Pricing's Next Top Model: A Game Theoretic Analysis Of 'Freemium' Pricing

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The Freemium pricing model, where a firm offers a service simultaneously at a free and a premium price, is skilfully examined by Greg Mangan using a game theory approach and then shown, under certainty, to be the firm's optimal pricing choice. The ease with which he extends his model and analyses its conclusions, from adding the case of imperfect information, to the discussion of in-game purchases and add-ons and the presence of network effects, is a testament to the insightful nature and the relevance to industry of the ideas expressed in this essay.

Introduction

The freemium pricing model, a simple portmanteau of free and premium, functions as it sounds - a firm offers a service simultaneously at a free and a premium price point. In the last decade or so, the freemium pricing model has been actively adopted by many firms, though primarily in the technology industry. Freemium services should be viewed as distinct from services that are `free-to-try' or that offer free samples, the important distinction being that the free service offered in a freemium model may be consumed in the long-term, beyond any form of trial period. In short, 'Users get basic features at no cost and can access richer functionality for a subscription fee' (Kumar, 2014).

This paper builds a simple game-theoretic model of firm and consumer behaviour under the freemium pricing model, and shows that it is generally an optimal choice for firms in the face of uncertainty over their customers' willingness to pay. Firstly though, a simple pricing game without a freemium option is presented as a motivator for the firm's decision. The model is then expanded to include freemium as a pricing strategy. Finally, the element of uncertainty is introduced into the game, where the firm is uncertain whether they are playing against a consumer who is willing to pay for their service or one who is not. An analysis of the model is then provided with some discussion of possible extensions.

Simple Pricing Game

Firstly, a simple pricing game without a freemium option is considered. There are two players in this sequential game, the Firm and the Consumer. The firm moves first, choosing to either offer a 'Premium' service or a 'Free' service. Providing the premium service involves a more development, refined service, but sunk costs (such as R&D) are incurred. The consumer then chooses to 'Use' or 'Don't Use' the service.

The firm prefers to have a paying customer than a non-paying customer, however it prefers to have a non-paying customer than no customer at all, such that:

$$Uf(Premium, Use)>Uf(Free, Use)>Uf(Free, Don't Use)$$

If the firm offers a Premium service and does not gain a customer it is assumed that it is at a financial loss due to sunk costs. No such sunk costs exist when operating under the Free model and so:

$$Uf(Free, Don't Use)> Uf(Premium, Don't Use).$$

The consumer prefers the outcome when it uses the service for free to either of the outcomes in which it does not use the service, and is indifferent between the latter two outcomes such that:

$$Uc(Free, Use)>Uc(Free, Don't Use)=Uc(Premium, Don't Use)$$

When the consumer uses the good in the case of the Premium model, they are charged a fee. Denote the payoff to consumer in this case, Uc(Premium, Use), as x. It is assumed that the Firm is playing against a consumer who is willing to pay for the service, in the sense that their personal valuation of the service - vi - is above the price paid - p - such that $p<vi$. The Consumer receives a positive payoff from using the premium service; for this game the case of $x=2\$$ is considered. This is arguably a valid assumption to make for any pricing model where perfect price discrimination does not occur; it simply assumes that a positive consumer surplus exists for this consumer.

The game is presented in extensive form in Figure 1. Solving for a stable state, the only pure strategy Nash equilibrium is the strategy profile (Premium, Use). For this strategy profile, neither player has an incentive to deviate as each is playing optimally given the others' action. This seems an intuitively plausible outcome. If the firm is playing against a consumer that is willing to pay for the premium service ($vi>p$) and who furthermore has a dominant strategy in the form of 'Use', it is logical that the firm will incur the sunk costs of providing the premium service, safe in the perfect information that the consumer will use the service.

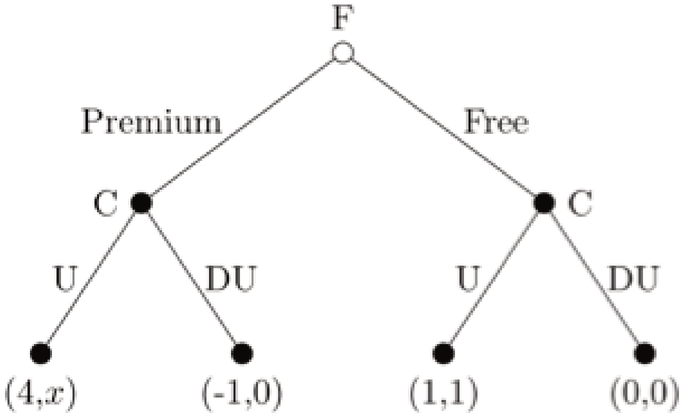

Figure 1: Simple Pricing Game

The Extended Pricing Game

Introducing Freemium

Building on the simple game presented, the game is expanded to include a choice for the firm to operate under a 'Freemium' pricing model, giving a new strategy set Af={Premium , Free, Freemium}. The consumer's actions are similar to those in the simple game, though now in the case of the firm choosing a freemium pricing model the consumer's choice to use the service is split in two: they may use the premium service (Fp) or use the free service (Ff). All preferences that the firm and consumer held in the simple game, over the actions 'Premium' and 'Free', and 'Use' and 'Don't Use' respectively, still hold in the extended game, including the assumption that the Consumer is willing to pay for the service (x=2). Preferences regarding outcomes involving the Freemium model will be justified below.

The firm now prefers to have a non-paying customer under the Freemium model than under the Free model. This is at the core of the rationale for the freemium business model. Customers using the free service are valued as there is potential for them to convert to paying customers(although this 'potential' is never realised within the confines of a single-shot game, it is still relevant for modelling). Under the Freemium model, paying customers are higher-valued than customers using the free service. This gives the relation Uf(Freemium, Fp) >Uf(Freemium, Ff)>Uf(Free, Use). An important assumption made is that the firm prefers to have a paying customer under the Premium pricing model than under the Freemium model. This is justified in that the firm may charge a higher price under the Premium model given that the customer’s only alternative is to not use the product, unlike the case of the Freemium model where the customer also has the alternative of using the free version of the freemium service should the firm consider charging a higher price, and so Uf(Premium, Use)> Uf(Freemium, Fp).

The consumer now prefers to use the free service under the Freemium model than to use the free service under the Free model, as a more valuable and developed (free) service is being offered in the case of the Freemium model, and so Uc(Freemium, Ff) > Uc(Free, Use). The consumer is again indifferent between the choice of business model when it does not use the service such that:

Uc(Freemium, Don’t Use)=Uc(Free, Don’t Use)=Uc(Premium, Don’t Use)

It also assumed that the consumer is indifferent between paying for the premium service under the Freemium model and paying for the premium service under the Premium model. Though it was assumed that the Firm may charge a higher price under the Premium model, a higher-valued service is being provided at this higher price. The slightly lower-valued premium service under the Freemium model comes at a slightly lower price, but it is assumed that the positive difference between consumer valuation and price paid (consumer surplus) is equal in both scenarios such that Uc(Premium, Use)=Uc(Freemium, Use) .

This game is presented in Figure 2. To solve this game via backward-induction, it is clear from the preferences that the consumer will always choose to use the product, and in the case of the Freemium pricing model the consumer will choose to pay to use the premium service. Knowing this, the firm will behave optimally by choosing to operate with a Premium pricing model, giving a subgame perfect equilibrium (Premium (Use, Use, Fp). This results in the pareto-optimal outcome whereby the firm chooses a Premium pricing model and the consumer pays for this service.

Now deviating from the Firm's ideal of operating in a market where their customer has a strong willingness to pay for their service, the case of a consumer who is unwilling to pay for the service is considered. This type of customer does not value the premium service at the price it is offered under either the Premium pricing model or the Freemium pricing model but rather at a value vi<p, receiving a negative payoff from consuming the premium good in either case. Their preferences may be modelled as specified for the other consumer type above in each outcome with the only exception being of a value of x= -1. This changes the subgame perfect equilibrium to (Freemium, Don’t Use,

Use, Ff)) with the equilibrium outcome now being that the Firm operates under a Freemium pricing model and the consumer uses the free service.

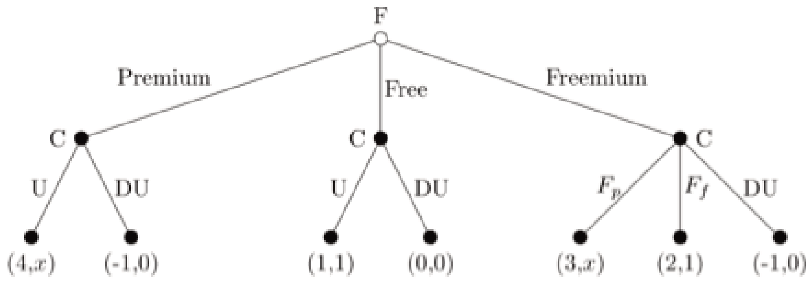

Figure 2: Extended Pricing Game with One Consumer

Imperfect Information

The extended game presented above showed that when the Firm knows that the Consumer is willing to pay, the Firm maximises their payoff by choosing a Premium pricing model. However, when the consumer does not value the service at the price charged, it is optimal for the Firm to choose the Freemium pricing model and gain a non-paying customer.

In reality, it is unusual for a firm to have complete information about the willingness to pay of all consumers in a market. To consider the more realistic scenario of uncertainty over the customer's willingness to pay, the extended game may be amended to include the case of imperfect information. In this game, the Firm does not know whether they are playing against the Customer who is willing to pay (x=2) or unwilling to pay (x=-1) for the service. Denote these Customer types as C1 and C2 respectively, so that the Customer's typeset is TC={ C1,C2}. Nature moves first and decides the type of the Consumer, and the Firm has some accurate prior belief (α) of the distribution over the Customer's types which is common knowledge. After Nature plays, the Firm forms a belief (p) as to the type of the Consumer that is playing, which is (given that there is no signalling involved or any intermediate decision nodes) trivially equal to α for this belief to be consistent.

This game is presented in extensive form in Figure 3. This game has a unique pure strategy perfect bayesian equilibrium (PBE) whereby the Firm chooses the Freemium model in the case of $\alpha<3/4$ (but chooses the Premium model for that of $\alpha>3/4$), Consumer C1 chooses strategy (Use, Use, Fp), Consumer C2 chooses (Don't Use, Use, Ff) (which coincide with the strategies each employed in the previous games) and the Firm has a consistent belief p= α . While the game was modelled as a two player game, and the consumer thought of as an individual, the model is of course applicable to the game of a firm playing against an entire market of consumers. In this case, α is simply reinterpreted as the proportion of consumers of type C1 in the market.

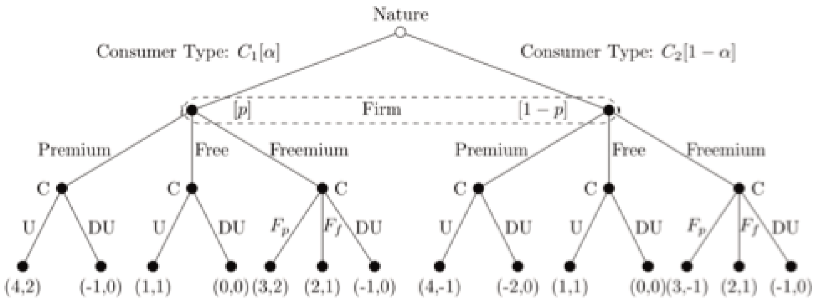

Figure 3: Extended Pricing Game with Two Consumer Types and Incomplete Information

Analysis and Extensions

The intuitive result of the extended game with imperfect information is such that if the Firm has strong reason to believe they are playing against the type C1 (i.e. if p=α >¾), it is optimal for them to choose the Premium pricing model, as in the case of the previous game in which the Firm is only playing against the consumer of C1. However, without this strong belief (i.e. if p=α <¾), the firm chooses optimally by operating with a Freemium pricing model, and Consumer C1 uses the premium service or else Consumer C2 uses the free service (depending on which Consumer type nature determines to be playing). This equilibrium outcome is notably applicable in the case of pure uncertainty (α=½).

The consumer's choices in the freemium model presented in this paper were limited to either using the premium service or using the free service. This is often how this pricing is operated; for example, Spotify customers can either pay for full access to a premium music-streaming service, or else use the ad-supported supported service at no cost. While this modelling approach captures the necessary element of choice for the consumer between premium and free under the freemium model, other industries take extended freemium model far beyond this binary choice. The mobile-gaming industry is one of the most prominent examples, and indeed successes, of the freemium model in action-as of May 2014, 92 per cent of mobile gaming revenue in the Apple App Store were generated under the freemium model (Lescop, 2014). In using the freemium model, gaming companies release games for free and then profit from sales of a plethora of in-game purchases for additional content such as levels, character customisations etc.

What is most fascinating about the mobile-gaming industry's approach to freemium is that consumers are allowed to 'set the price and even determine the precise characteristics of the product,' by choosing any combination of in-game purchases and add-ons, which constitutes a 'form of near-perfect price discrimination' (Holmes, 2013). In terms of policy implications, this could have huge relevance to competition in regulated industries. One such approach of adopting the freemium model for regulation could see regulators involved in determining minimum standard of service of the free tier, and then allowing firms to structure their premium pricing in a much less regulated way. With a firm's premium services now effectively competing with their own free service, they could conceivably be incentivised to offer multiple price points for additional services above those available for free (as in the mobile gaming industry) which ultimately benefits the consumer in terms of choice and could lead to a highly competitive market outcome.

One extension to the model presented in this paper that would be particularly interesting to consider is that of network effects. The freemium model is most prominent in the technology sector, and most technology services increase in value to the consumer as the consumer base grows. Spotify offers a social experience to listening to music, and mobile games generally involving playing with and against other users of the game. In both of these cases, the service is much more valuable when the service has an active user base, and more valuable to a specific user when that user's friends are also users of the service. In terms of modelling, this would need to be modelled as an n-player game with 1 firm and $n-1 \geq 2$ players, where individual players' payoffs from using the service are positively correlated with the total number of players choosing to use the service (whether by paying or as a free user). In this model, the customer who is willing to pay for the service would strictly prefer the freemium model as it results in the greatest number of users of the service (as those willing to pay would pay, those not willing would use the free service) and so the equilibrium outcome would almost certainly involve the firm choosing the freemium pricing model.

Conclusion

This paper presented a game-theoretic analysis of the firm's decision concerning their pricing model. Starting with a simple case of the choice between 'Free' and 'Premium' pricing, the model was extended to include the choice of the 'Freemium' pricing model. In the face of pure uncertainty, it was shown that firm chooses optimally by operating with a freemium pricing model. The use of the freemium pricing model is on the rise.

Though mainly employed by technology firms at present, it would be interesting to see how it functions if adopted by other industries, and especially if incorporated by regulators into an approach to competition policy.

References

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Appendix

Calculating the Perfect Bayesian Equilibrium

Let α denote the prior belief that consumer is of type C1.

Let p denote the Firm's belief that the consumer is of type C1.

The firm's expected payoffs are calculated as follows:

UF(Premium | p) = 5p-1

UF(Free | p) = 1

UF(Freemium | p) = 2+p

From this it is clear that for any p in [0,1] the pure strategy 'Free' is strictly dominated by 'Freemium' for the Firm.

The 'Premium' strategy will be preferred to the 'Freemium' strategy in the case of:

UF(Premium | p) > UF(Freemium | p) which is true only when p>¾

For the firm's belief p to be consistent, it need only be consistent with the prior beliefs i.e. p= α

The unique pure strategy Perfect Bayesian Equilibrium is defined as:

Firm: Freemium if 0 ≥ α < ¾ or Premium if ¾ < α ≤

Consumer C1: (Use, Use, Fp)

Consumer C2: (Don't Use, Use, Ff)

Beliefs: p= α