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THE CONTRIUBTION OF BLACK, MERTON AND SCHOLES TO FINANCIAL ECONOMICS

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The introduction of the Black-Scholes Formula revolutionised the world of financial economics and lead to the creation of new fields, new markets and new types of securities. In this paper, Iain Nash examines the impact which the formula has had on the realm of economics and in particular the options market. Through his analysis of the combined contribution of Fisher Black, Robert Merton and Myron Scholes to the creation of this formula he clearly highlights the fundamental role such pioneers and visionaries play in the progression of a discipline.

Introduction

> *'Robert C. Merton and Myron S. Scholes have, in collaboration with the late Fischer Black, developed a pioneering formula for the valuation of stock options. Their methodology has paved the way for economic valuations in many areas. It has also generated new types of financial instruments and facilitated more efficient risk management in society'.*[1]

The contribution of Black, Merton and Scholes to the study of financial economics is a broad and profound one. They are most commonly associated with the development of the first complete options pricing formula. However, the impact of this formula extended to the creation of: 'a new field within finance, known as derivatives and offered a new perspective on related areas including corporate finance, capital budgeting and financial markets and institutions' (Arrow 1999: 230). Arrow also discusses how the work of Black, Merton and Scholes has influenced the development of mathematical and computer science,

[1] 1997 Nobel Prize for Economics – Press Release
Available at http://nobelprize.org/nobelprizes/economics/laureates/1997/press.html

as well as revolutionising private industry practice (Arrow, 1999). Few economists can claim to have had such an impact upon such a diverse range of fields, in both practical and academic terms.

A Brief History of Options

The theory of option pricing can be traced back to Bachelier (1900), whose PhD thesis presented an asset pricing model described by Duffie (1998) as quite similar to that of the Black-Scholes approach. This was one of the first papers which dealt with what would now be described as 'derivatives'. Arrow describes the thesis, while having somewhat flawed economic and mathematical inclusions, as being one of the primary motivations behind the work of Black, Merton and Scholes (Arrow, 1999). There had been work conducted on the area by Sprenke [1961], Ayres [1963], Boness [1964], Samuelson [1965], Baumol, Malkiel and Quandt [1966] and Chen [1970]. However, none of the formulae developed by these authors were 'complete' as they all contained arbitrary and non-estimable parameters and as such, they had no practical value; however, they did help create insights which were used by Black and Scholes in their paper (Black and Scholes, 1973).

Elton et al. define an option as 'a contract entitling the holder to buy or sell a designated security at or within a certain period of time at a particular price' (Elton et al 2007: 576). Two of the most common and simple options are 'calls' and 'puts'. A call gives the holder the right to purchase a security at a predetermined price, while a put gives the holder the right to sell a security at a fixed price. Many complicated options can be considered (and valued) as combinations of various calls and puts.

Options can be further broken down into 'European' and 'American' options. These are not actually based on geographical factors but rather European options can only be exercised at a set, predetermined maturity date, while American options can be exercised at any time up until the maturity date. The Black-Scholes model is only able to calculate European style options (Black and Scholes, 1973). For the calculation of American options, other pricing methods such as the Binomial Options Pricing Model must be used.[2]

Options are traded in two forms of markets: organised exchanges such as the Chicago Board Options Exchange and the New York Board of Trade, or in

[2] A good treatment of this model is given in Cuthbertson & Nitzsche (2001).

Over the Counter (OTC) markets. Organised exchanges follow standard terms and are centralised, while OTC markets are composed of networks of institutions, traders and investment banks and there is no central locus. Options markets differ from other forms of markets in that they are 'zero supply' markets (Arrow, 1999). Each trade must have a seller and a counterparty, and for each option sold there must be an option purchased. OTC markets are more dynamic then organised exchanges, as the lack of standard trading practices allow the investment banks and other types of specialist institutions to create custom products for clients. In the OTC market, it is also possible to create a 'one sided' trade if the broker is able to construct a synthetic portfolio (which are discussed later in this paper) as a counterparty.

Black-Scholes Formula

The history of the development of the Black-Scholes formula is covered extensively in Black (1989) but a brief outline is given below. In essence, Black and Scholes combined the intuition of the Capital Asset Pricing Model (CAPM) with stochastic processes. This lead to the development of the Partial Differential Equation (PDE) that was to become the Black-Scholes formula. Black had established the initial form of this equation in 1969; however, he was unable to solve it at the time.

The solution to Black's PDE was the result of both mathematical and theoretical foresight. Black had teamed up with Myron Scholes in M.I.T. and they had begun working to solve the equation. However, it was Robert Merton who had developed a model using arbitrage for solving hedged portfolios. This model allowed Black and Scholes to solve the model differentially, and thus the famous model was born.

The intuition behind the model is based on the premise of creating a synthetic hedged portfolio to determine the price (premium) of put and call options. We follow the process as put forward by Arrow (1999: 233):

1. A call option is taken on an underlying stock:
   As the underlying stock price rises, the value of the call rises as it is more likely to lie above the strike price of the call.

2. A short position is then taken to offset the call:
   As the underlying price of the stock rises, the short position will decrease in value. This will partially offset the call. Naturally, this process works in reverse if the price of stock

falls, with the call option now offsetting the increased value of the short sale.

3. The positions are modified to create a perfect hedge: The exact holding of the call and short strategies are modified to create a net return of zero. Any change in the price of underlying stock will be countered by either the call or the short sale.

This allows the calculation of the unique arbitrage free price of the portfolio and the change in the price of the option as a function of the change in the price of the underlying security. Creating a synthetic[3], perfectly hedged portfolio allows the PDE to be solved. What is special about the Black-Scholes formula is that it only depends on factors which are easily observable and available in the market place. Arrow lists these as: the strike price of the option, the current date, the current stock price, the risk free rate and the volatility of the underlying stock which is measured by its standard deviation (Arrow, 1999). Other models developed at the time depended upon factors that cannot be estimated such as the expected return of the asset or its risk premium.

The Black-Scholes Formula is given below, as presented by Elton et al. (2007: 592):

$$C = S_0 N(d_1) - \frac{E}{e^{rT}} N(d_2)$$

$$d_1 = \frac{\ln(S_0/E) + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

$$d_2 = \frac{\ln(S_0/E) + (r - \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

Where r is the risk free rate, C is the current value of the option, S_0 is the current price of the stock, E is the exercise price of the option, $e = 2.7183$, t is the time remaining before the expiration date as a fraction of the year, σ is the standard deviation of the stock, $ln(S_0/E)$ is the natural logarithm of S_0/E and $N(d)$ is the

[3] These synthetic portfolios are theoretical constructs rather then actual products. This is proved by contradiction, as if it were possible to easily create these portfolios, people would hold these rather then options.

value of the cumulative normal distribution.

Thanks to this formula, for the first time people were able to easily and accurately calculate options prices. Given that the formula was published around the same time that the Chicago Board Options Exchange (CBOE) opened, the Black-Scholes formula was a corner-stone in the development and transformation of the options market from a thinly traded, inactive and illiquid market to one which traded over 944 million options in 2007.[4]

The Impact of the Black-Scholes Model

Duffie notes how the Black-Scholes model is taught to every MBA student, most graduate and many undergraduate students who study either finance or economics (Duffie, 1998). Many people attribute the explosive growth of options markets to the model. It is generally acknowledged that the Black-Scholes formula was the catalyst which led to the development of 'derivatives', a whole new field in finance. While some people attribute the development to the options pricing papers which preceded the 1973 paper, the advent of the formula allowed the derivative market to develop as traders and brokers were now able to accurately price and measure risk (Arrow,1999; Konishi and Dattareya, 1996).

One of the most basic (and widely used) manifestations of the Black-Scholes model is the ability for speculators to 'bet' on the direction of equities. By using either puts or calls, speculators are able to profit by correctly estimating which direction a stock is going to move. By executing these transactions through options markets a speculator can realise the following benefits:

1. Lower cash outlay:
   By purchasing an option, a speculator does not have to actually own the underlying security. Given that option prices are often only fractions of the cost of the security, the speculator can make the same bet many times for the price of the underlying security.

2. Short selling:
   Until the advent of options markets, a speculator was unable to profit from the expected depreciation of a stock. However, with the advent of derivatives a speculator can

[4] http://www.cboe.com/AboutCBOE/History.aspx

now profit by either using a put or a futures contract.

3. Hedging:
   Investors, firms and other market participants are able to protect themselves from being exposed to risk associated with the movement of equities by using derivatives.

Transactions such as these are now commonplace in the market but did not exist at the time when Black, Merton and Scholes were conducting their research. The Black-Scholes model also allowed traders to work out the markets perception of a stock volatility through the process of Implied Volatility. Implied Volatility is a dynamic measure and is: 'a matter of turning the option on its head and finding out what other traders and market-makers think volatility should be' (Sutton, 1990: 36). Essentially, Implied Volatility allows speculators to see how the market has viewed past volatility and determine how this view has changed over the sample. The results from this measure differ from the historical pricing of volatility, a more rudimentary and simplistic method of analysing the volatility of a stock. Kim et al. (2007) conclude that Implied Volatility provides an unbiased forecast of future volatilities and furthermore, that implied volatility is much more closely related to the volatility of excess returns then that of gross returns when the interest rates are highly volatile.

Other Contributions

The Black-Scholes formula also had a major impact on the field of financial risk management and in the valuation of corporate liabilities. Duffie (1998) discusses how the practice of financial risk analysis has cloned the method of the Black-Scholes formula:

> 'The idea that the option can be priced by finding a trading strategy that replicates its payoff is frequently used to hedge a given security, or even to hedge a given cash flow that is not traded as a security. If one is to receive an untraded option payoff, for example, the risk inherent in that payoff can be eliminated by selling the replicating strategy previously described' (Duffie, 1998: 419).

This practice has become particularly widespread in the investment banking community and allows them to create packages containing virtually any combination of options and offset the risk by using either synthetic or dynamic portfolios.

The Black-Scholes formula has also become one of the key tools used in the pricing of corporate loans (Jarrow and Turnbull, 1995). Corporate debt is harder to price then government debt as corporations can end up in bankruptcy and default on their obligations; this results in corporate and government debt having different term structures. Banks then use a slightly modified form of the Black-Scholes formula in an arbitrage-free model that is based on the spread between the two term structures to calculate the price of bond.

Conclusion

The work of Black, Merton and Scholes has impacted upon numerous fields in the social sciences. Their solving of the PDE has lead to the development of a new field in financial economics and many of the modern day financial engineering techniques are based upon the insights of their original formula. In addition, the practices of short selling and hedging, as well as the introduction of lower cash outlay per transaction has lead to an increase in the efficiency in many financial markets. The 1973 paper is clearly one of the key foundations of modern financial economics.

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