Extending the Mean-Variance Framework to Test the Attractiveness of Skewness in Lotto Play

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Catriona Purfield
Dept. of Economics
Trinity College
Dublin 2
Ireland

cpurfild@tcd.ie

Patrick Waldron
Dept. of Economics
Trinity College
Dublin 2
Ireland

pwaldron@tcd.ie

Abstract

Economic theory proposes that consumers are primarily concerned with increasing the mean and reducing the variance of the payoff when choosing between products the return to which is uncertain. This approach fails to explain the popularity of Lotto and other forms of gambling. The highly skewed prize distribution of the Lotto game suggests a case for extending the theory of choice in mean-variance space to include a third dimension, skewness. Empirical examination of Lotto sales supports the case for the inclusion of skewness and other, non-monetary, variables in a demand function.

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Lotto is a pari-mutuel game in which players usually choose 6 from 39 or more numbers through on-line terminals located in retail outlets. Players whose six numbers match those drawn share a large, *jackpot*, prize, but the probability of doing so is low; players matching most but not all of the numbers drawn usually win smaller prizes. Only a small fraction of prize funds are usually devoted to these higher probability outcomes while the bulk of the prize fund is allocated to the low probability jackpot prize. If the jackpot prize is not won, then it is rolled over and added to the jackpot for the following draw. Consequently, the game has the potential to produce very large jackpots.

Lotto is a popular game throughout much of the western world. Clotfelter and Cook (1989, p.92) note that in any given week "about one-third of adults play (Lotto); over the course of the year participation broadens to encompass one-half or more of the adult population." In the fiscal year 1995–1996, US per capita expenditure on Lotto was US$30, while in Ireland, the subject of the empirical analysis in this paper, per capita expenditure was IR£59 in 1996.

As with other gambling products, Lotto is designed to ensure that in aggregate the consumer loses. The expected value of a Lotto ticket is less than its price[1] and the variance of the value of the Lotto ticket is high. Table 1 shows that in 1995, the average payout rate was 55% of cumulative Lottery sales in the US while it was 96% in casinos (Christiansen, 1996, p.98). Therefore Lotto play cannot be a feature of risk averse behaviour, despite the widespread evidence of the game's pop-

[1]The expected return is negative.

ularity. Alternative explanations must be sought to explain the popularity of Lotto. Several of these explanations are reviewed in Section I.

One of the more popular hypotheses used to explain choice under uncertainty is that consumers' utility depends, positively, on the mean and, negatively, on the variance of wealth. Such a consumer would not purchase a Lotto ticket, since Lotto is a particularly poor value gamble, its expected value being lower and its variance higher than in other gambles. Section II provides the theoretical motivation for extending the mean-variance approach to include skewness in the utility function of a Lotto player. It is proposed that the skewness of the Lotto prize distribution counteracts the unattractive mean and variance of returns associated with the Lotto game.

The distribution of the value of a Lotto ticket assuming a single prize fund and uniform number selection is derived in Section III. In addition, the impact of changing the parameters of the Irish Lotto game on this distribution is examined, and it is shown that while the changes introduced in 1992 and 1994 cannot be explained within the mean-variance framework, they are more easily rationalised using the mean-variance-skewness approach.

Finally, in Section IV, we use semi-weekly sales data from the Lotto game in Ireland to estimate a demand function for Lotto and to test how demand responds to draw-by-draw variations in the moments of the distribution of returns. The model also allows us to test the extent to which non-monetary factors, such as addiction and fun, motivate play.

Section I: Economic Explanations for Gambling

The conventional assumption that consumers are *globally* risk averse expected utility maximisers fails to explain the existence of Lotto, except for the rare occasions when large rollovers or other bonuses make it a fair game, or of any gambling products. An expected utility maximising consumer who participates in a normal Lotto draw must be *locally* risk *loving* at some levels of wealth, but may still be *locally* risk *averse* around his or her present level of wealth. There is, of course, no guarantee that such a customer will be able to find a unique (or any finite) utility maximising strategy. Alternatively, it may be that a Lotto player's preferences do not satisfy the expected utility axioms.

These problems have led others to propose alternative ideas for modelling gamblers' preferences. One set of explanations, not necessarily at variance with the expected utility hypothesis, concentrates on the moments of the probability distribution of the gambler's payoff. It has also been suggested that other, non-monetary, aspects of a bet ('fun', 'addiction', *&c.*) may influence the gambler's utility. In this section, we will consider each approach in turn.

Using the standard mean-variance approach, it can be argued that consumers who are locally risk averse around their current level of wealth will look for bets whose payoffs have lower variance.[2] While the expected return and variance of a bet are of concern, this two-moment approach is insufficient to explain Lotto play within the expected utility

[2]Locally risk loving consumers will prefer higher variance.

framework and it is necessary to consider the skewness of the distribution of returns. This three-moment approach has already been suggested by the work of Tsiang (1972), Kraus and Litzenberger (1976), Kraus and Litzenberger (1983) and Waldron (1991) among others, but such an analysis has not yet been applied to Lotto. Indeed, to date, the impact of only the first moment of the distribution of returns has been examined in the context of Lotto (see for example Sprowls (1970), Gulley and Scott (1993), and Farrell, Hartley, Lanot and Walker (1996)). Yet, as noted by Cook and Clotfelter (1993), players are attracted by the large maximum values and asymmetric distributions of the payoffs associated with Lotto. To account for such preferences, this paper considers not only the mean, but also the variance and skewness of the distribution of returns from a Lotto ticket. We show below that by changing the format of the Lotto game, the Irish Lotto operators have been appealing to consumers with a preference for higher skewness. This is consistent with the view put forward by Arditti (1967, p.21), Tsiang (1972, p.359) and others, and expanded on in Section II, that real world attitudes to risk are such that higher skewness is desirable.

Outside the expected utility framework, it has been argued that consumers may be motivated to maximise utility with respect to other, non-monetary, variables. An example of one such motivation is addiction in consumption, as developed by Becker and Murphy (1988) and Becker, Grossman and Murphy (1994). In their models of myopic or rational addiction, past or future consumption influences current consumption. They propose that if past consumption influences current consumption, then a good is addictive. Empirical evidence presented in Section IV suggests myopic addiction in Lotto play. Equally plausible is that Lotto players derive fun from play. A stronger than expected positive relationship between Lotto rollovers and sales may indicate that it is more fun to play Lotto when everyone else is playing.[3] Even risk averse consumers may play Lotto because participation generates some non-pecuniary positive utility.[4]

Douglas (1995), however, notes that investment dynamic is the principal motivating agent for play, investment referring to gambling motivated by the desire to increase wealth. Empirical evidence also favours investment based explanations for Lotto play. Kallick-Kaufmann (1979, p.20) found that the largest proportion of American state lottery participants have traditionally played the game 'to make money' and few play for 'excitement' or 'challenge'. Likewise, a survey conducted by the Irish national lottery found that the 'hope of winning a big prize' was the principal motivating factor for play while 'fun and excitement' was ranked third (DKM Economic Consultants Ltd., 1996).

Section II: The Importance of Skewness

According to the expected utility hypothesis as proposed by von Neumann and Morgenstern (1947), individuals should choose between goods with uncertain prospects, and therefore by implication between

[3]Equally, increased advertising and media coverage in Lotto rollover weeks may also explain the strong relationship between rollover and Lotto sales.

[4]See Kanto, Rosenqvist and Suvas (1992) for an exposition of how fun was incorporated in the utility curve estimate for gamblers at a Finnish harness racetrack.

gambling products, as if their preferences were complete, consistent, transitive and independent, so as to maximise expected utility. Individuals in a gambling market should therefore choose between competing portfolios of gambling products so as to maximise expected utility. If this is so, then the utility function, u, of an individual operating in a gambling market should furthermore possess the characteristics which according to Arrow (1970) are desirable in an expected utility function, that is:

1. $u'(W) > 0$, *i.e.* positive marginal utility of wealth;
2. $u''(W) < 0$, *i.e.* decreasing marginal utility of wealth;
3. $d\left[-u''(W)/u'(W)\right]/dW \leq 0$, *i.e.* non-increasing absolute risk aversion; and
4. $d\left[-Wu''W(W)/u'(W)\right]/dW \geq 0$, *i.e.* increasing relative risk aversion.

A Lotto player's expected utility of wealth can be approximated by a Taylor expansion about the mean of wealth:

$$E\left[u\left(\tilde{W}\right)\right] = u\left(E\left[\tilde{W}\right]\right) + \frac{1}{2}u''\left(E\left[\tilde{W}\right]\right)\mathrm{Var}\left[\tilde{W}\right] + \frac{1}{6}u'''\left(E\left[\tilde{W}\right]\right)\mathrm{Skew}\left[\tilde{W}\right] + \ldots \tag{1}$$

Traditional analysis of the expected utility derived from gambling considers only the first two terms of Equation 1 (see Quandt (1986)).[5] However, Tsiang (1972) notes that as the ratio of risk to wealth increases

[5] Golec and Tamarkin (1997) fit a three moment truncation of expected utility to horse race betting.

the mean-variance analysis becomes less accurate as an approximation of expected utility. An appropriate approximation must then consider higher moments, in particular the third, otherwise expected utility is biased downward. Since Lotto offers a high probability of losing the sum staked in return for a low probability of winning a larger sum, the ratio of risk to wealth is relevant. Therefore the influence of the skewness of the distribution of returns must also be considered in the expected utility function of a Lotto player.

Arditti (1967) offers a compelling argument for the inclusion of skewness in the utility function. He shows that a necessary condition for the properties of positive marginal utility of wealth, or $u' > 0$, and non-increasing absolute risk aversion, or

$$\frac{d}{dW}\left(\frac{-u''}{u'}\right) = \frac{-u'u''' + (u'')^2}{(u')^2} \leq 0, \tag{2}$$

is that

$$u''' \geq \frac{(u'')^2}{u'} > 0. \tag{3}$$

Thus Arrow (1970)'s conditions of positive marginal utility of wealth and non-increasing absolute risk aversion imply that an individual would accept a lower expected return from a gambling product which has higher skewness but the same variance and higher moments as an alternative product. Thus, if the notion of increasing absolute risk aversion is disregarded, it must be acknowledged that a risk averse individual would have a preference for higher skewness in addition to an aversion to dispersion of the probability of returns. Then, dropping higher order terms in

Equation 1,[6] a three moment approximation to a Lotto player's expected utility of wealth is

$$E\left[u\left(\tilde{W}\right)\right] = u\left(E\left[\tilde{W}\right]\right) + \frac{1}{2}u''\left(E\left[\tilde{W}\right]\right)\text{Var}\left[\tilde{W}\right] + \frac{1}{6}u'''\left(E\left[\tilde{W}\right]\right)\text{Skew}\left[\tilde{W}\right] \tag{4}$$

where the coefficient of the skewness of wealth is positive.

Skewness has heretofore largely been ignored in the explanation of gambling behaviour. This is because most gambling products are characterised by a relatively symmetric distribution of returns which render skewness irrelevant. Yet in the UK and Australia the football pools offer similar large prizes but fail to gain the market penetration that characterises Lotto. This suggests that skewness may be irrelevant as a choice parameter. However, high transaction costs dispel the benefits of the skewed prize fund offered by the football pools. These transaction costs comprise of information costs (to play the pools knowledge of football is advantageous) and accessibility costs (football pools are seasonal and operators rely on door-to-door canvassing to attract players).[7] In contrast, Lotto is a simple game of pure chance that requires no prior knowledge. It is available either on a weekly or semi-weekly basis and its distribution network is based on retail agents who sell tickets on their premises for

[6]This analysis ignores terms of the fourth and higher order in the individual's expected utility function. Such an approach is justified because "comparable *a priori* behaviouristic arguments for general investor attitudes toward the fourth and higher moments have not been made" (Kraus and Litzenberger, 1976, p.1087).

[7]Nearly 90% of sales of the Littlewood Pools organisation are received through the hands of agents as opposed to postal service or standing order purchases (Douglas, 1995, p.141).

commission through on-line terminals. The geographic coverage of such outlets is generally wide — over 90% of the UK population is reported to live or work within two miles of a lottery outlet (Camelot Group Plc, 1996, p.21) — while opening hours of such agents are generally flexible. As a result of its low transaction costs, Lotto makes it feasible for players to realise their preferences for skewed prize distributions.

The popularity of Lotto reveals a preference for large maximum prizes, so that utility appears to be increasing in the jackpot or prize variable. Players should exhibit behaviour that is consistent with the hypothesis that they maximise utility with reference to three variables: mean, variance and skewness, in each case holding the values of the other two parameters constant. Provided that the underlying expected utility function is an increasing concave function which displays non-increasing absolute risk aversion, consumer utility will be increasing in expected mean return, decreasing in variance and increasing in skewness. This suggests that the demand for Lotto should be related in the same way to each of these moments.

Section III: The Distribution of the Value of a Lotto Ticket

The expected value of a Lotto ticket, making the crucial simplifying assumptions (which we retain here) that individuals choose their numbers uniformly and that there is just a single (jackpot) prize, was first derived by Sprowls (1970) and has subsequently been used by Cook and Clotfelter (1993), Gulley and Scott (1993) and Scoggins (1995). In practice, single pooled prize funds do not occur and all the evidence suggests that number selection is non-uniform.[8] However, Farrell et al. (1996) prove that the most important theoretical properties of expected value are unaffected by the assumptions of non-uniform selection and fixed prizes.

None of the articles cited here consider any of the moments higher than the mean. Therefore this paper considers in addition to the mean, the variance and skewness of the distribution of returns, retaining the simplifying assumptions of a single jackpot prize pool and of uniform number selection. As far as we are aware, this is the first paper to consider variance and skewness for Lotto.

The jackpot prize is assumed to be the revenue from sales minus the proportion of sales withheld plus any rollover from the previous draw, R. If N is the number of tickets sold, and τ is the proportion of sales withheld by the lottery operator for distribution to nominated good causes and for operational expenses, then the value of the jackpot prize, J, is given by

$$J(\tau, R, N) = R + (1 - \tau) N. \tag{5}$$

To a player who has purchased one of the N tickets sold, at the moment when sales have closed but the draw has not yet taken place, the probability that his or her combination of numbers is drawn is determined by the game matrix. Players usually select six numbers from 36 or more, the winning numbers being drawn without replacement. Thus, if π is the probability that the combination selected by this individual is drawn, then $(1 - \pi)$ is the probability that it is not drawn. Therefore, the probability

[8] Clotfelter and Cook (1989, p.86) note that in one lotto drawing in Maryland, more than 3,200 players had selected the combination 1,2,3,4,5,6. If that combination had won, the jackpot winners would have received a mere $193.50 each.

that the jackpot is not won, q, is given by $q = (1 - \pi)^N$ and the probability that the jackpot is won, p, is given by $p \equiv 1 - q = 1 - (1 - \pi)^N$.

Let V be the random variable denoting the value after the draw has taken place of our player's ticket. Then the probability distribution of the total Lotto payout, which is the product of the number of tickets sold, N, and the value of the typical ticket, V, is described by

$$VN \equiv \begin{cases} J & \text{if the jackpot is won (with probability } p\text{)} \\ 0 & \text{if the jackpot is not won (with probability } q\text{).} \end{cases} \tag{6}$$

Thus VN has expected value

$$\begin{aligned} E[VN] &= J \times p + 0 \times q \\ &= Jp. \end{aligned} \tag{7}$$

The nth moment about the mean of VN is

$$\begin{aligned} E[(VN - E[VN])^n] &= (J - Jp)^n p + (0 - Jp)^n q \\ &= J^n (q^n p + (-p)^n q). \end{aligned} \tag{8}$$

Since N is non-random,[9] the expected value of the prize money per ticket sold can be obtained as in Gulley and Scott (1993), Scoggins (1995) and Farrell and Walker (1996) by dividing across by N in Equation 7:[10]

$$E[V] = p\frac{J}{N} = \left[\frac{R}{N} + (1 - \tau)\right] p. \tag{9}$$

[9] Fifteen minutes prior to the Lotto draw terminals cease selling tickets, so that at the time of draw the number of tickets sold is known.

[10] As noted by Farrell and Walker (1996), Equation 9 is effectively the inverse supply function for the market. Alternatively, Equation 9 may be interpreted as the conditional expectation of V, conditional on N (and R). In the long run, the unconditional expectation of V is, of course, equal to $1 - \tau$.

However, as has been shown in Section II, the variance and higher moments of the prize distribution are also of concern. We derive the variance of the total Lotto payout by setting $n = 2$ in Equation 8:

$$\begin{aligned} \mathrm{Var}[VN] &= J^2\left(q^2p + p^2q\right) \\ &= J^2pq, \\ &= J^2p(1-p), \end{aligned} \tag{10}$$

using the fact that $p + q = 1$. The variance of prize money per ticket sold is therefore given by

$$\begin{aligned} \mathrm{Var}[V] &= \frac{1}{N^2}\mathrm{Var}[VN] \\ &= \frac{J^2p(1-p)}{N^2} \\ &= \left[\frac{R}{N} + (1-\tau)\right]^2 p(1-p). \end{aligned} \tag{11}$$

The third parameter of the distribution of returns, skewness, is obtained by setting $n = 3$ in Equation 8:

$$\begin{aligned} \mathrm{Skew}[VN] &= J^3\left(q^3p + (-p)^3 q\right) \\ &= J^3pq\left(q^2 - p^2\right) \\ &= J^3p(1-p)(1-2p). \end{aligned} \tag{12}$$

Since

$$\mathrm{Skew}[VN] = N^3\mathrm{Skew}[V], \tag{13}$$

the skewness of the prize money per ticket sold is

$$\mathrm{Skew}[V] = \left(\frac{J}{N}\right)^3 p(1-p)(1-2p)$$

$$= \left[\frac{R}{N} + (1-\tau)\right]^3 p\,(1-p)\,(1-2p)\,. \tag{14}$$

From Equations 9, 11 and 14 it is clear that the probability distribution of the value of a Lotto ticket depends on four pre-determined parameters,[11] R, τ, N and p. Since π, N and p are related by the identities:

$$\pi = 1-(1-p)^{\frac{1}{N}}\,, \tag{15}$$

$$N = \frac{\ln(1-p)}{\ln(1-\pi)} \tag{16}$$

and

$$p = 1-(1-\pi)^N\,, \tag{17}$$

the moments can in fact be written in terms of R, τ and any two of π, N and p. It seems most natural to view the moments as functions of the predetermined R, of the operator's or regulator's choice variables, τ and π, and of p, which can be viewed as a scale-invariant measure of sales. Of particular interest is the way in which changes in p, arising from changes in sales, effect the mean, variance and skewness of the value of a Lotto ticket. The relationships when there is a rollover are slightly different from the relationships without a rollover. They can be analysed by examining the partial derivatives of each moment with respect to p, holding R, τ and π constant.[12] Figs. 1–3 summarize our findings where in non-rollover draws ($R=0$)

$$p_0 = \frac{1}{2}, \tag{18}$$

[11]While these four parameters are pre-determined at the time of the draw, clearly the value of sales, N, will be influenced in an economic sense by the values of the other three.

[12]Details of these calculations are available on request from the authors.

$$p_1 = \frac{1}{2} - \frac{1}{\sqrt{12}} \approx 0.21 \tag{19}$$

and

$$p_2 = \frac{1}{2} + \frac{1}{\sqrt{12}} \approx 0.79, \tag{20}$$

while in rollover draws $(R > 0)$

$$p_0 < \frac{1}{2}, \tag{21}$$

$$0.21 < p_1 < \frac{1}{2} \tag{22}$$

and

$$0.79 < p_2 < 1. \tag{23}$$

Moving from left to right as p increases in Figs. 1–3 will have a similar impact as increasing N or π.

Farrell and Walker (1996) show that when there is no rollover $(R = 0)$ the expected value of a Lotto ticket increases towards $1 - \tau$ with increasing p and in the limit is equal to $1 - \tau$. This is because the probability that there is no winner is larger the smaller is p. Thus the expected value of the Lotto ticket rises with p because adding a further ticket decreases the probability of a rollover, the benefit of which current players cannot appropriate in subsequent draws. When there is a rollover $(R > 0)$, the same authors show that the expected value of a Lotto ticket is always higher than in non-rollover draws irrespective of the value of p. As was the case in the absence of rollovers, adding an additional player increases $E[V]$ to a point, but now players have an additional pool of prize money to play for and the higher is p the more likely it is that the prize will have multiple winners. Therefore, for modest rollovers the expected value will attain a maximum at some positive finite p, but for sufficiently large rollovers, the relationship will be monotonically decreasing and in the limit approaches $1 - \tau$ from above. The relationship between the expected value of a Lotto ticket and Lotto sales obtained by Farrell and Walker (1996) is depicted in Fig. 1.

Equation 11 and Fig. 2 show that, when there is no rollover $(R = 0)$, the variance of prize money per ticket sold is a concave quadratic function of p. For $p < p_0 = 0.5$, variance is increasing; it attains its maximum at p_0 and then decreases monotonically for $p > p_0$. This is because the smaller is p the larger is the probability that there is no prize winner, but adding a further player increases the probability that the jackpot is won and therefore increases the variance. Beyond p_0 it is increasingly likely that the jackpot prize will be won and adding further players then reduces the variance.

When there is a rollover $(R > 0)$, the variance is higher than in regular draws. Fig. 2 illustrates that $\text{Var}[V]$ again increases to a maximum at p_0 (where now $p_0 < 0.5$) and then decreases monotonically to zero as p approaches one. $\text{Var}[V]$ attains a maximum more rapidly that in non-rollover draws because the value of rollover prize money per ticket sold decreases as p increases.

Equation 14 shows that, when there is no rollover $(R = 0)$, there is a non-monotonic, cubic, relationship between $\text{Skew}[V]$ and p. For $0 < p < 0.5$, the skewness of the value of a Lotto ticket is positive, while it is negative for $0.5 < p < 1$. However the relationship between $\text{Skew}[V]$ and p has two turning points. $\text{Skew}[V]$ is increasing in the range $0 < p < p_1$, decreasing for $p_1 < p < p_2$ and increasing again for $p_2 < p < 1$.

When there is a rollover the relationship between Skew $[V]$ and p is similar. Skew $[V]$ remains positive in the region $0 < p < 0.5$ and negative between $0.5 < p < 1$. However, the turning point p_1 moves right in the presence of rollovers, with $0.21 < p_1 < 0.5$. Likewise, Skew $[V]$ decreases to the point p_2, but $0.79 < p_2 < 1$. Thereafter Skew $[V]$ increases to zero as 'sales' exceed p_2. Fig. 3 shows that in the presence of rollovers the critical turning points of Skew $[V]$ lie to the right of the turning points in the absence of rollovers.

It is possible to examine empirically how changes in p effect the mean-variance-skewness approximation of expected utility in the Irish context because the Lotto game format or *game matrix* changed on two occasions, first with effect from August 22, 1992 from a 6/36 to a 6+bonus/39 design and then with effect from September 24, 1994 to a 6+bonus/42 design. The motivation outlined by Director of the Irish National Lottery was that under the original format:

> "Sales were stagnating because of lack of exciting jackpots.
>
> The frequency of rollovers had greatly reduced as the jackpot was being won in 3 out of 4 draws[13] and no growth in jackpot size was possible." (Bates, 1993, p. 2)

Both of these changes to the game matrix increased the available number of combinations; reduced π, the probability that a particular combination is drawn; and reduced the proportion of the available combinations covered in a draw for a given level of sales, thereby also reducing p, the

[13]As can be seen from Table 2, this figure exhibits some generous rounding: only 59% of jackpots were won under the 6/36 regime.

probability that the jackpot is won. This increased the occurrence of rollovers and therefore the maximum game payoff. Table 2 summarises how changing Lotto format influenced p in rollover and non-rollover draws.

Equation 4, Table 2 and Figs. 5–7 can be used to illustrate the mixed effects of changing game format on the mean, variance and skewness of the value of a Lotto ticket. Both changes reduced the expected value of a Lotto ticket, and, since expected utility is positively related to expected wealth, therefore impacted negatively on expected utility, *ceteris paribus.* The variance of the value of a Lotto ticket increased, also causing expected utility to decrease, *ceteris paribus.* Thus, using a mean-variance approximation, changing game format unambiguously reduced expected utility which should have resulted in a decrease in Lotto demand.

Lotto demand under each game format has, however, grown unabated. This apparent anomaly is explained by expanding the mean-variance approximation of expected utility to include the skewness of the value of a Lotto ticket. Changing game format changed p on average in a way that increased the skewness of the value of a Lotto ticket, for both rollover draws and non-rollover draws. Since expected utility of wealth is related positively to skewness, it appears that the increase in skewness outweighed the negative impact of changing game format on mean and variance.

Section IV: The Determinants of the Demand for Lotto in Mean-Variance-Skewness Space

A : Data Description

The purpose of this time series analysis of Lotto sales is to determine the importance of the mean, variance and skewness of the value of a Lotto ticket as decision parameters for players. The role of other non-monetary factors is also examined. The data available for analysis are comprised of draw-by-draw sales for the twice weekly Lotto draw in Ireland for the period May 1990 to the end of 1995 (SALES) and the corresponding Lotto rollovers for the same period (ROLLOVER).[14] Lotto, introduced in Ireland in 1988, first held only weekly Saturday draws. In May 1990 a Wednesday draw was added.[15] On average 50% of sales revenue is returned as prizes ($\tau = 0.5$). As already noted, there have been two changes in game format. Using this information, and Equations 9, 11 and 14, $E[V]$, $\text{Var}[V]$ and $\text{Skew}[V]$, abbreviated EV, VAR and SKEW respectively, can be calculated for each draw. $1 - E[V]$ can be viewed as the price of a Lotto ticket, so $E[V]$ is essentially the price variable in this demand analysis.

In the context of the Irish lottery there is considerable variation in the expected value of a Lotto ticket on a draw-by-draw basis as illus-

[14] Note that in practice ROLLOVER includes £1m. and £0.5m. bonuses which are frequently added to the jackpot prize for the normal draw, or offered as a jackpot prize for a free extra draw, on holiday weekends and other special dates. These additions, funded from unclaimed prizemoney, were introduced in order to improve sales.

[15] The first Wednesday draw was on May 30, 1990. There are 584 observations extending from Saturday, May 26, 1990 to Saturday, December 30, 1995, inclusive. No draw was held on Christmas Day, Wednesday, December 25, 1991.

trated in Fig. 5: on numerous occasions the Lotto draw even appears to have been a fair bet ($E[V] > 1$). As can be seen from Equations 9 and 17, this variation is derived from three principal sources. Firstly, the occurrence of rollover draws causes the 'price' of a Lotto ticket to decrease, even becoming negative on occasions. Secondly, changes in the game matrix have reduced p, the probability that the jackpot is won, which indirectly raised the price of a Lotto ticket by increasing the occurrence of rollover draws. Thirdly, variations in sales alter the probability that the jackpot is won and the rollover per ticket, and thus also effect $E[V]$.

Taking advantage of the draw-by-draw variations in the price variable, it is possible to estimate the demand function for Lotto because other factors that influence demand such as income, gender and race are relatively constant in comparison. Since by definition draw-by-draw changes in R, N and π also generate variation in the variance and skewness of the value of a Lotto ticket it will be possible for the first time to estimate the impact of these variables on Lotto demand.

Figs. 4–7 plot the raw data against time and especially evident is the considerable variation in all variables due to the aforementioned factors. Since each of these time series exhibits considerable 'seasonal' variation between Wednesday and Saturday draws and since the series also contain a number of outlying observations, the first step in the analysis is to investigate formally whether they are stationary. Lotto sales tend to be above trend on Saturdays and below trend on Wednesdays, suggesting that the detrended series are negatively autocorrelated and that we should be testing for the presence of a seasonal root.[16] To determine the existence or otherwise of seasonal and unit roots, the procedure developed by Hylleberg, Engle, Granger and Yoo (1990) was modified to test for the semi-weekly roots. We are satisfied that SALES, EV, VAR and SKEW are stationary.[17]

B : Model Estimation and Analysis

From Section II it is hypothesised that expected utility and Lotto demand are related positively to changes in the mean, negatively to changes in the variance and positively to changes in the skewness of the value of a Lotto ticket. However, there is a simultaneous relationship between the variables EV, VAR, SKEW and SALES. The moments are endogenous to the model and are therefore correlated with the error terms in any estimated Lotto demand equation. This invalidates any estimation of Lotto demand using ordinary least squares (OLS) because the resulting parameter estimates are biased and inefficient. Therefore a two-stage least squares regression is used to estimate consistent and efficient parameters of Lotto demand. In the first stage, the predicted values are obtained by regressing EV, VAR and SKEW on all the predetermined variables in the following equation:[18]

$$\begin{aligned}\text{NTHMOMENT} &= b_0 + b_1\text{TIME} + b_2\text{SAT} + b_3\text{ROLLOVER} \\ &\quad + b_4\text{ROLLSQ} + b_5\text{ROLLCUB} + b_6\text{MILLIONX}\end{aligned}$$

[16] Regressing the sales series on a dummy variable for this Wednesday/Saturday effect gives an R^2 value of 23%.

[17] See Purfield and Waldron (1999) for further details.

[18] See Table 3 for variable definitions.

$$
\begin{aligned}
&+b_7\text{EXTRA} + b_8\text{MILLONE} + b_9\text{MILLTWO} \\
&+b_{10}\text{GUARANT} + b_{11}\text{KLWICZ} + b_{12}\text{MILL1} \\
&+b_{13}\text{MILL2} + b_{14}\text{MILL3} + b_{15}\text{GUARCEQ2} \\
&+b_{16}\text{GUARCEQ3} + b_{17}\text{STARTUP} + b_{18}\text{CARRYEQ1} \\
&+b_{19}\text{CARRYEQ2} + b_{20}\text{CARRYEQ3} + b_{21}\text{CARRYEQ4} \\
&+b_{22}\text{CARRYEQ5} + b_{23}\text{CARRYEQ6} + b_{24}\text{EXTRA1} \\
&+b_{25}\text{EXTRA2} + b_{26}\text{EXTRA3} + b_{27}\text{THIRTYN} \\
&+b_{28}\text{FORTYTWO} + b_{29}\text{UKLOTTO} + b_{30}\text{SALES(-1)} \\
&+b_{31}\text{MILLX(-1)} + b_{32}\text{OBS542} + b_{33}\text{STARTROLL} \\
&+b_{34}\text{HALFMILL} + b_{35}\text{MILLXWED} + b_{36}\text{WEDEXTRA} \\
&+b_{37}\text{LGXMAS92} + b_{38}\text{GUARMILLX}. \qquad (24)
\end{aligned}
$$

The first stage regressions for EV, VAR and SKEW have $\bar{R}^2$ values of 98%, 88%, and 87% respectively and using the fitted values from these equations should be adequate to purge the demand equation below of the stochastic element.

In the second stage of estimation, the endogenous variables that appear in Equation 25 are replaced with the fitted values calculated in the first stage regressions and an OLS regression of SALES on these fitted values and a subset of the predetermined variables is conducted. The model is summarised by the following equation:

$$
\begin{aligned}
\text{SALES} \quad = \quad & b_0 + b_1\text{TIME} + b_2\text{SAT} + b_3\text{EV} \\
&+b_5\text{VAR} + b_6\text{SKEW} + b_7\text{EV(-1)} \\
&+b_8\text{VAR(-1)} + b_9\text{SKEW(-1)} + b_{10}\text{THIRTYN} \\
&+b_{11}\text{FORTYTWO} + b_{12}\text{CARRYEQ1} + b_{13}\text{CARRYEQ2}
\end{aligned}
$$

$$
\begin{aligned}
&+b_{14}\text{CARRYEQ3} + b_{15}\text{CARRYEQ4} + b_{16}\text{CARRYEQ5}\\
&+b_{17}\text{CARRYEQ6} + b_{18}\text{KLWICZ} + b_{19}\text{GUARANT}\\
&+b_{20}\text{SALES(-1)} + b_{21}\text{SALES(-2)} + b_{22}\text{SALES(-3)}\\
&+b_{23}\text{SALES(-4)} + b_{24}\text{MILLONE} + b_{25}\text{MILLTWO}\\
&+b_{26}\text{EXTRA} + b_{27}\text{GUARMILLX} + b_{28}\text{MILL1}\\
&+b_{29}\text{MILL2} + b_{30}\text{MILL3} + b_{31}\text{GUARCEQ2}\\
&+b_{32}\text{GUARCEQ3} + b_{33}\text{STARTROLL} + u \qquad (25)
\end{aligned}
$$

The results of this procedure are presented in Table 4. The model explains the variation in sales well, the R^2 being 97%. LM tests for serial correlation and functional form are satisfactory.

The majority of the coefficients in the model are significant at the 1% level. The positive and significant coefficient on TIME indicates that sales increase draw-by-draw. Lotto players in the Irish market do not yet seem to have switched to other products as the Lotto game has aged. This contrasts with the findings of Gulley and Scott (1993) who find that Lotto players in Kentucky, Massachusetts and Ohio reduce expenditure through time. The statistically significant coefficient on SAT confirms that sales are higher on Saturday than on Wednesday drawings. The coefficients on THIRTYN and FORTYTWO show that Lotto sales increased as a result of Lotto game format change, while the negative coefficient on STARTROLL indicates that Wednesday rollovers were initially slow to generate extra sales. The variable KLWICZ captures the positive impact on Lotto sales of a betting syndicate organised by Stefan Klincewicz to purchase all combinations of numbers in May 1991. Likewise, the positive coefficients on various bonus prize variables (GUARANT, MILLONE, MILLTWO and EXTRA) reflect the positive influence on Lotto demand of bonus additions to and guaranteed minimum values of the prize fund. The significant and positive coefficients on lagged values of the dependent variable show that consumption from three of the four previous draws impacts positively on current consumption thus confirming that Lotto play is myopically addictive.

To evaluate whether or not Lotto consumers' preferences for the moments of their payoff are as outlined in Section II, we examine the influence of the mean, variance and skewness of the value of a Lotto ticket on Lotto demand. In Section II it was argued that demand should be increasing in EV (or, equivalently, decreasing in price). The estimated coefficient on EV indicates that the data are consistent with this hypothesis: a one unit increase in the expected value of a Lotto ticket increases Lotto sales by IR£2.4m., *ceteris paribus.* This contrasts with the findings of Cook and Clotfelter (1993), who also use two-stage least squares to estimate the effect of expected value on Lotto demand and find a negative relationship. However, this is most likely due to the fact that the Lotto jackpot and $E[V]$ in their regressions are strongly positively correlated. Using weekly UK Lotto sales, Farrell et al. (1996) find that Lotto demand is positively related to $E[V]$. Likewise Gulley and Scott (1993) find that Lotto demand in Kentucky, Massachusetts and Ohio is negatively related to $1 - E[V]$, the price of a Lotto ticket. DeBoer (1986), who conducted a panel study of seven state lotteries in the US, also found the same relationship.[19]

For the purposes of the model outlined in Section II, the most important results are the coefficients on the variance and skewness variables. Section II proposed that the property of diminishing marginal utility of wealth implies that the coefficient on the second term in a Taylor approximation of expected utility is negative. Empirically, the negative coefficient on VAR confirms that this is indeed the case: a one unit increase in the variance of the value of a Lotto ticket decreases demand by IR£1.2m., *ceteris paribus.* This is as expected because consumers are concerned not only with the expected return from risky products but also with the variance of the return.

Section II also proves that positive marginal utility of wealth and non-increasing absolute risk aversion imply that the coefficient of skewness in a Taylor approximation of expected utility is positive. The hypothesis is confirmed by the data: a one unit increase in skewness of the value of a Lotto ticket increases demand by IR£1.8m., *ceteris paribus.*

Using the coefficient on EV it is possible to evaluate whether the National Lottery operator is maximising the revenue contributed to the state exchequer. To calculate the revenue maximising elasticity, it is necessary to account for variable costs, which sum to 10% of total sales

[19]However, there is evidence that the relationship between Lotto demand and expected value is non-linear. The significant coefficients on CARRYEQ1–CARRYEQ6 show that as EV changes in successive rollovers, sales initially fall below trend before rising above trend, *ceteris paribus.* This convex relationship may reflect the fact that it is more fun to play Lotto when a wider proportion of the population are playing or it may reflect some advertising or media effect. Likewise, the positive coefficients on MILL1, MILL2, MILL3, GUARCEQ2, and GUARCEQ3, denoting when rollovers interact with various bonus prizes, also convey a non-linear relationship between expected value and sales.

in Ireland. From Clotfelter and Cook (1989, p.282) we calculate that with an average payout rate of 50% and variable costs totalling 10%, net revenues are maximised if elasticity is -1.25. However, the observed price elasticity calculated at sample means is -0.67.[20] Since sales are inelastic with respect to the price of a Lotto ticket, the policy implication is that the expected value of a Lotto ticket should be reduced either by making the game more difficult or by increasing the take-out rate.

Section V: Conclusion

From our estimated model it would appear that Irish Lotto players are maximising expected utility in accordance with the propositions of Arrow (1970) as outlined in Section II. However, our analysis makes use of various simplifying assumptions. Firstly, we assume that Lotto players choose their numbers in a uniform manner. Secondly, there is assumed to be only one prize pool which goes to one ticket holder; no account is taken of shared prize pools. Thirdly, the probability distribution considered is that of the *average* value of a Lotto ticket, and not that of the value of a *particular* ticket, which would have to take into account the effect of possibly sharing the jackpot with other winners. Finally, the weekly take-out rate is assumed to be equivalent to the long-run take-out rate but empirical evidence suggests that in Ireland, the proportion of turnover devoted to prize money varies on a draw-by-draw basis. However, in the absence of further information from the Irish National Lottery operator on the

[20] Since the elasticity with respect to expected value is asymptotically equivalent to the price elasticity, at large levels of sales. The authors wish to thank Ian Walker for his assistance in this point.

prize fund algorithm it is impossible to correct for this mis-specification. Future work will attempt to relax the first three assumptions.

The Lotto game appeals to a wide audience and is one of the most popular betting mediums in the US and Europe. However, standard utility theory, where products are evaluated in mean-variance space, offers little explanation for the popularity of the Lotto game. Here it is proposed that, if consumers possess a utility function that is increasing and displays non-increasing absolute risk aversion, then they will in addition to mean and variance be concerned with the skewness of the value of a gambling product. The low probability large prize structure associated with the Lotto game is an ideal medium for testing this hypothesis. Using semi-weekly observations from the Lotto game in Ireland, we find that in addition to the mean and variance, the skewness of the value of a Lotto ticket is also a significant parameter which impacts positively on demand. We find that non-monetary factors such as addiction and fun also impact demand.

Table 1: Payout Ratios and Net Expenditure on Gambling Products in the US, 1995.

Product	Retained	Payout	Turnover IR£m	Net Expend IR£m
Pari-Mutuel	21%	79%	10,988	2,312
Lotteries	45%	55%	23,988	9,405
Casinos	4%	96%	260,842	11,120
Bookmaking	4%	96%	1,602	62

Source: Christiansen (1996), authors' own estimates.

Table 2: Effect of Changing Game Format on p in the Irish Context

	Theoretical values			
Game Format	**p for Non-Rollover Draws**	**p for Rollover Draws**	**p for All Draws**	**Actual values**
6/36-2	0.71	0.80	0.74	0.59
6/39-2	0.59	0.69	0.64	0.51
6/42-2	0.47	0.54	0.51	0.44

Notes: These figures are based on all Lotto draws up to the end of 1995. The theoretical values are computed from Equation 17 using the relevant value of π for each subperiod and the actual level of sales and averaged across draws; there are also significant variations (not shown) between the theoretical values of p for Wednesday and Saturday draws. The actual values are just the proportion of draws in each subperiod in which the jackpot was actually won. The differences between the theoretical and actual values of p reflect the fact that in practice non-uniform selection substantially reduces the probability that the jackpot is won.

Table 3: List of variables and their definitions

Variable	Description
Const	Intercept parameter
Time	Draw number, running from 1 to 584
Sat	Dummy variable for Saturday drawings
Thirtyn	Dummy variable for 6 from a 39 number draw
Fortytwo	Dummy variable for 6 from a 42 number draw
Klwicz	Dummy variable for the Klincewicz draw
Guarant	Dummy variable for guarantee of small prizes
Millone	Dummy variable for jackpot guarantees of £1 million
Milltwo	Dummy variable for jackpot guarantees of £2 million
Extra	Dummy variable for extra draws
Variable(-n)	The nth lagged value of the named variable
Ev	The expected value of a Lotto ticket
Var	The variance of the value of a Lotto ticket
Skew	The skewness of the value of a Lotto ticket
Carryeq1-6	Dummy variables for number of sucessive rollovers
Mill1-3	millionx×carryeq1-3
Guarceq2-3	guarant×carryeq2-3
Rollover	Equal to previous jackpot if unwon, zero otherwise
Rollsq-cub	The squared and cubed values of the rollover
Millionx	Dummy variable for £1 million bonus draws
Startup	Dummy variable for first 13 Wednesday draws
Startroll	startup×rollover
Extra1-3	extra×carryeq1-3
UKLotto	Dummy variable from the start of the UK Lotto
Obs542	Dummy variable for interaction of Millionx, Mill3 and Carryeq3
Halfmill	Dummy variable for £500,000 bonus to jackpot
Millxwed	millionx×(1 − sat)
Wedextra	extra×(1 − sat)
Guarmillx	millionx×guarant
lgxmas92	Dummy variable for outlying Christmas observation (Dec. 20, 1992)

Table 4: Two-Stage Least Squares Regression of Lotto Demand

Dependent variable is SALES

Regressor	Coefficient	Standard Error	T-ratio[Prob]
CONST	40890.00	71471.40	0.57212[.567]
TIME	757.16	120.58	6.2792[.000]
SAT	311109.50	36192.90	8.5959[.000]
THIRTYN	231937.30	29816.80	7.7787[.000]
FORTYTWO	429198.30	64306.30	6.6743[.000]
STARTROLL	-0.31	0.046	-6.8186[.000]
KLWICZ	548440.60	117810.90	4.6553[.000]
GUARANT	245542.00	38595.30	6.3620[.000]
MILLONE	285422.80	60647.80	4.7062[.000]
MILLTWO	548021.30	81720.20	6.7061[.000]
EXTRA	266553.30	28852.80	9.2384[.000]
SALES(-1)	-0.0079	0.096	-0.082185[.935]
SALES(-2)	0.043	0.016	2.7342[.006]
SALES(-3)	0.026	0.012	2.1954[.029]
SALES(-4)	0.044	0.010	4.2243[.000]
EV	2352693.00	109167.80	21.5512[.000]
VAR	-1160777.00	288477.10	-4.0238[.000]
SKEW	1823179.00	622261.20	2.9299[.004]
EV(-1)	188729.50	309109.30	0.61056[.542]
VAR(-1)	-608892.50	336014.30	-1.8121[.071]
SKEW(-1)	179171.10	695015.30	0.25779[.797]
CARRYEQ1	-212128.40	24815.50	-8.5482[.000]
CARRYEQ2	-185951.90	42766.50	-4.3481[.000]
CARRYEQ3	-16400.60	53370.20	-0.30730[.759]
CARRYEQ4	192924.80	64643.20	2.9845[.003]
CARRYEQ5	571435.40	83357.60	6.8552[.000]
CARRYEQ6	1247473.00	123053.70	10.1376[.000]
MILL1	135475.90	52558.10	2.5776[.010]
MILL2	257096.60	65845.20	3.9046[.000]
MILL3	412719.20	61542.90	6.7062[.000]
GUARCEQ2	508678.60	94121.60	5.4045[.000]
GUARCEQ3	326974.70	124206.10	2.6325[.009]
GUARMILLX	-431854.40	89405.90	-4.8303[.000]

R-Squared	0.97	F-statistic $F(32, 550)$	658.4199[.000]
R-Bar-Squared	0.97	S.E. of Regression	83021.30
Residual Sum of Squares	379×10^{10}	Mean of Dependent Variable	1651655.00
S.D. of Dependent Variable	505999.80	Value of IV Minimand	647×10^{9}
DW-statistic	1.72		

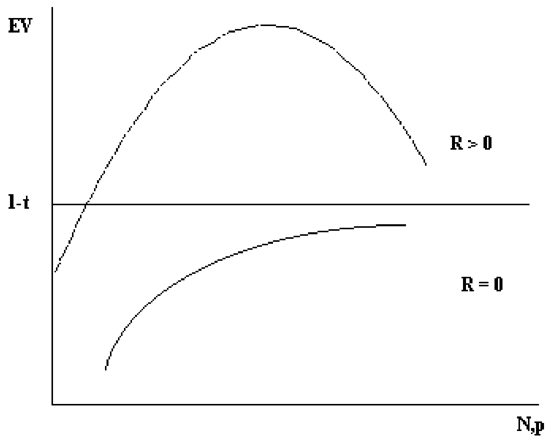

Fig. 1: Relationship between the Expected Value of a Lotto Ticket and Lotto Sales, 1990–1995 (ev).

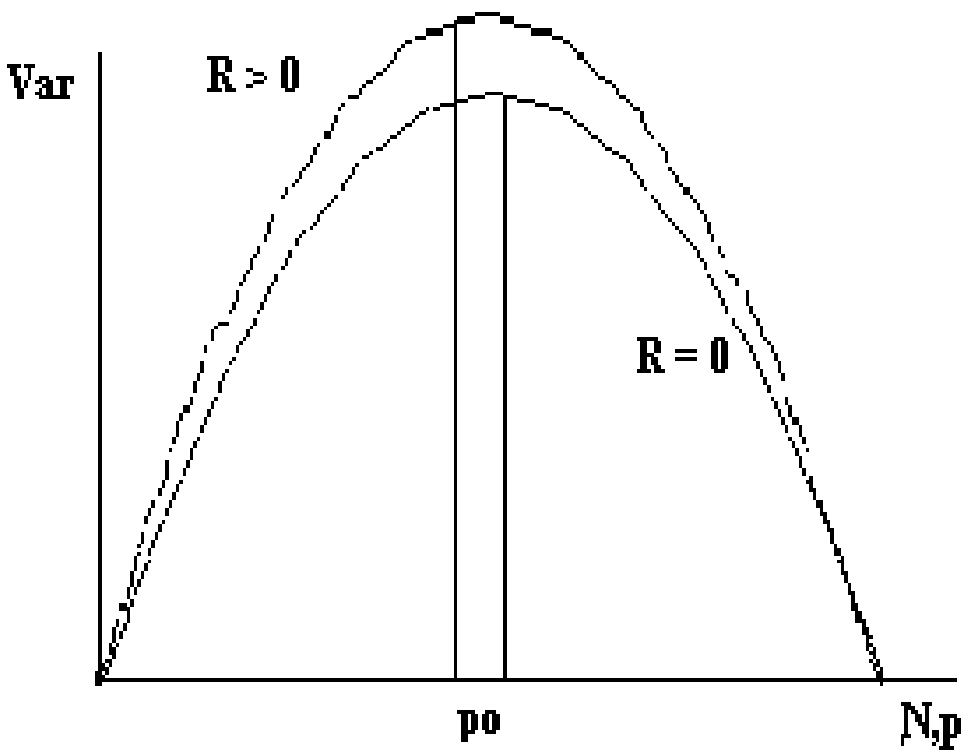

Fig. 2: Relationship between the Variance of the Expected Value of a Lotto Ticket and Lotto Sales, 1990–1995 (var).

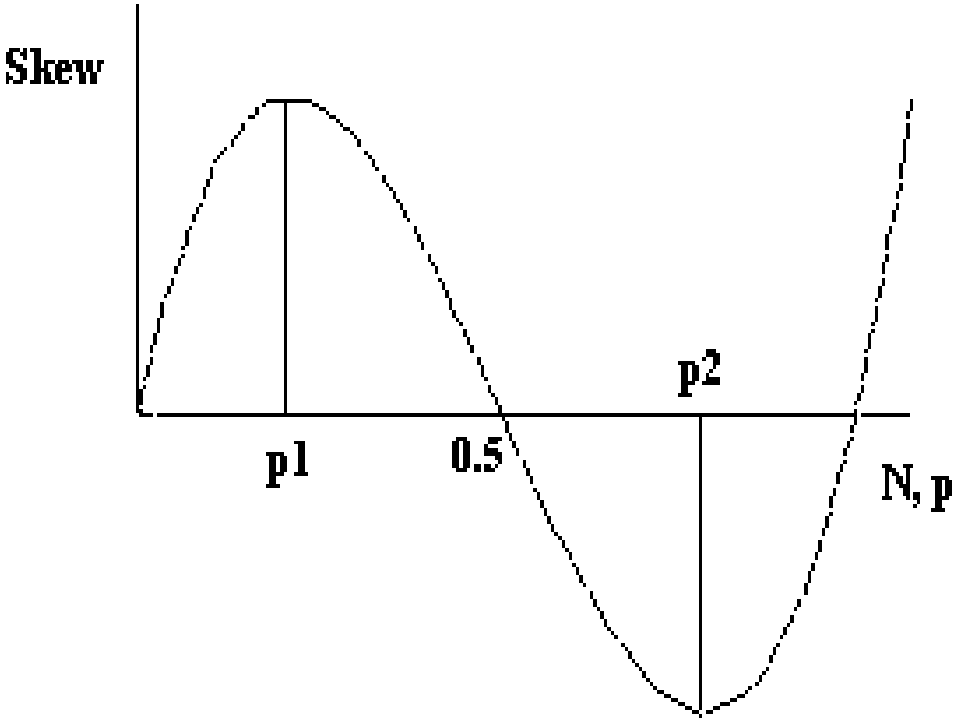

Fig. 3: Relationships between the Skewness of the Expected Value of a Lotto Ticket and Lotto Sales, 1990–1995 (skew).

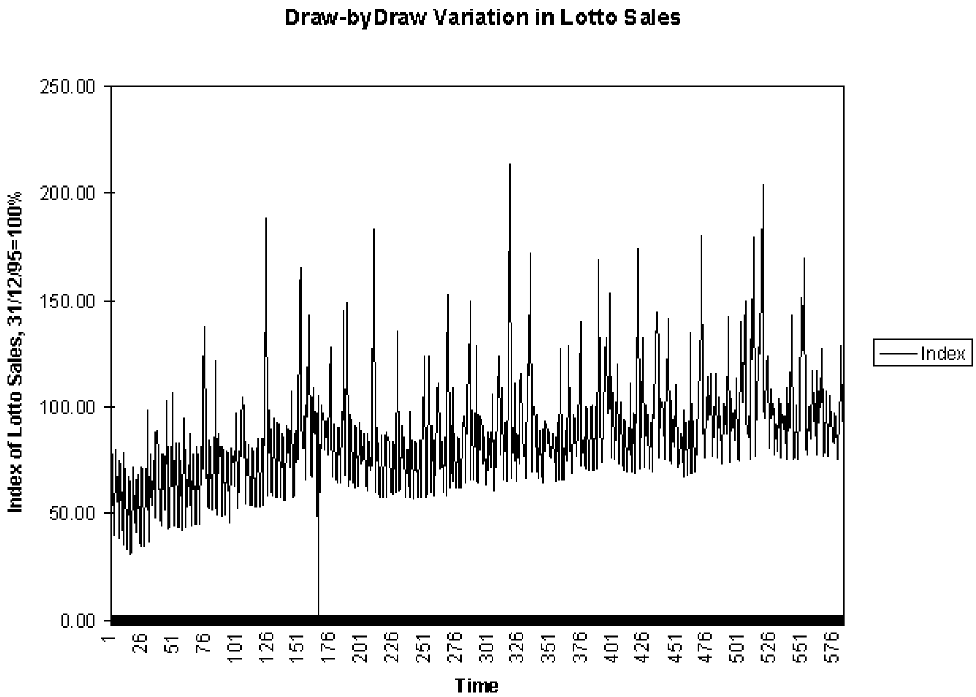

Fig. 4: Draw-by-draw Index of Lotto sales, 1990–1995 (sale index).

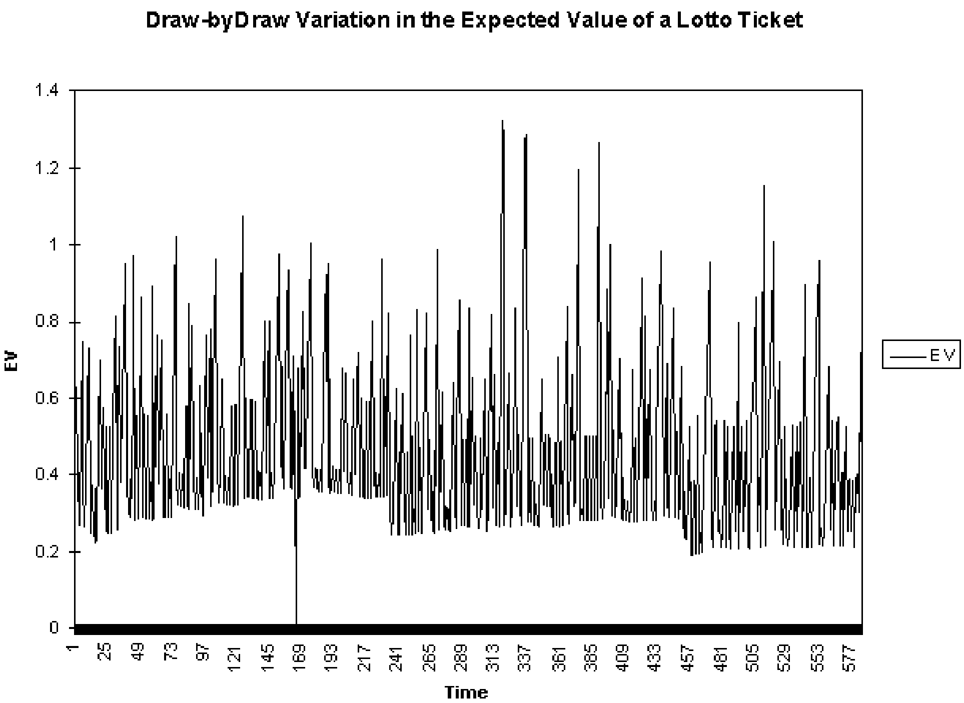

Fig. 5: Draw-by-draw Variation in the Expected Value of a Lotto Ticket, 1990–1995 (ev).

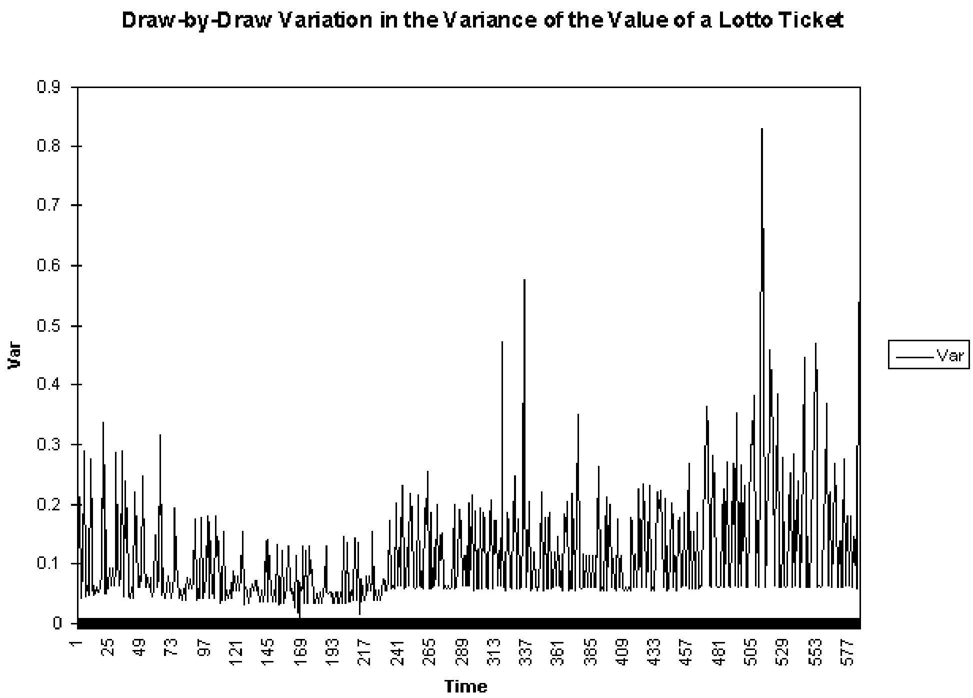

Fig. 6: Draw-by-draw Variation in the Variance of a Lotto Ticket, 1990–1995 (var).

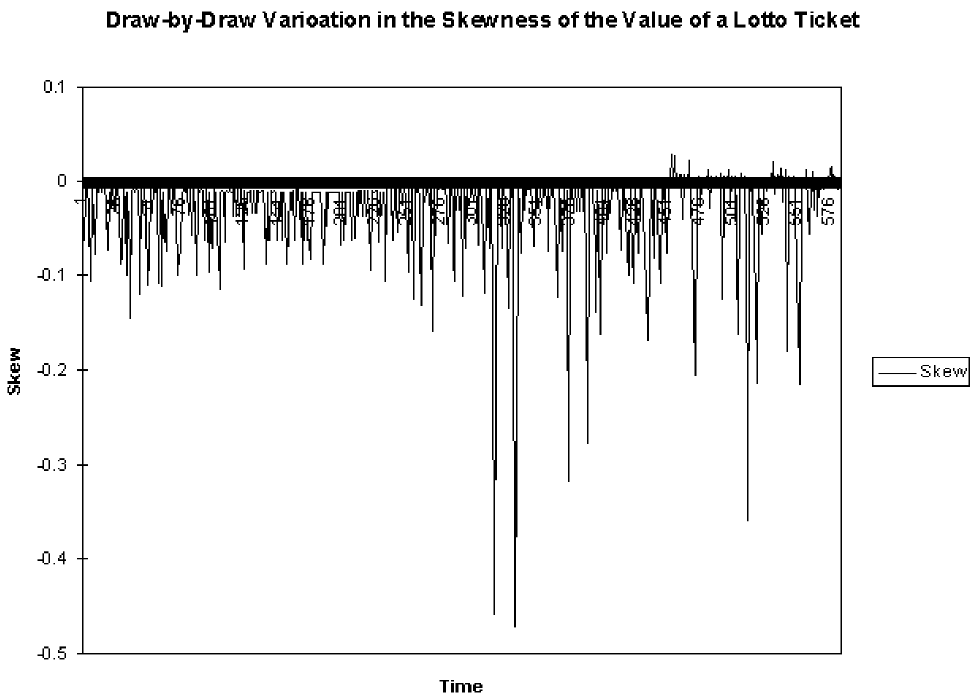

Fig. 7: Draw-by-draw Variation in the Skewness of a Lotto Ticket, 1990–1995 (skew).

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