Asset Price Bubbles and Volatility

Dudley Cooke

Trinity College Dublin

Reading

- RB Ch. 10, EGBG Ch. 18, and CN Ch. 17.
- Garber, P., 1990. Famous First Bubbles, Journal of Economic Perspectives 4, 35–54.
- Evans, G., 1991. Pitfalls in Testing for Explosive Bubbles in Asset Prices, American Economic Review 81, 922-930.

Outline

- Things we'll cover:

1. Dividend-Discount model (Gordon model) of asset prices.
2. Variance bounds tests for asset prices.
3. Tests for bubbles in asset prices.

Examples of Historical Bubbles and Economics

- Well-known bubbles (and possibly fun book; google "niall ferguson"):

1. Mississippi and South Sea Bubbles, 1719-1720.
2. Wall Street Crash, 1920.
3. Tulipmania, 1636-1637.
4. Stock market crash, 1987.
5. Nasdaq/dot-com bubble, 1999-2000.

- For the course, the most important thing is that we will think of the price of an asset as relating to the net present value of it's dividend payout.

Dividend Growth Model of Asset Prices

- Call P_t the price of an asset and D_t it's dividend payout.
- The asset purchased today for P_t can be sold tomorrow at P_{t+1}.
- The rate of return, r_{t+1}, is then the following.

$$r_{t+1} = \frac{D_t + \Delta P^e_{t+1}}{P_t}; \ \Delta P^e_{t+1} \equiv P^e_{t+1} - P_t$$

- That is, the rate of return has two components:

1. The dividend in the period the asset was held (this was set to equal zero in the previous lecture).
2. The capital gain/loss due to the change in price.

Dividend Growth Model of Asset Prices

- Using the gross return, we have,

$$r_{t+1} = \frac{D_t + \Delta P_{t+1}^e}{P_t} \Leftrightarrow 1 + r_{t+1} = \frac{D_t + P_{t+1}^e}{P_t}$$

$$\Rightarrow P_t = \frac{D_t + P_{t+1}^e}{1 + r_{t+1}}$$

- With rational expectations, $P_{t+1}^e = \mathbb{E}_t P_{t+1}$, and so,

$$\mathbb{E}_t (P_t) = \mathbb{E}_t \left(\frac{D_t}{1 + r_{t+1}} + \frac{P_{t+1}}{1 + r_{t+1}} \right) = P_t$$

- Note: the stock price at time t is observable; hence $\mathbb{E}_t (P_t) = P_t$.

Dividend Growth Model of Asset Prices

- To make things simple, assume constant returns; that is, $r_{t+j} = r \ \forall j$. Rolling forward,

$$P_t = \frac{D_t}{1+r} + \frac{\mathbb{E}_t(P_{t+1})}{1+r} \Rightarrow P_{t+1} = \frac{D_{t+1}}{1+r} + \frac{E_{t+1}(P_{t+2})}{1+r}$$

- Substituting one into the other,

$$P_t = \frac{D_t}{1+r} + \mathbb{E}_t \left[\frac{D_{t+1}}{1+r} + \frac{E_{t+1}(P_{t+2})}{1+r} \right] \frac{1}{1+r}$$

$$\iff P_t = \frac{D_t}{1+r} + \frac{\mathbb{E}_t(D_{t+1})}{(1+r)^2} + \frac{\mathbb{E}_t(P_{t+2})}{(1+r)^2}$$

- We have applied the law of iterated expectations; i.e., $\mathbb{E}_t E_{t+1}(P_{t+2}) = E_t(P_{t+2})$.

Dividend Growth Model of Asset Prices

- We can keep going with this method and we end up with a forward-looking equation for prices.

$$P_t = \sum_{j=0}^{N-1} \left(\frac{1}{1+r} \right)^{j+1} \mathbb{E}_t \left(D_{t+j} \right) + \left(\frac{1}{1+r} \right)^N \mathbb{E}_t \left(P_{t+N} \right)$$

- This says that the price of an asset is related to all future expected payouts, plus an additional term.

- How do we deal with the $\left(\frac{1}{1+r} \right)^N \mathbb{E}_t \left(P_{t+N} \right)$ term? For now, we assume the following.

$$\lim_{N \longrightarrow \infty} \left(\frac{1}{1+r} \right)^N \mathbb{E}_t \left(P_{t+N} \right) = 0$$

- We will think of this as a "no bubbles" solution.

No Bubbles Assumption

- Why do we assume $\lim\limits_{N \longrightarrow \infty} \left(\frac{1}{1+r}\right)^N \mathbb{E}_t \left(P_{t+N}\right) = 0$?

- If this term is positive and $D_{t+j} = 0 \ \forall j \geqslant 1 \Rightarrow P_t > 0$, but with no payout, how could $P_t > 0$? We rule that out as a possibility.

- This term cannot be negative.

- So,

$$P_t = \sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^{j+1} \mathbb{E}_t \left(D_{t+j}\right)$$

- Notice the relation with the previous lecture. There we used, $P_t = \delta \mathbb{E}_t \left(D_{t+1}\right)$. We can see this model as generalizing the previous one.

Dividend Growth Model of Asset Prices

- For now, we also assume a constant dividend growth rate, g.

$$\mathbb{E}_t \left(D_{t+j} \right) = \left(1 + g \right)^j D_t$$

- If we use the solution to the model,

$$P_t = \sum_{j=0}^{\infty} \left(\frac{1}{1+r} \right)^{j+1} \mathbb{E}_t \left(D_{t+j} \right) \text{ and } \mathbb{E}_t \left(D_{t+j} \right) = \left(1 + g \right)^j D_t$$

$$\Rightarrow P_t = \left(\frac{D_t}{1+r} \right) \sum_{j=0}^{\infty} \left(\frac{1+g}{1+r} \right)^j \Longleftrightarrow P_t = \left(\frac{1}{r-g} \right) D_t > 0$$

1. Prices are a multiple of the current dividend.
2. This multiple depends on both expected future growth in the dividend and the expected rate of return of the stock.

More on Fair Value

- If we rearrange the condition above, we get a dividend-price ratio which we can use to calculate the fair value of an asset.

$$\frac{D_t}{P_t} = r - g$$

- This could be the rental-price ratio for Irish housing (we can calculate this from ESRI data, say).

- It is well-known that the price of Irish housing has risen dramatically and so we expect that $\frac{D_t}{P_t}$ has fallen.

- Then we can ask if Irish houses are over-valued. Note that this depends on comparing $\frac{D_t}{P_t}$ with r, which can be viewed as the expected return on the investment - this has also fallen.

- This should be reassuring if we use an arbitrage argument.

Extensions to the Basic Model

- As before, we can also extend things to bring the model closer to the data.
- Some obvious possibilities are:

1. Dividends that fluctuate around a steady growth trend.
2. Time-varying expected returns.

- However, any of these extensions needs to help better capture the observed volatility in asset prices.

Ex-Ante and Ex-Post Outcomes

- Is the model we have seen any good at capturing observed asset price volatility?
- First consider a variable, X_t. We have,

$$X_t = \mathbb{E}_{t-1}(X_t) + \varepsilon_t$$

- $\mathbb{E}_{t-1}(X_t)$ is the ex-ante expectation and ε_t is news/innovations.
- From this we have the following,

$$Var(X_t) = Var[\mathbb{E}_{t-1}(X_t)] + Var(\varepsilon_t) + 2Cov[\mathbb{E}_{t-1}(X_t), \varepsilon_t]$$

- This should be familiar.

Ex-Ante and Ex-Post Outcomes

- What happens if we want to forecast the value of X_t?
- If $Cov(\mathbb{E}_{t-1}(X_t), \varepsilon_t) \neq 0$, we can systematically improve our forecast of X_t.
- However, if we assume Rational Expectations, orthogonality, i.e. that $Cov(\mathbb{E}_{t-1}(X_t), \varepsilon_t) = 0$, must imply,

$$Var(X_t) > Var[\mathbb{E}_{t-1}(X_t)]$$

- What this says, is that the variance of the ex-post outcome is bigger than variance of ex-ante expectation.
- With this result, we go back and consider our model.

Asset Price Volatility and "Variance Bounds"

- We have the following model solution, which says stock price is ex-ante expectation of the discounted sum of future dividends.

$$P_t = \sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^{j+1} \mathbb{E}_t \left(D_{t+j}\right)$$

- In our model of dividend growth, RE implies,

$$Var\left(P_t\right) < Var\left[\sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^{j+1} \mathbb{E}_t \left(D_{t+j}\right)\right]$$

- That is, the variance of stock prices is smaller than the variance of present value of the dividend. However, this does not hold in the data.

- Shiller (1981, AER) - seminal paper - finds that asset prices are too volatile. Also see Shiller (2003, JEP).

Problems with the Approach

- General problems (that is, the above result is not that obvious):

1. Price series need to be stationary (that is, an $I(0)$ series) for this test to work.
2. Successive prices might be correlated.
3. Sample size is too small to draw good inference.

- Problems with the model:

1. Dividend expectations cannot be directly observed.
2. There is lots of variation in P_t that never turn out to be justified by later variations in D_t.

Asset Price Bubbles

- So far $\{P_t\}_0^\infty$ may vary, but it is tied to fundamentals. Is that the case?

- Call, $\delta = 1/(1+r)$. Then,

$$P_t = \delta \left[\mathbb{E}_t \left(P_{t+1} \right) + \mathbb{E}_t \left(D_{t+1} \right) \right] \Rightarrow P_t = \sum_{j=1}^{\infty} \delta^j \mathbb{E}_t \left(D_{t+j} \right)$$

- To test for the possibility of bubbles, we distinguish two solutions. Above, is the "fundamentals solution". However, we also allow for a "bubbles solution", which we denote,

$$P_t = P_t^F + B_t$$

- The bubbles solution measures the deviation from the fundamental price. It also satisfies the fundamentals solution.

Asset Price Bubbles

- Since the bubbles solution also satisfies the fundamentals solution, there is a restriction on the dynamic behavior of the bubble, B_t.

$$P_t = \delta \mathbb{E}_t (D_{t+1}) + \delta^2 \mathbb{E}_t (D_{t+2}) + ... + B_t$$

- This implies an equality between the two must hold.

$$\mathbb{E}_t (P_{t+1}) = \mathbb{E}_t \left[\delta \mathbb{E}_{t+1} (D_{t+2}) + \delta^2 \mathbb{E}_{t+2} (D_{t+3}) + ... + B_{t+1} \right]$$
$$= \delta \mathbb{E}_t (D_{t+2}) + \delta^2 \mathbb{E}_t (D_{t+3}) + ... + \mathbb{E}_t (B_{t+1})$$

Asset Price Bubbles

- Further manipulating the previous conditions, we have the following:

$$\delta \mathbb{E}_t \left(P_{t+1} \right) + \delta \mathbb{E}_t \left(D_{t+1} \right)$$
$$= \delta \mathbb{E}_t \left(D_{t+1} \right) + \left[\delta^2 \mathbb{E}_t \left(D_{t+2} \right) + \delta^3 \mathbb{E}_t \left(D_{t+3} \right) + ... + \delta \mathbb{E}_t \left(B_{t+1} \right) \right]$$

- This is the same as,

$$\delta \left[\mathbb{E}_t \left(D_{t+1} \right) + \mathbb{E}_t \left(P_{t+1} \right) \right] = P_t^F + \delta \mathbb{E}_t \left(B_{t+1} \right)$$

- I have used $P_t = \delta \left[\mathbb{E}_t \left(P_{t+1} \right) + \mathbb{E}_t \left(D_{t+1} \right) \right] = P_t^F + \delta \mathbb{E}_t \left(B_{t+1} \right)$ here.
- From this, we can conclude,

$$P_t = P_t^F + B_t$$
$$\text{but } ... P_t = P_t^F + \delta \mathbb{E}_t \left(B_{t+1} \right)$$

Asset Price Bubbles

- In general, we have,

$$P_t = P_t^F + B_t \text{ and } P_t = P_t^F + \delta\mathbb{E}_t\left(B_{t+1}\right)$$

- But both cannot solve $P_t = \delta\left[\mathbb{E}_t\left(P_{t+1}\right) + \mathbb{E}_t\left(D_t\right)\right]$, i.e. the original equation.

- It all only works out if,

$$\mathbb{E}_t\left(B_{t+1}\right) = \left(\frac{1}{\delta}\right)B_t$$

$$\Longleftrightarrow \mathbb{E}_t\left(B_{t+n}\right) = \left(\frac{1}{\delta}\right)^n B_t$$

- That is, if the best forecast of all future values of the bubble depends on its current value (only).

Asset Price Bubbles

- We say that the bubble solution satisfies an 'Euler equation' but violates the transversality condition (the same as the terminal condition imposed before).
- Because B_t is arbitrary, $P_t = P_t^F + B_t$ is not a unique solution.
- Thus, the bubble solution is valid if it is expected to grow at the rate of return required for investors to hold the stock. Investors (rationally) do not care if they pay for the bubble as it pays out the market rate of return $\Rightarrow$ self-fulfilling expectations (e.g. Tulips).
- Note that roughly, we think of there being two types of bubbles; deterministic or stochastic.

Tests of Rational Bubbles

- Consider Shiller's variance bounds test for asset-price volatility when there is a bubble.
- Why is it a bad idea if the data contain a bubble?
- The terminal price, P_N (in the data), contains the bubble element, say, and so,

$$P_t^* = \sum_{j=0}^{N} \delta^j D_{t+j} + \delta^N P_{t+N}$$

- That is, the calculated price is equal to present value of dividends plus actual market price at end of dataset.

Bubbles and Variance Bounds

- The variance bound is the same as before,

$$Var\left(P_t\right) \leq Var\left(P_t^*\right)$$

- But now we also know the following,

$$P_t = P_t^F + B_t; \ \ \mathbb{E}_t\left(B_{t+N}\right) = \left(\frac{1}{\delta}\right)^N B_t$$

$$\Rightarrow P_{t+N} = P_{t+N}^F + B_{t+N}$$

$$\Rightarrow \mathbb{E}_t\left(P_t^*\right) = P_t^F + \delta^N \mathbb{E}_t\left(B_{t+N}\right) = P_t^F + B_t$$

- The conclusion is that, even with a bubble, $P_t = \mathbb{E}_t\left(P_t^*\right)$.

A First Test of Deterministic Bubble

- Flood and Garber (1980, JPE) consider a deterministic bubble process to understand German hyperinflation.
- Their approach is based on the following condition.

$$P_t = P_t^F + B_0/\delta^t$$

- Where B_0 is value of bubble at beginning of sample, and the null is, $H_0 : B_0 = 0$ (if δ known).
- But $\delta^{-1} > 1 \Rightarrow$ regressor δ^{-t} explodes $\Rightarrow$ inference is complicated.
- Despite this, their main result is that there is not a bubble.

A Better Test for Bubbles

- West (1987, QJE) takes a different approach. He calculates one parameter in two different ways to see if they are consistent.

- The idea is the following:

1. Assume there is not a bubble $\Rightarrow$ parameter estimates (confidence interval) should be the same.

2. Assume there is a bubble $\Rightarrow$ parameter estimates should "differ".

- The strength of the approach is that there is no need to specify the bubble process $\Rightarrow$ any (bubble, dividend) can be detected.

West's (1987) Test

- West (1987) runs the following regression,

$$P_t = \delta \left(P_{t+1} + D_t \right) + u_{t+1} \tag{1}$$
$$u_{t+1} = -\delta \left[\left(P_{t+1} + D_{t+1} \right) - E_t \left(P_{t+1} + D_{t+1} \right) \right]$$

- He also assumes, as AR(1) for dividends;

$$D_t = \alpha D_{t-1} + v_t \tag{2}$$

- Without proof, this implies,

$$P_t = \sum_{j=1}^{\infty} \delta^t E_t \left(D_{t+j} \right) = \left(\frac{\delta \alpha}{1 - \delta \alpha} \right) D_t + \varepsilon_t \tag{3}$$

West's (1987) Test

- Now, the question is, how do we estimate $\delta\alpha / (1 - \delta\alpha)$?

1. Indirect: regression estimates of δ in (1) and α in (2).
2. Direct: P_t on D_t in (3).

- H_0: no bubbles $\Rightarrow$ (1) and (2) yield equal results.
- With Shiller's data $\Rightarrow$ rejects $H_0 \Rightarrow$ bubbles are present in the data.

Tests Using Cointegration Results

- Diba and Grossman (1988, AER) ask the following question: why not just consider the stationary properties of the time series (i.e. asset prices and observed fundamentals)?

- Now the idea is the following:

1. If stock prices are not more explosive than dividends $\Rightarrow$ bubble not present.

2. If stock prices are explosive compared to stock prices $\Rightarrow$ bubble present.

- Use a unit root test.

- However, if P_t and D_t have unit roots, it could simply be that, $P_t, D_t \sim CI \Rightarrow$ no bubble. Also implies, $(\Delta P_t, \Delta D_t) \sim I(0)$.

- Ultimately, they reject the bubbles hypothesis.

Evans (1991, AER) and Periodically Collapsing Bubbles

- Suppose there is positive growth in price, but it 'erupts' (i.e. grows rapidly) and then collapses to its (positive) mean rate again.
- If we test $P_t \sim I(1)$ (using the usual Dickey-Fuller test) this may not imply there is a bubble, just an $I(1)$ variable. Furthermore, if the bubble collapses periodically, the test won't work.
- Evans (1991) approach is to create artificial data with a bubble and perform the typical tests. He can't detect a bubble.
- The conclusion is that the Diba and Grossman (1988) finding does not imply a bubble doesn't exist (a negative type of result).

Roundup

- Things we've covered:

1. Dividend-Discount model of asset prices with constant returns and growth.
2. Variance bounds tests for asset prices.
3. Numerous tests for bubbles processes in asset prices (and why variance bounds tests may not be appropriate if that is the case).