THE EDGE OF CHAOS – AN ALTERNATIVE TO THE RANDOM WALK HYPOTHESIS

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The Random Walk Hypothesis claims that stock price movements are random and cannot be predicted from past events. A random system may be unpredictable but an unpredictable system need not be random. The alternative is that it could be described by chaos theory and although seem random, not actually be so. Chaos theory can describe the overall order of a non-linear system; it is not about the absence of order but the search for it. In this essay Ciarán Doran O'Fathaigh explains these ideas in greater detail and also looks at empirical work that has concluded in a rejection of the Random Walk Hypothesis.

Introduction

This essay intends to put forward an alternative to the Random Walk Hypothesis. This alternative will be that systems that appear to be random are in fact chaotic. Firstly, both the Random Walk Hypothesis and Chaos Theory will be outlined. Following that, some empirical cases will be examined where Chaos Theory is applicable, including real exchange rates with the US Dollar, a chaotic attractor for the S&P 500 and the returns on T-bills.

The Random Walk Hypothesis

The Random Walk Hypothesis states that stock market prices evolve according to a random walk and that endeavours to predict future movements will be fruitless. There is both a narrow version and a broad version of the Random Walk Hypothesis.

Narrow Version:

The narrow version of the Random Walk Hypothesis asserts that the movements of a stock or the market as a whole cannot be predicted from past behaviour (Wallich, 1968). This would suggest that an investor cannot beat the market, yet there are many stories of those who have. This can partially be attributed to the fact that those who do not beat the market, i.e. those who perform worse than the market are slow to publicise their failures.

Broad Version:

The broader version of the Random Walk Hypothesis expands on the narrow version, claiming that '…in a well functioning market, all known information has already [been] discounted' (Wallich, 1968: 160). Similar to the Efficient Market Hypothesis, this broad version, if correct, demonstrates even further that one cannot out-perform the market. If new information becomes available, any past information becomes irrelevant, as it has already caused any market movements that it was capable of.

Possible Flaws with in the Broad Version RWH

The broad version of the Random Walk Hypothesis assumes that all information is reflected immediately in the prices of stocks and this means that past information has no effect on stock price movements in the future. There are two flaws with the fundamental assumptions in this theory.

Firstly, there is the assumption that all investors have the same access to the same information. This is surely not the case. Reliable and detailed information is usually obtained from a paid service, or through the employment of analysts - resources not available to every investor. Also, there is the assumption that all investors act upon the information at the same time. Investors receiving information up to several hours after it becomes public may still act on it. Naturally, this affect prices.

Secondly, the effect of information obtained at time t may not be fully understood without information at time t+1. This could lead to the effects of information from the past compounding with the effects of information subsequentially released. Thus, the past information can still have an effect on the market prices.

Random vs. Unpredictable

Before elaborating on the nature of a chaotic and complex system, it is necessary to distinguish between two concepts: random and unpredictable. These two words are generally used interchangeably, and generally this does not pose a problem in everyday usage. However, when discussing the nature of a particular system and deciding how best to analyse it, it is appropriate to be pedantic. The best way to distinguish the subtleties between these two concepts is by illustration of their application to a given system. For the purposes of example, the weather shall be used, not least because it was the study of this system that led to the discovery of chaos theory.

Unpredictable:

The weather is unpredictable. By this it is meant that it would not be possible to gauge at this moment what the weather will be in six hours' time. Perhaps it will be raining, perhaps not. A meteorologist, it would be assumed, would be able to take into account more information in making what essentially amounts to an educated guess. The odds of being correct would increase. However, he would not be guaranteed to be correct. Also, as we extend the timeline, the likelihood of being correct would decrease.

Random:

The weather is not random however. If, at midday today, it is seven degrees and raining, it is fairly certain that it will not be twenty four degrees and sunny six hours later. This is because there are *deterministic relationships* at play here. There are many variables that affect the weather, many of which may not be taken into consideration when making predictions about future weather conditions.

The Distinction:

It seems then that a clear distinction can be made. Unpredictable events or systems can be described as those that we are unable to forecast, or are only able to partially forecast, due to a lack of information. Random systems are systems in which no deterministic relationship exists.

Chaos Theory

Chaos is a non-linear deterministic process, which looks random (Hsieh, 1991). The explanation of the concept is, ironically, not overly complex. There are several characteristics, which, once properly understood, lead to a functional understanding of the idea. They are:

- Sensitive dependence on initial conditions.
- Apparent randomness disguising deterministic relationships.
- Strange Attractors (also known as Chaotic Attractors or Fractal Attractors).
- Fractal Dimension.

Sensitive Dependence on Initial Conditions:

The first characteristic of chaotic systems that will be discussed is possibly the most important. This is due to the fact that the chaotic nature of a system's evolution arises from it. In a standard statistically modelled system, one expects that, if the independent variable is altered by some proportion, then there will be a similar or predictable change in the dependant variable. This is the reason that such systems are so widely used: they facilitate prediction of events.

However, in a chaotic system, an infinitesimally small change in the initial conditions can cause the model to evolve in a completely different fashion. This phenomenon was discovered by a meteorologist, Edward Lorenz, while he was running a weather-predicting model In 1961, he wished to re-examine a certain portion of the results and, in the interest of expedience, he used the data from a read-out which he had obtained previously for the beginning of that sequence, rather than re-running the entire model. The system evolved in a completely different fashion from his earlier models. The reason, he discovered, was that during the initial run, the computer had used figures to six decimal places but he had only printed out figures to three decimal places. A change of just over a *thousandth of a significant figure* completely altered the model. This has very obvious implications for economics or finance systems if they are indeed found to be chaotic.

Apparent Randomness Disguising Deterministic Relationships:

The best explanation of this concept is through example. Consider a roulette wheel. The outcome is believed to be random and certainly seems that way on first observation. However, the result has several influencing factors: the speed and number of rotations of the wheel, the spin on the ball as it leaves the croupier's hand, the force the croupier uses to throw the ball, etc. So what seems random is in fact deterministic. If the initial conditions were known, a better forecast of the result could be obtained. In the final section of this essay, further examples will be drawn from market data.

Strange Attractors:

So, it would seem that if an economic or financial system is chaotic, then it cannot be modelled. However, this is not the case. This is a crucial aspect of the theory; the point at which order arises from disorder. While the *positions* of data at a specific time cannot necessarily be predicted, quite accurate models of the overall *behaviour* of the system can be created.

An attractor is the equilibrium level of a system, but should not be confused with an econometric equilibrium, which is a narrow form of an attractor. An attractor is the level or value a system attempts to regain after external effects have abated (Peters, 1991).

A strange (or chaotic) attractor is present in a system that tends towards a set of possible values. The possible values are infinite in number but limited in range. Chaotic attractors are not periodic, i.e. they do not have any repetition regardless of the length of the timeline (Peters, 1991). Attractors are labelled as strange or chaotic when they have a non-integer dimension.

Fractal Dimension:

The most basic way to understand fractal dimension is as a measure of how chaotic a system is; the closer to the higher integer the dimension is between, the more chaotic the system. Again, chaotic does not mean random. On a more complex scale, fractal dimension is a statistical quality, giving a measure of how completely a fractal fills space.

Empirical Examples Demonstrating Random Market Hypothesis Failures

Real Exchange Rates:

In his 1999 paper, In Choi examines whether the Random Walk Hypothesis is observed for real exchange rates. He uses the log-differenced US real monthly exchange rates and certain other major currencies. In his paper he sets out a null hypothesis that a random walk is observed. The alternative hypothesis is that there is serial correlation present. He does not propose any specific model for the correlation, thus allowing both linear and non-linear dependence (Choi, 1999).

The rates used are the US real exchange rates versus the Canadian dollar, French franc, German mark, Japanese yen, British pound and Swiss franc, for the periods 1960:1 to 1993:11 (Choi, 1999). Several tests were run for each currency, and the results were mixed. In his conclusions, he states that '…for the full sample, the null is rejected at conventional significance levels for Japan, Switzerland and Britain' (Choi, 1999: 306). Here the Random Walk Hypothesis is rejected, with the possibility arising of the presence of a non-linear system.

A Chaotic Attractor for the S&P 500:

In 1991, Edgar E. Peters examined the S&P 500 index in order to ascertain whether or not there was a chaotic attractor present. The dynamic observable used was the log-linear deflated S&P 500. However, it was not the changes in the values that were recorded but the absolute values themselves. The reason for this is that using simply the percentage changes in prices may destroy the delicate non-linear structure present in the data (Peters, 1991). The results, again, help to refute the Random Walk Hypothesis.

Firstly, he found that the fractal dimension of the detrended S&P 500 is approximately 2.33 (incidentally, this is the same fractal dimension as cauliflower). If the data were completely random, the dimension would have been an integer. Random data, as stated above, fills any space available to it. He states that '…the attractor is "chaotic", with a positive Lyapunov exponent' (Peters, 1991: 61).

Lyapunov exponents measure the loss in predictive power experienced by non-linear systems over time, by measuring the divergence of nearby trajectories over time. A positive exponent indicates expansion while a negative one indicates contraction (Peters, 1991). The positive Lyapunov exponent indicates that the system is subject to 'sensitive dependence on initial conditions', another feature of chaotic systems, as mentioned above. When using Lyapunov exponents as a measure of divergence from initial trajectories, it is common to simply use the largest one. This is known as the Maximal Lyapunov exponent, MLE.

Not only does this example refute the Random Market Hypothesis, but it also supports the presence of chaotic behaviour in the market. Due to sensitive dependence on initial conditions, attempts to model its behaviour stochastically could lead to extremely large errors, which grow exponentially as time goes on.

Returns on T-Bill Rates:

In a different paper in 1991, Larrain rightly asserts that if the past interest rates affect the future evolution of interest rates, then the Random Market Hypothesis is false. If this turns out to be the case, then a genuine and justified use of investment tools and strategies can be made to generate a profit (Larrain, 1991).

According to Larrain, both fundamentals and technical analysis can be used to determine future interest rates. Moreover, the relationships are non-linear in nature (ibid.). This not only discredits the Random Market Hypothesis, but gives further credence to the idea that markets are chaotic in nature.

Before presenting the results, the issue of a series being random some of the time, and deterministic the remainder of the time is addressed. Instead of asking whether or not the T-bill series is random, he examines the process which creates the series to ascertain if it may, at times, have a native 'random-number' generator (ibid.).

The conclusions reached by Larrain are as follows. Firstly, there is a non-linear structure in the series for T-bill rates and secondly, this non-linear structure, while not explicitly guaranteeing mathematical chaos, does allow for the possibility of it arising, under certain market conditions (ibid.).

Conclusions and Remarks

After examining the possibility of chaos theory being an alternative to the Random Market Hypothesis, the following conclusions can be drawn from the academic literature and empirical data:

1. The Random Walk Hypothesis is not correct in its narrow or broad form as is shown by the empirical data.

2. It is possible that chaos theory could be used to describe some markets but is not to be adopted as the complete alternative.
3. Where the situation arises that a system is not chaotic, it may very well be nonlinear, and so still requires that we do not assume randomness.

In recent years there have seen an increasing tendency for economists to explore the ideas of chaos theory in search of explanations for events in the markets and the economy as a whole However, the concept is still in its infancy relative to many of the alternative quantitative tools which have been in use since econometrics first came to prominence, at the beginning of the twentieth century. With continued research, there is a lot of potential to gain a better understanding not just of the markets, but in many other fields of economic theory, and to establish patterns and models in areas which were believed to be operating in disorder and randomness.

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