An Empirical Investigation of the Arbitrage Pricing Theory in relation to the Irish Market

John Power
Senior Sophister

The growth of the world's financial markets, both in size and sophistication, has been matched by the growth of the literature which attempts to predict their future movements. John Power tests empirically whether the Arbitrage Pricing Theory - a model tailor made for the US - is relevant in the context of a small, peripheral market such as Ireland's.

Introduction

The last 40 years have witnessed an extraordinary amount of innovation and remarkable progress in the realm of asset pricing theory. Indeed the financial literature over the past 30 years seems to be dominated by the subject. However, research has invariably tended to focus on larger economies (most notably the United States) to the neglect of economies such as Ireland. This paper will make an attempt to remedy the situation by carrying out an empirical test on the applicability of the Arbitrage Pricing Theory (APT) to Irish securities. The broad aim of the project is both to test whether the generally accepted factors of APT given by U.S. researchers can adequately describe the Irish market, and to construct other factors that may be specifically applicable to Ireland.

APT - General Issues

By the mid 1970's, despite its apparent empirical success, the consensus amongst financial economists upon the easy testability of the Capital Asset Pricing Model (CAPM) began to breakdown. In particular Roll's critique (which proved that the methodology used in its empirical investigation was of very limited power), heralded a new era of reflection amongst financial economists. This reflection not only stimulated tighter methods in the empirical investigation of the CAPM, but also a new, more radical school of thought - the APT (Ross, 1977). In essence, Ross proposed that security returns were sensitive not to just one type of non-diversifiable risk, but to a variety of different types of risk inherent in the economy.[1]

Issues Associated with its Empirical Investigation

Unfortunately it is difficult to carry out empirical tests of the theory. The broadness of the theory (i.e. the existence of many determining factors rather than just one) is both its principle strength and weakness, as the authors make no mention as to the nature of these factors.

[1] See appendix 1 for a more rigorous definition of the APT

Statistical methods, known as factor analysis, involving the simultaneous numeric estimation of the loadings and sensitivities are quite often used, however this approach is intellectually unappealing as the actual nature of the factors is not specified.

The second approach is more intuitive, but is, however, rather ad hoc in nature. In this case, on the basis of economic theory, the "a priori" specification of the general factors (F's) is established. Since stock prices are merely expected discounted cash-flows/dividends, possible explanatory factors should affect either the expected cashflow of the firms, or the discount rate. In general only unanticipated deviations (innovations) serve as proper factors, as the forecastable element will have been previously built into prices.

For the purposes of my tests I shall follow the Fama/McBeth (1973) procedure of time series and cross-section regression to obtain the factor sensitivities and loadings. The factor sensitivities (bj's) are obtained through time series regression of a number of stock returns on the factors. The bj estimates are then used as the explanatory variables in a cross section regression on the returns, over a period of time (normally one month). This regression is repeated over several periods. The coefficients of the bjs in these cross-section regressions are estimates of the λi's, or risk premiums, associated with the factors. Whether these factors are priced can be examined by t-testing the means of the sample λi's.

What Factors Should be Used?

In their study Chen, Roll and Ross (1986), found that the risk of changes in the default premium (spread between the return on corporate bonds and the risk-free rate), risk of change in the term structure of interest rates (spread between the return on long term bonds and treasury bills), risk of unanticipated inflation, risk that the long run expected growth rate of profits for the economy will change (using changes in industrial production as proxy for this), are priced factors. Burmeister and McElroy (1988) introduce a residual market factor into their model, as a proxy for unobserved influences. This factor is measured as the residuals of a regression of the state variables against the excess return for the market index (Rm - Rf).[2] Finally, other commentators have used changes in unemployment as another measure of risk in the economy.

With regards to an Irish analysis, I will apply all of the above factors, nevertheless, following blindly the guidelines of American researchers would be foolhardy. The nature of the Irish economy warrants special attention.

[2] These residuals can be used directly as a factor, as it is assumed that the associated factor sensitivity with the market is 1.

Firstly, as a small open economy with a high propensity to trade, an "international" factor would seem appropriate. Risk of changes in the exchange rate could be used as one possible factor to model this. In order to resolve the problem of which exchange rate to choose, I have decided to use an aggregated trade weighted exchange rate. The Central Bank produces such an index (effective exchange rate), which, to the best of my knowledge, is trade weighted exchange rate with our principal trading partners.[3]

Secondly, again stemming from its "open" nature, it has been well documented that the Irish stock exchange is highly integrated with London. Thus, I use the FT-100 index as a possible measure of risk in the British economy. Furthermore the use of a British index, where thin trading is not as common as it is in Ireland, may help explain some of the systematic risk components of Irish securities.

Finally, the non-existence of a corporate bond sector precludes the use of measuring default risk. Instead, I propose the use of change in the commercial bank lending rate as a potential factor, as unanticipated changes in this rate could conceivably change the structure by which future cashflows are discounted.

Data Sources

I obtained stated variables and stock price data from three sources.

The monthly Central Statistics Office publication, Economic Series, contains a monthly wholesale price index, and a seasonally adjusted industrial production index. I decided to use this index as it removes some of the "predictable" component of industrial production.

Central Bank Quarterly bulletins provide monthly data on the effective exchange rate, the return to long term government bonds (15 years to maturity), the return to 91 day exchequer bills (serves as a proxy for the risk-free rate), and average commercial bank lending rates.

Data-stream provides monthly data not only on stocks, but also on the number of unemployed (seasonally adjusted), the ISEQ index, the FT-I00 index and bond data.

Finally, general data on Irish firms, such as market capitalisation and industry type, was obtained from the *Sunday Business Post* newspaper.

[3] The CB does not publish the weights used.

The Empirical Investigation

Preliminaries

Before I could proceed with the tests, it was necessary to manipulate the data. It is necessary to strip out any predictable components in the state variables. Most writers propose that the monthly change in price indices, unemployment, industrial production and exchange rates be treated as an innovation. For simplicity I will use this technique. Thus, in order to calculate the F's associated with these variables, I used the formula:

$$Fj(t) = \log^{e} Ij(t) - \log^{e} Ij(t-1)$$

where

Ij(t) - value of index j in period t

Fj(t) - value of factor j in period t

An analogous procedure was used to calculate the monthly returns of the price indices and stock prices down-loaded from Datastream.

With regards the term structure factor, Chen et al advise the use of the following factor:

$$F(t) = LGB(t) - EB(t-l)$$

where:

LGB(t) is the return on a long term government bond in period t

EB(t) is the return on an exchequer bill in period t

Finally, in order to calculate residual market risk as espoused by Burmeister and Mcelroy, a time series regression was run on the excess return of the ISEQ index against the above factors. The excess monthly return in month t, Et was calculated by the following equation:

$$Et = Rit - ((I + Ebt)^{1/12} - 1))$$

where Rit refers to the return on the ISEQ index in month t.

The Tests[4]

I divided my data into 2 sub-samples - period I (Jan 1985 - Dec 1990) and period 2 (Jan 1991 - June 1996), with the goal of performing 2 separate, distinct tests of the APT in both periods. I aimed to use the first 36 months of each sub-sample to

[4] Note: All regressions were carried out using SPSS. Tests of significance which will be referred to throughout the rest of this section relate to simple t -tests, where the null hypothesis states that the coefficient is insignificant (=0). Thus, significance at the 10% level implies that I reject the null hypothesis with a 90% chance of being correct.

perform the time series regression and the final 24-30 months to perform monthly cross-sectional regressions.

The First Sub-sample

The grouping of stocks into portfolios is necessary to avoid an "errors in variables" problem associated with the bj estimates in the cross-section regressions. Unfortunately, for the first period I was not afforded such a luxury. Datastream provided coherent monthly price data on about 40 stocks during this period. Portfolio formation on the basis of such limited sample would be unwarranted, especially given the severe multicollinearity problem[5] in the subsequent cross-sectional regression. Thus, I decided not to form portfolios, but to take 20 "main movers"[6] across different industries, of differing market capitalisation.

Main Results[7]

The meaning of the factor sensitivity coefficients can be interpreted as the quantity of the associated risk inherent in a particular stock. Thus, for example, a realisation for residual market risk of 1% per month will raise James Crean's monthly rate of return by 0.56%, when all other factor realisations are zero. The coefficient for the London index is, in general, well behaved, in the sense that fairly stable significant quantities of that risk are inherent in most of the securities. Residual market risk follows a similar pattern, where significant, quantities of term structure risk seem to be low. Exchange rate risk tends to be present (and in large quantity) only in larger firms. For the most part the other factors fail to have a significant, consistent impact on other stocks returns.[8]

The results of the cross section regression are summarised in Appendix 3. The data failed to establish a significant, consistent (factor loading) risk premium for any of the factors. The constant, which should measure the risk-free rate, was statistically significant on only three occasions in the 24 month period, giving rates of 10%, 10% and -2%. The value of the other risk premia were quite erratic, for example, when the risk premium associated with the FT-100 was priced, it gave rates of -22%, -11%, 10% and 20%. Given these ambiguities, which were

[5] Problem of few observations and several variables.

[6] The "Main-moving" criteria was rather ad hoc, it involved the visible inspection of the 40 stock prices, and the selection of the 20 that tended to frequently change price.

[7] See appendix 2 for a summary of the time series regression results.

[8] The unemployment and industrial production factor are not included, as their factor sensitivity estimate failed to be significantly different from zero for all firms in the time series regression - I re-estimated the regressions without using these factors.

prevalent in the other premia estimates, and given the spurious validity of the variables used to construct them, I did not test the average value of the risk premia for significance.

The Second Sub-sample[9]

For the second period, I had access to data on 60 stocks. I believed I could perform viable portfolios on the basis of such a number of stocks. Portfolios were formed on the basis of industry sector, and weighted using market capitalisations. The portfolios formed were: leading industrials (high market capitalisation), leading industrials (low market capitalisation), three other industrial portfolios ranked on market capitalisation, the two main banks, other financial property companies, and finally, other stocks (consisting mainly of exploration companies of low market capitalisation).

Main Results

Results were once again of limited significance.[10] The exchange rate sensitivity was the only coefficient that proved to be significant, it being present in large negative quantities in three portfolios, thus implying high sensitivity to exchange rate fluctuations for these portfolios. The cross-section regressions[11] were disappointing, yielding only one interesting result - all factor loadings were statistically significant for three periods - the same three periods! Although a formal test is impossible, by visual inspection one can see that the loadings tend to be fairly stable. However, we must be reminded that the data used to generate these results is of limited quality.

Concluding Thoughts

Clearly the APT model in its proposed form is refuted by the above results. None of the factors were consistently significant in explaining stock returns. Indeed, the results of the first time series regression implicitly support the CAPM (the aggregate market factor excluding the hypothesised factors and the London index being highly significant).

Before we consign APT to the financial economists' graveyard, a number of concerns should be voiced. Firstly, the usual problem of modelling expected returns, not actual returns appears. Similarly, specifically with regards to the

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[9] For the second sub-sample it was necessary to reduce the number of explanatory factors, so as to avoid multicollinearity in the cross-section regressions. For this reason I decided not to use the least significant factors in explaining excess return on the market index, these factors once again proved to be unemployment and industrial production.

[10] See appendix 4 for summary.

[11] See appendix 3 for summary.

APT, the forecastable component of any of the explanatory factors should be excluded. My analysis did not really account for either of these issues.

Secondly, there were only nine observations, for seven explanatory variables in my second series cross-sectional regressions. The problem of multicollinearity arises.

However, this problem was necessary to construct portfolios. The construction of portfolios is needed for reliable variables in the cross-section regression. Immediately, one is struck by the question of how one makes an efficient trade-off?

Finally one particular problem of theoretical inconsistency, succinctly underlined in the second period, merits discussion. Due to the currency crisis of 1992, the "risk-free" rate reached over 30% in some periods, clearly distorting the results on term structure and perhaps interest rates. However, it is precisely this type of situation that the APT tries to model - currency risk - a specific risk to the economy as a whole that is probably undiversifiable.[12] My own analysis somewhat accommodated this risk. In my second period model, sensitivity to exchange rates was the best factor of a bad lot. Indeed, by extending the analysis, I could assert that, in three periods, investors demanded an 11% premium due to this risk - this is entirely plausible. However, the other parameters of the model were unable to cope with such an environment.

To sum up - yes, APT in its proposed form does not adequately model the Irish market, but no, this should not lead to an absolute rebuttal of the theory. If anything, I have shown that research efforts should be redoubled, so as to resolve some of the problems my study has elicited in dealing with a market such as Ireland.

[12] Unless of course, investors can internationally diversify by converting their holdings into other currencies - this is unlikely.

Appendix 1

The generation of the APT equilibrium equation requires only the 2 rather innocuous assumptions that markets are in equilibrium (in the financial sense that there are no arbitrage opportunities), and that the return generating process can be described by the following equation:

$$\mathbf{R_k = E_k + bk_1F_1 + bk_2F_2 + bk_3F_3 + ... + bk_qF_q + \varepsilon k} \quad \textbf{(1)}$$

where:

R_k = return on security k

F_q = value of the qth risk factor in the economy that impacts on R_k

$E(F_q) = 0$ (i.e on average, we do not expect the factor to be different from zero)

bk_q = the sensitivity of stock k's return to the qth factor (factor sensitivity)

E_k = the expected value of R_k if the factors have a zero value

εk = "classical" random error term with the usual properties[13]

When all arbitrage possibilities are exhausted, the expected return on a well diversified portfolio p of securities will be a linear combination of the b coefficients:

$$\mathbf{E(Rp) = \lambda o + \Sigma bpi\lambda i} \quad \textbf{(2)}^{14}$$

where:

λo - risk-free rate

λi - the risk premium associated with the ith factor (factor loading)

Appendix 2

Leading industrials

	Exchange rates	FTSE100	Inflation	Interest rates	Term structure	Residual mkt risk
James Crean	-2.02x	0.98xx	2.02	-0.18	-0.01x	0.56xx
CRH	-1.1	0.66xx	2.25	0.4	0	0.73xx
Fyffes	0.07	1.02x	-2.54	0.24	0.01	0.79xx
Smurfit	1.57	1.23xx	-5.57x	0.11	0.01x	1.05xx
Waterford Wedgewood	-0.43	1.03xx	-4.49	-0.25	-0.01	0.45

[13] Including $E(\varepsilon k)=0$, $Cov(\varepsilon k,\varepsilon j)=0$, $Cov(\varepsilon k,Fj)=0$; for all k & j

[14] The derivation of the APT is outside the remit of the paper. hb I refer the interested reader to Roll (1977) for a rigorous treatment.

Second line industrials
-High market capitalisation (>£40m)

	Exchange rates	FTSE100	Inflation	Interest rates	Term structure	Residual mkt risk
European leisure	24.06	3.78	41.66	6.93	0.3xx	6.6xx
Fitzwilliam	4.89xx	1.02xx	-5.59	-2.51	0.04xx	1.31xx
Flogas	2.93xx	0.84xx	-2.07	0.72	0.01	0.54xx
Jones Group	0.21	1.07xx	5.75	-0.02	0.01	0.91xx
Ryan Hotels	-2.73	0.95xx	5.35	-0.52	-0.02	o.44
Unidare	2.85x	0.62xx	-3.69	0.9	0.01	0.43x

Second line industrialists
-Low market capitalisation (<£40m)

	Exchange rates	FTSE100	Inflation	Interest rates	Term structure	Residual mkt risk
Abbey	0.14	1.54xx	12.75	1.1	0.03	1.13
Arnotts	-0.09	0.72x	3.62	0.32	-0.01	0.12
Ennex international	0.57	0.99xx	11.89x	-0.48	0.01	1.02xx
Heitoin Holdings	0.52	1.41xx	4.52	-1.07x	0.02xx	1.1
Clondalkin Group	-0.13	0.93xx	3.27	-0.34	0	0.79xx

Banks

	Exchange rates	FTSE100	Inflation	Interest rates	Term structure	Residual mkt risk
Allied Irish Banks	-0.09	0.45xx	-6.31x	0	0.8	0.8xx
Bank of Ireland	-1.99	0.34	-1	-0.01	0.5	0.5xx

Other Financials

	Exchange rates	FTSE100	Inflation	Interest rates	Term structure	Residual market risk
Anglo-Irish Bank	1.88	0.58	-0.25	0.01	0.85	0.85xx
	0.58	1.55xx	-3.67	-0.34xx	0xx	1.02xx

x significant at the 10% level
xx significant at the 5% level

Appendix 3

Number of periods where the lambda (risk-premium) coefficient was significant:

	No. of Periods Jan 88-Dec 90	No. of Periods Jan 94-Dec 96	Per. a Jan 1994 - Dec 1996	Per. b Jan 94-Dec 96	Per. c Jan 94-Dec 96
Exchange Rates	2	3	0.13	0.11	0.11
FTSE 100	4	3	-0.38	-0.49	-0.13
Inflation	6	3	-0.16	-0.20	-0.13
Interest Rates	2	3	0.70	0.67	0.42
Term Structure	1	3	-300.00	-314.00	-244.00
Residual Mkt Risk	4	3	1.13	1.85	1.32
Constant	3	3	-0.04	-0.16	-0.16

Note: Significance of the coefficients is evaluated at the 10% level.

Appendix 4

"b" coefficient (factor sensitivities) estimates of time regression 2 (Jan 1991 - Dec 1993):

	Exchange rates	FTSE100	Inflation	Interest rates	Term structure	Residual market risk
Leading industrialists (1)	-3.27xx	-0.33	0.41	-0.43	0	-0.17
Leading industrialists (2)	-3.02xx	-0.22	-0.42	0.08	0	0.09
Industrialists (3)	-1.73	-0.43	1.75	0.88	0	-0.11
Industrialists (4)	-0.89	0.31	1.08	0.07	0	0.26
Industrialists (5)	-1.6	0.28	-1.69	0.32	0	-0.01
Banks	-2.58	-0.12	1	-0.07	0	0.1
Other financials	0.27	0.17	0.09	-0.12	0	0.07
Market/Property	-4.12	0.41	3.61	1.39	0	-0.3
Others	-1.43	-0.23	0.86	-0.08	0	0.09

(1) - Market Cap.>£200m
(2) - Market Cap. <£20m
(3) - Market Cap. £70m-£100m
(4) - Market Cap. £30m-£70m
(5) - Market Cap. <£30m

xx significant at the 5% level
x significant at the 10% level

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