UNIVERSITY OF DUBLIN

TRINITY COLLEGE

FACULTY OF ARTS, HUMANITIES AND SOCIAL SCIENCES

DEPARTMENT OF ECONOMICS

Senior Freshman

BESS, TSM, PPES

QUANTITATIVE METHODS
FOUNDATION SCHOLARSHIP EXAMINATION

SAMPLE PAPER

Exam Instructions:
Please answer any 3 of the 4 questions.
Each question carries equal weight.

Materials Permitted for this Examination: Standard calculator

You may not start this examination until you are instructed to do so by the Invigilator

Q1. **Economic Applications of Cramer's Rule**

(a) Explain what Cramer's rule is and how it is important for the analysis of economic models. Use Cramer's rule to find the solution of the following system of equations:

$$\begin{aligned} 7x_1 - x_2 - x_3 &= 0 \\ 10x_1 - 2x_2 + x_3 &= 8 \\ 6x_1 + 3x_2 - 2x_3 &= 7 \end{aligned}$$

(b) Consider the National-Income model:

$$\begin{aligned} \text{Y} &= C + I_0 + G_0 \\ C &= a + b(Y - T) \\ T &= d + tY \end{aligned}$$

where $a > 0$, $0 < b < 1$, $d > 0$, $0 < t < 1$, and Y is output, C is consumption, I_0 is investment, G_0 is government consumption, T are taxes, and t is income tax rate.

(i) Identify the endogenous and exogenous variables and explain their meaning. Write the system in matrix form and solve it using Cramer's rule.

(ii) Derive and discuss the impact of a government spending stimulus package on the equilibrium levels of Y, C, and T.

Q2. **Implicit-Function Theorem**

(a) Discuss the usefulness of the Implicit-Function theorem for comparative-static analysis. Describe which are the conditions that ensure the existence of an implicit function at point $(y_0, x_{10}, ..., x_{m0})$.

(b) For each of the following equations $F(y, x) = 0$, is an implicit function $y = f(x)$ defined around the point $(y = 3, x = 1)$?

(i) $x^3 - 2x^2y + 3xy^2 - 22 = 0$

(ii) $2x^2 + 4xy - y^4 + 67 = 0$

If your answer is affirmative, find dy/dx by the implicit-function rule and evaluate it at the given point.

(c) Consider the following national-income model without taxes:

$$\begin{aligned} Y - C(Y) - I(i) - G_0 &= 0 \quad (0 < C' < 1; I' < 0) \\ kY + L(i) - M_0 &= 0 \quad (k = \text{positive constant}; L' < 0) \end{aligned}$$

Analyse the comparative statics of the model when money supply changes and subsequently when government expenditure changes. Discuss your results.

Q3. **Utility Maximization and Consumer Demand**

(a) Given the utility function $U = U(x, y) = (x + 2)(y + 1)$ and the following budget constraint $xP_x + yP_y = B$ with $P_x = 4$, $P_y = 6$, and $B = 130$. Write the Lagrangian function for the utility maximization problem. Derive the first order conditions. Derive the expression for the Lagrange multiplier and discuss its meaning in the context of this problem.

(b) Using the first order conditions in part (a) derive the marginal rate of substitution between goods x and y and describe its meaning. Plot the indifference curves and the budget constraint. Discuss what happens when P_x falls.

(c) Derive the second order conditions and interpret the sign of the bordered Hessian. Discuss your results.

Q4. **Elasticity and Inflection Points**

(a) Consider a generic demand function $q = D(p)$

(i) Explain what is the (price) elasticity of demand and prove that it can be written as

$$\epsilon_{qp} = \frac{d(\ln q)}{d(\ln p)}$$

(ii) Find the elasticity of demand when $q = k/p$ and interpret your result.

(iii) Prove that given any two functions $y = g(w)$ and $w = h(x)$ it is true that $\epsilon_{yx} = \epsilon_{yw}\epsilon_{wx}$

(b) What are *inflection points* and how can we identify them? Discuss using examples.

(c) What are the differences and similarities between the mathematical and the economic approach to problem solving? Discuss (feel free to use examples).