, what is the impact on the Market Equilibrium of Y increasing from 0 to 20?
\(\Rightarrow\) Set \(Q_D = Q_s\)

\[
200 - 2P + \frac{1}{2}Y = 3P - 100
\]

\[
5P = 300 + \frac{1}{2}Y
\]

\[
P = 60 + \frac{1}{10}Y
\]

\(\Rightarrow\) Knowing \(P\), find \(Q\)

\[
Q = 3(60 + \frac{1}{10}Y) - 100
\]

\[
= 80 + \frac{3}{10}Y
\]

As \(\uparrow Y\) \(\Rightarrow\) \(\uparrow P\) by \(\frac{1}{10}\) of \(\uparrow Y\), and \(\uparrow Q\) by \(\frac{3}{10}\) of \(\uparrow Y\)

What is equilibrium \(P\) and \(Q\) when \(Y = 20\)

\[
P = 60 + \frac{1}{10}Y
\]

\[
P = 60 + \frac{1}{10}(20) = 62
\]

\(i.e\ \uparrow P\) by \(\frac{1}{10}\) of \(20 = 2\)

\[
Q = 80 + \frac{3}{10}Y
\]

\[
Q = 80 + \frac{3}{10}(20) = 86
\]

\(i.e\ \uparrow P\) by \(\frac{3}{10}\) of \(20 = 6\)
Qd = 200 – 2P + \( \frac{1}{2} \) Y  
Qs = 3P – 100

Finding Intercepts:
S (Q, P): (-100, 0) and (0, \( \frac{33}{3} \))

Y=0:
D1 (Q, P): (200, 0) and (0, 100)

Y=20:
D2 (Q, P): (210, 0) and (0, 105)