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NETWORK DYNAMICS, PREFERENTIAL ATTACHMENT AND MARKET LIBERALISATION

ENDA HARGADEN

Senior Sophister

Today's markets are constantly in flux, with firms forced to continually grow and adapt in order to survive in the increasingly interconnected global environment. In this dynamic context, dominant firms frequently emerge holding large shares of their respective markets. By focusing on the preferential attachment of revenue growth to larger firms, Enda Hargaden takes a novel approach to this topic. He expands on existing measures of market concentration, combining these measures with cutting edge network theory. The model he proposes demonstrates significant monopolistic pull, with the result that the initially dominant firm will eventually overpower the other market players. The implications of these findings for future industrial regulation are considerable.

Introduction

The Hirschman-Herfindahl Index (HHI) is a formula for analysing market concentration. This paper borrows from network theory and proposes a model demonstrating that by using the HHI as a benchmark, implementing barriers to entry in growing markets with nonlinear distribution of said growth can be preferable to complete freedom of entry. The advantage of using the HHI over measures such as the concentration ratio is that it gives greater weight to larger firms. It is often used as a proxy measure of competition; however, it fails to take barriers to entry into account, an important consideration for any comprehensive evaluation of competitive market forces within an industry. This paper explores a further problem of simple HHI analysis. It examines a market that displays continuous growth and preferential attachment, demonstrating that such a market will display significant 'monopolistic pull', dependent upon: the initial market shares, the level of growth of the market and the weight of the attachment. These findings are particularly relevant for policy in the case of an industry confronted by liberalisation from previous monopolisation. Other influential factors discussed include the number of 'footloose' customers and the total number of firms. Interestingly, the results of the model suggest that too many small firms competing in an industry in which one firm has a large share of the market, may well result in a monopoly outcome.

The HHI

The HHI is a standard measure of market concentration used by the EU Commission, the US Department of Justice's Antitrust Division and the Irish Competition Authority, among others. The HHI of an industry is the sum of the squares of its market shares. Formally, where there are n firms competing in the industry and x_i is the percentage market share of firm i:

$$HHI = \sum_{i=1}^{n} x_i^2$$

This returns a value of $0 < HHI \leq 10{,}000$, where perfect competition is represented by a level close to zero[1] while 10,000 implies total perfect monopoly.

It is important to note that the HHI is a measure of market concentration in an industry and is thus indicative of firms' ability to raise prices above their competitive level. The HHI itself does not explicitly consider the observed prices pertaining in the market. An assumption is required that increased market power will have a detrimental impact on consumer welfare. This is common practice; the Irish Competition Authority's official guidelines on mergers state that firms are: 'assumed to pursue maximum profits' (Competition Authority 2002: 5). The assumption that undesirable price increases are ex ante more likely in markets with higher HHIs is retained for this paper.

Network Theory

Network theory primarily concerns itself with the study of graphs which contain points (or nodes) and lines (or edges) connecting the nodes. The number of edges emerging from a particular node represents that node's degree of connectivity.

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[1] As at least one firm has a market share >0, the HHI is strictly positive.

For decades, sociologists have used these graphs to illustrate social concepts such as cliques. More recently, physicists and mathematicians have developed network theory for more inferential use and to study networks as stochastic objects with probability densities. Furthermore, the recent past has seen developments in dynamic network theory–the study of networks which develop over time–and it is the concept of dynamic networks which will be utilised in this paper and perhaps which contains the greatest potential for future use within economics.

Network theory is less obscure than the reader may first imagine. Much of the general population are familiar with the 'six-degrees of separation' concept; the hypothesis that that each person can be connected by at most six intermediate acquaintances (or nodes). Although this concept is not strictly true, it provides a useful insight into the 'small world phenomenon'.[2] The network of the world-wide web has been the source of much research. Albert et al. (1999) found that two randomly chosen web pages, out of a possible 800 million, were just nineteen 'clicks' away from each other on average.

Network theory has many real world applications. Preventing the spread of HIV/AIDS can be classed as a problem that requires sufferers to pass the infection onto less than one person on average; that the average infected node creates less than one edge to an uninfected node. Google's system of listing search results, PageRank, is a network-based algorithm whereby pages are ranked in accordance with how many other sites link to them, those links in turn weighted toward sites that are highly linked themselves. The combination of social networking websites' friend-lists and profile views provide invaluable data previously unobtainable regarding the level and distribution of the connectivity of social networks.

The internet has also spawned research into clustering or 'preferential attachment'. Websites become popular based largely on how many sites link to them, and being popular in turn encourages more links. Over time this preferential attachment clusters people to a small number of very popular sites. A similar effect has also been observed in academic citations (Price, 1976). Barabasái and Albert (1999) show that continuously growing networks displaying preferential attachment produce scale-free networks that are dominated by few nodes of very high relative degree. If we consider a market as a network where consumers are connected via purchases (edges) to firms (nodes),

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[2] Research from the University of Virginia has shown that assuming an edge is created whenever two actors appear in the same production; the average number of actors (nodes) between Kevin Bacon and 250,000 other actors studied was just 2.96. See http://oracleofbacon.org/ for details.

the Barabasái and Albert result can become extremely important. The next section models a market in such a way.

The Model

Begin with a vector $R_t^n = (r_t^i, \ldots, r_t^n)$ of the revenues of firms *1* through to *n* at time *t*. By defining total market revenue:

$$\overline{R}_t = \sum_{j=1}^{n} r_t^j$$

We can create a vector of market shares by dividing each firm's revenue by total market revenue:

$$x_t^i = \frac{r_t^i}{\overline{R}_t}$$

We now allow for constant growth in market revenue at rate γ. The crux of the model is that this growth in revenue γ is distributed disproportionately; there exists a preferential attachment of it to larger firms. This is not incompatible with standard economic analysis. The HHI itself applies exponential additional importance to higher market shares. The nonlinear distribution is at a weight w[3]. Specifically:

$$r_t^i = r_{t-1}^i + \frac{(x_{t-1}^i)^\omega}{\sum_{j=1}^{n}(x_{t-1}^j)^\omega}\gamma\overline{R}_{t-1}$$

$$x_t^i = \frac{r_{t-1}^i}{\overline{R}_t} + \frac{(x_{t-1}^i)^\omega}{\sum_{j=1}^{n}(x_{t-1}^j)^\omega}\gamma\frac{\overline{R}_{t-1}}{\overline{R}_t}$$

[3] w could take several forms. For this model, w should be considered a scalar whose magnitude is >1 and that w is constant for all firms at all time periods. The condition that w>1 is necessary for the clustering effect of this model to occur. As w represents the rate at which new revenue tends to larger firms, an extension of this essay could be to include the price of firm i's product as a determinant of $w_{i.}$

$$\textit{But:}\qquad r^i_{t-1} = x^i_{t-1}\overline{R}_{t-1}$$

$$\textit{and}\qquad \frac{\overline{R}_{t-1}}{\overline{R}_t} = \frac{1}{1+\gamma}$$

$$x^i_t = x^i_{t-1}\frac{1}{1+\gamma} + \frac{(x^i_{t-1})^\omega}{\sum_{j=1}^n (x^j_{t-1})^\omega}\frac{\gamma}{1+\gamma}$$

$$\textit{Defining:}\quad \chi^i_t \equiv \frac{(x^i_{t-1})^\omega}{\sum_{j=1}^n (x^j_{t-1})^\omega}$$

$$\textit{Therefore:}\qquad x^i_t = \frac{1}{1+\gamma}[x^i_{t-1} + \gamma\chi^i_t]$$

Firm i's market share depends positively on the exogenous growth rate of revenue γ, last period's market share, and the impact of preferential attachment. Crucially, the impact of preferential attachment depends not just on a firm's market share but also their market share relative to the market share of other firms in the industry. To investigate if an equilibrium exists, we set:

$$\Delta x^i_t = 0$$

$$\Delta x^i_t = \frac{\gamma}{1+\gamma}[\chi^i_{t-1} - x^i_{t-1}]$$

$$= 0 \iff \chi^i_{t-1} - x^i_{t-1}$$

$$\iff \frac{(x^i_{t-1})^\omega}{\sum_{j=1}^n (x^j_{t-1})^\omega} = x^i_{t-1}$$

$$\iff (x_{t-1}^{i})^{\omega} = x_{t-1}^{i} \sum_{j=1}^{n} (x_{t-1}^{j})^{\omega}$$

$$\iff (x_{t-1}^{i})^{\omega-1} = \sum_{j=1}^{n} (x_{t-1}^{j})^{\omega}$$

$$\iff x_{t-1} = e_j$$

$$(x_{t-1})^T = [1, 0, 0 \ldots 0]$$

Steady-state is observed when one firm has total market share, i.e. when complete monopoly is observed. The practical implications of this result will be discussed in the policy considerations section.

Numerical Example

To provide a more illustrative example, allow $\gamma = 0.2$, $w = 2$ and $T = [60\ 10\ 10\ 10\ 10]$. In this case, the market shares evolve over time such that:

Time	Firm 1	Firm 2	Firm 3	Firm 4	Firm 5	HHI
1	0.6	0.1	0.1	0.1	0.1	4000
2	0.65	0.0875	0.0875	0.0875	0.0875	4351
3	0.697	0.0757	0.0757	0.0757	0.0757	5088
4	0.74	0.0649	0.0649	0.0649	0.0649	5648
5	0.7783	0.0554	0.0554	0.0554	0.0554	6181
10	0.9055	0.0236	0.0236	0.0236	0.0236	8222
15	0.9613	0.0097	0.0097	0.0097	0.0097	9244
20	0.9843	0.0039	0.0039	0.0039	0.0039	9689

This simple numerical simulation provides a striking example of the monopolistic pull the model presents. Revenue growth of 20% is not extraordinary and the preferential attachment of w=2 is substantially lower than power laws observed in some real world scenarios. Nonetheless after 5 periods of this model's evolution, the dominant player's market share increased from 60% to 77.83% and the HHI increased by 54.5%. By the tenth period, the dominant player's market share exceeded 90% and the HHI had more than doubled to over 8000.

Policy Considerations

The model demonstrates that if a market displays continuous growth and preferential attachment–a well-documented occurrence – it will also display significant 'monopolistic pull' dependent upon the initial market shares, the level of growth of the market and the weight of the attachment. This is particularly relevant for industries that face liberalisation from previous monopolisation. It is regularly observed that previous monopolists maintain dominant market shares: ComReg (2007: 12) report that nine years after telecommunications deregulation, Eircom's market share was still 69%.

The initial market share of the dominant firm and consequently the number of firms a market can bear in this context is ultimately related to the 'foot-loose' fraction of consumers, ρF who are inclined to move from the dominant firm.[4] By definition, new entrants must compete for these customers among themselves and so average market share falls as the number of firms increase. As already noted, this increases the effect of preferential attachment in our model.

Specifically, regulators wishing to avoid clustering must choose n firms in the market such that:

$$\frac{\rho F}{n} = 1 - \rho F$$

$$n = \frac{\rho F}{1 - \rho F}$$

$n > 0$, so no desirable free market solution exists if $\rho F < 0.5$. Values of ρF observed in real life suggest that n should rarely exceed 10. In this model, $n > 10$ would portray an example the oft-cited 'race to the bottom' effect, where

[4] Editor's note: The 'footloose' fraction of customers may well be dependent upon the level of 'switching costs', which can 'lock' customers into a particular firm or product. This concept is explored in the next essay on predatory pricing. Interestingly, these costs can be of a psychological nature.

additional competition hastens a monopolistic outcome.

Under the preferential attachment distribution of new revenue, ultimately the largest firm will conquer the market. In this regard, the model is intended to be illustrative rather than precise. The lower the dominant firm's initial market share, the weaker the monopolistic pull. Furthermore, the longer it takes to conquer a market, the longer rival firms have to respond and the longer competition authorities have to attempt to rectify undesirable outcomes. Thus the initial liberalisation of and specifically the number of firms allowed to enter the market can be vital to the market outcome and these outcomes may not be consistent with those predicted by Contestable Market Theory, among others. The model points to the use of alternative post-liberalisation policies such attempting to lower w or increasing ρF.

Conclusion

This essay sought to integrate the findings of the field of scale-free networks into the context of economic regulation. The model presented showed that a growing market with preferential attachment displays significant monopolistic pull. The results and conclusions deriving from the model conflict somewhat with those of contestable market theory, or at least question the use of simplistic static HHI analysis for market liberalisation policies. It is interesting to consider how future advances in the study of networks may provide insights into and help shape regulatory policy in the future. It is hoped this essay will encourage some future application of network theory into economics.

Bibliography

Albert, R., Jeong, H. and Barabási, A.L. 1999. *The Diameter of the World Wide Web*. Viewed at: http://xxx.lanl.gov/PS_cache/cond-mat/pdf/9907/9907038v2.pdf

Barabási, A.L. and Albert, R. 1999. 'Emergence of Scaling in Random Networks'. *Science* 286:509-512.

The Competition Authority 2004. 'Notice In Respect of Guidelines for Merger Analysis'. Viewed at: http://www.tca.ie/controls/getimage.ashx?image_id=114

Commission for Communications Regulation. 2007. *Irish Communications Market: Quarterly Key Data – December 2007*.
Viewed at: http://www.comreg.ie/_fileupload/publications/ComReg07106.pdf

Demsetz, H. 1968. 'Why Regulate Utilities?' *Journal of Law and Economics* 11:1:55-65.

Newman, M.E.J. 2001. 'Clustering and preferential attachment in growing markets'. *Physical Review* E 64:025102.
Newman, M.E.J., Barabási, A.L. and Albert, R. (eds.) 2006. *The Structure and Dynamics of Networks*. Oxford: Princeton University Press.

Price, D.S. 1976. 'A General Theory of Bibliometric and Other Cumulative Advantage Processes'. *Journal of the American Society for Information Science* 27:5-6:292-306.