Purchasing Power Parity: The Irish Experience Re-visited

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Abstract

This paper looks at issues surrounding the testing of purchasing power parity using Irish data. Potential difficulties in placing the analysis in an $I(1)/I(0)$ framework are highlighted. Recent tests for fractional integration and nonlinearity are discussed and used to investigate the behaviour of the Irish exchange rate against the United Kingdom and Germany. Little evidence of fractionality is found but there is strong evidence of nonlinearity from a variety of tests. Importantly, when the nonlinearity is modelled using a random field regression, the data conform well to purchasing power parity theory, in contrast to the findings of previous Irish studies, whose results were very mixed.

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by

Derek Bond, Michael J. Harrison and Edward J. O'Brien[1]

1 Introduction

Purchasing power parity (PPP) has become a major subject of research in applied economics. In part, this is due to its crucial role in both the theory of exchange rates and international finance. Recent surveys include Rogoff (1996), Sarno and Taylor (2002), and Taylor and Taylor (2004). The empirical analysis has generally kept pace with developments in time series econometrics. Two major areas of research are the mean reversion characteristics of the real exchange rate [see Cashin and McDermott (2004)] and its nonlinear representation [see Sarno (2005)]. However, the mainstream literature in the area has as yet to fully utilise two recent developments in econometric theory, namely, long memory models and random field-based inference. These could provide useful additional tools for investigating both mean reversion and nonlinearity in PPP analysis.

From the econometrics literature, it is clear that nonstationarity and nonlinearity are closely related. It has been well known for many years that it is difficult to distinguish statistically between difference stationary series and nonlinear but stationary series; see Perron (1989) and Harrison and Bond (1992). Recent works in this area include Lee, et al. (2005), Hong and Phillips (2005), and Basci and Caner (2005). Increasingly, the analysis uses the fractional integration framework rather than the 'knife-edge' $I(1)/I(0)$ approach to consider the interaction between nonlinearity and nonstationarity. For example, Diebold and Inoue (2001) and Perron and Qu (2004) investigate the effects of nonlinearity on the estimation of the fractional integration parameter, while Hsu (2001) and Krämmer and Sibbertsen (2002) examine the impact of long memory on estimates and tests of structural change. Other recent work by Dolado, et al. (2005), Gil-Alana (2004) and Mayoral (2005) has devised new test procedures for fractionality and/or nonlinearity. However, in most cases the form of the nonlinearity needs to be known.

The aim of this paper is to use two recent developments in econometric theory discussed in Bond, et al. (2005b) to explore the time series characteristics of simple empirical interpretations of PPP theory using Irish data. The first of these is the Dolado, et al. (2002) fractional augmented Dickey-Fuller (FADF) test; the second is the random field regression approach to the investigation of nonlinearity due to Hamilton (2001). The structure of the paper is as follows. Section 2 provides some background, briefly describing the theory of PPP and the few previous Irish studies. Section 3 explains popular approaches to modelling nonlinearity, the random field regression

[1]The authors wish to thank Jonathan H. Wright, Assistant Director, Division of Monetary Affairs, The Federal Reserve Board, Washington DC 20551, for providing some of the data used in this study.

model of Hamilton (2001) and the concept of fractional integration. Section 4 contains an account of the Dolado, et al. (2002) FADF test, as well as the methods of inference employed in Hamilton's approach to nonlinearity. Section 5 gives details of the data and the precise approach adopted in the analysis in this paper, while the results are presented and discussed in Section 6. Finally, Section 7 concludes by considering how the methodology might assist in the development of the general discussion of PPP.

2 Background

A simple statement of the purchasing power parity hypothesis is that national price levels should be equal when expressed in a common currency. More formally, if s_t is the logarithm of the nominal exchange rate (expressed as units of foreign currency per unit of domestic currency), p_t and p_t^* are the logarithms of the domestic and foreign price levels, respectively, and q_t is the logarithm of the real exchange rate in period $t = 1, 2, ..., T$, then for all t,

$$q_t = s_t + p_t - p_t^*. \tag{1}$$

It follows that q_t must be stationary for long run PPP to hold. If the mean of q_t, $E(q_t)$, is zero, we have absolute PPP, whereas if $E(q_t) \neq 0$, we have relative PPP. Most of the empirical studies of PPP have either been concerned with testing whether q_t has a mean reversion tendency over time or whether s_t, p_t and p_t^* move together over time.

This latter work has generally been concerned with models whose simplest form is

$$s_t = \alpha_0 + \alpha_1 p_t + \alpha_2 p_t^* + \epsilon_t, \tag{2}$$

where ϵ_t is white noise. Early studies were concerned with whether the estimated values of the parameters of various versions of Equation (2) were as predicted; see, for example, MacDonald and Taylor (1992). As awareness of time series dynamics increased, the issue changed to one of whether Equation (2) is a cointegrating regression. Papers such as those by Thom (1989), Wright (1994) and Kenny and McGettigan (1999) take such an approach with Irish data, using the now well-known Engle-Granger (1987) two-step method or Johansen (1988) approach to cointegration. The results of these Irish studies have been confusing. In some cases, PPP could not be accepted, whereas in others it could not be rejected. Nonrejection seemed most common when either prices were split into their component parts or other variables were included in the model. For instance, Kenny and McGettigan (1999) distinguished between prices in the traded and nontraded sectors; Wright (1994) considered interest rate differentials, along with the variables in Equation (2).

In recent years, the emphasis has generally shifted from considering models of the form of Equation (2), to considering directly the behaviour of $\{q_t\}_{t=1}^{T}$, the sequence of real exchange rate values. Within the $I(1)/I(0)$ framework, most of the initial studies failed to reject the hypothesis of real exchange rates being $I(1)$ for recent periods of flexible exchange rates.[2] This failure to reject the possibility of unit roots in real exchange rate series implies a lack of mean reversion, which undermines the PPP hypothesis. The explanation often given for this nonrejection is the recognised low power of traditional unit root tests, such as the standard Dickey-Fuller test. To overcome this problem, two general approaches have been adopted. The first has been the construction and use of long series of exchange rate data and more powerful asymptotic tests; see Taylor (2002). The second, using panel data, attempts to estimate the half life of the mean reversion of the real exchange rate; see Cashin and McDermott (2004). Another explanation has been that the real exchange rate is time varying and requires the use of other factors in its modelling; see Lane and Milesi-Ferretti (2002), who identify relative output levels, terms of trade and the net foreign assets in their linear model for the Irish real exchange rate. There is, though, a third possibility that is receiving increasing attention, and this is described in some detail in the following section.

3 Nonlinearity and Nonstationarity

The alternative explanation that has been gaining ground in the literature suggests the possibility that real exchange rate generating processes are in fact nonlinear. It is argued that nonlinearities arise because of transactions costs in international arbitrage; see Sarno (2005) for further details and discussion of the argument.

3.1 Smooth transition autoregressive models

The standard way to model the nonlinearities has been to use smooth transition autoregressive (STAR) models; see Teräsvirta (1994). Assuming that the real exchange rate is a stationary process, the STAR representation can be written as

$$q_t = \varphi' \mathbf{z}_t + \theta' \mathbf{z}_t G(\gamma, c, \tau_t) + \epsilon_t, \tag{3}$$

where $\epsilon_t \sim iid(0, \sigma^2)$, $\mathbf{z}_t = [1\, q_{t-1} \, \dots \, q_{t-p}]'$, and φ and θ are $(p+1)$-vectors of parameters. The transition function $G(\gamma, c, \tau_t)$ determines the degree of mean reversion and is itself a function of γ, the slope coefficient, c the location parameter and τ_t the transition variable. Normally τ_t is set to be a lagged value of q_t.

There has been little discussion about the choice of specification of the transition function G. It is generally accepted, following Taylor, et al. (2002), that its form is exponential:

$$G(\gamma, c, \tau_t) = 1 - \exp\left[-\gamma(\tau_t - c)^2\right], \tag{4}$$

and the resultant model is known as the exponential smooth transition autoregressive (ESTAR) model. The reason for this choice is that it is felt that

[2] In the literature there is some confusion between unit root testing and testing for a random walk. The unit root hypothesis includes the random walk hypothesis but a unit root might exist for reasons other than that the series in question is a random walk. Data may be generated by a more complex nonstationary dynamic process.

the movement of the real exchange rate is symmetrical. However others, such as Sen and Baharumshah (2003), argue that the asymmetric logistic function (and hence the LSTAR model) should also be considered, i.e.,

$$G(\gamma, c, \tau_t) = [1 + \exp[-\gamma(\tau_t - c)]]^{-1}, \tag{5}$$

on the grounds that there is little empirical evidence to support the use of ESTAR models.

A more general alternative to the ESTAR model is the LSTAR2 model:

$$G(\gamma, c, \tau_t) = \left[1 + \exp\left[-\gamma \prod_{k=1}^{2} (\tau_t - c_k)\right]\right]^{-1}. \tag{6}$$

The use of the LSTAR2 model overcomes the problem that, as $\gamma \to \infty$, Equation (4) becomes linear. However, there is a very different alternative method available.

3.2 Random field regression models

This other approach to modelling nonlinearity is provided by random field regression. Dahl (2002) showed that the random field approach has relatively better small sample fitting abilities than a wide range of parametric and nonparametric alternatives, including LSTAR and ESTAR models. The idea of using random field models to estimate and test for nonlinear economic relationships was introduced by Hamilton (2001) and is as follows.

If y_t is a stationary process, $\epsilon_t \sim nid(0, \sigma^2)$, and $\mathbf{x}_t$ is a k-vector, that may include lagged dependent variables, then the basic model is

$$y_t = \mu(\mathbf{x}_t) + \epsilon_t, \tag{7}$$

where the form of the conditional expectation functional, $\mu(\mathbf{x}_t)$, is unknown and assumed to be determined by the outcome of a random field. Hamilton suggests representing $\mu(\mathbf{x}_t)$ as consisting of two components. The first is the usual linear component, while the second, a nonlinear component, is treated as stochastic and hence unobservable. Both the linear and nonlinear components contain unknown parameters that need to be estimated. Following Hamilton, the conditional mean function is written as

$$\mu(\mathbf{x}_t) = \alpha_0 + \boldsymbol{\alpha}_1' \mathbf{x}_t + \lambda m(\bar{\mathbf{x}}_t), \tag{8}$$

where $\bar{\mathbf{x}}_t = \boldsymbol{g} \odot \mathbf{x}_t$, $\boldsymbol{g}$ is a k-vector of parameters and $\odot$ denotes the Hadamard (element-by-element) product of matrices. The function $m(\bar{\mathbf{x}}_t)$ is referred to as the random field. If the random field is Gaussian, it is defined fully by its first two moments. If $\mathbf{H}_k$ is the covariance matrix of the random field, with a typical element $H_k(\mathbf{x}, \mathbf{z}) = E[m(\mathbf{x})' m(\mathbf{z})]$, Equation (7) can be rewritten as

$$y_t = \alpha_0 + \boldsymbol{\alpha}_1' \mathbf{x}_t + u_t, \tag{9}$$

where

$$u_t = \lambda m(\bar{\mathbf{x}}_t) + \epsilon_t, \tag{10}$$

or in matrix form

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}, \tag{11}$$

where $\boldsymbol{\beta} = [\alpha_0 \, \boldsymbol{\alpha}_1']'$. It follows that

$$\mathbf{u} \sim N(0, \lambda^2 \mathbf{H}_k + \sigma^2 \mathbf{I}_T). \tag{12}$$

Treating equations (11) and (12) as a generalised least squares problem, the associated profile maximum likelihood function can be obtained and estimated. The only problem is that the form of the covariance matrix is unknown. Hamilton derives $\mathbf{H}_k$ as a simple moving average representation of the random field based on $\boldsymbol{g}$, using an L_2-norm measure. He shows that even under fairly general misspecification, it is possible to obtain consistent estimators of the conditional mean. Additional results on the consistency of the parametric estimators obtained from this approach are given in Dahl, et al. (2005).

3.3 Long memory models

Related to the issues of nonlinearity and nonstationarity is the concept of long memory. However, long memory has not played a central role in the discussion of PPP, despite being used extensively in other areas of exchange rate analysis, such as the forward rate anomaly [see Bond, et al. (2006)], and being used in the early and heavily cited works by Diebold, et al. (1991) and Cheung and Lai (1993). The papers by Cheung and Lai (2001) and Robinson and Iacone (2005) are two of the few recently published works that apply the concept to PPP.

A series $\{y_t\}_{t=0}^{\infty}$ is said to be integrated of order d, denoted by $I(d)$, if the series has to be differenced d times before it is stationary. In the classical analysis, d is an integer and the majority of investigation has involved the $I(1)/I(0)$ framework. That is, either $\Delta y_t = y_t - y_{t-1}$ or y_t is stationary. In fractional integration analysis, the restriction that d is an integer is relaxed. This leads to a more general formula for an integrated series of order d given by

$$\Delta^d y_t = y_t - d y_{t-1} + \frac{1}{2!} d(d-1) y_{t-2} - \ldots + \frac{(-1)^j}{j!} d(d-1) \ldots (d-j+1) y_{t-j} + \ldots, \tag{13}$$

which is $I(0)$. In the case where $0 < d < 1$, it follows that not only the immediate past values of y but values from previous time periods influence the current value. If $0 < d < 0.5$, then the series $\{y_t\}_t^{\infty}$ is stationary; and if $0.5 \leq d < 1.0$, then $\{y_t\}_t^{\infty}$ is nonstationary. Both estimation and inference in the case where d is not an integer is more complex than in the standard integer d case [see Bond, et al. (2005b)] and this could be an explanation for the lack of uptake of the concept in the analysis of PPP.

The issue of trying to accommodate the possibility of both nonlinearity and nonstationarity has been the subject of some recent research. In particular, Haug and Basher (2004), have used the rank test proposed by Breitung (2001) to test for nonlinear cointegration, while Hong and Phillips (2005) have developed a modified version of the RESET test that has power against both nonlinear cointegration and the absence of cointegration.

4 Fractional ADF and Random Field Inference

As mentioned in Section 1, the first of the two recent tests whose usefulness in helping to explore PPP empirically is to be investigated is the FADF test introduced by Dolado, et al. (2002). This is a simple-to-implement parametric test that should be attractive to practitioners. The second is based on the random field regression approach to nonlinearity introduced by Hamilton (2001) and Dahl and González-Rivera (2003). The various methods of handling this approach are more complex than FADF testing, but they are attractive because, unlike STAR models, they do not rely on any specific nonlinear functional form being specified prior to estimation.

4.1 The FADF test

The Dolado, et al. (2002) approach to testing for fractionality is based on the distribution of the t-statistic on ϕ from the generalised ADF regression

$$\Delta^{d_0} y_t = \phi \Delta^{d_1} y_{t-1} + \sum_{i=1}^{p} \zeta_i y_{t-i} + \upsilon_t, \tag{14}$$

where υ_t is a hypothesised white noise error. For testing purposes, Dolado, et al. (2002) set d_0 equal to 1. The test of the null hypothesis $H_0\colon \phi = 0$ is then a test that the series $\{y_t\}_{t=0}^{\infty}$ is $I(1)$ against the alternative hypothesis that the series is $I(d_1)$. They showed that if $0.5 \le d_1 < 1.0$, the t-statistic for ϕ under H_0 follows an asymptotic normal distribution, while if $0 < d_1 < 0.5$, the t-statistic follows a nonstandard distribution of fractional Brownian motion. However, they also showed that in the practically realistic case in which d_1 is unknown, the t-statistic has an asymptotic normal distribution for $0 \le d_1 < 1$, provided that a $T^{-\frac{1}{2}}$-consistent estimator of d_1 is used.

4.2 Random field regression

The additive random field function used by Hamilton suggests that a simple method of testing for nonlinearity is to check if λ, or λ^2, is zero or not. Hamilton showed that if $\lambda^2 = 0$ and the nonlinear model is estimated for a fixed $\boldsymbol{g}$, the maximum likelihood estimate $\widetilde{\lambda}$ is consistent and asymptotically normal. Thus a test based on the use of the standard normal probability table is possible, though it is computationally complex for reasons discussed by Hamilton (2005) and Bond, et al. (2005a). Given the assumption of normality and the linearity of Equation (7) under the null hypothesis that $\lambda^2 = 0$, a simpler alternative uses the Lagrange multiplier principle. Hamilton showed that provided the covariance function of the random field can be derived, for a fixed $\boldsymbol{g}$ (Hamilton uses the mean of its prior distribution), this only requires a single linear regression to be estimated. Using a covariance function based on the L_2-norm, Hamilton (2001) derived the appropriate score vectors of first derivatives, for $k = 1, 2, .., 5$, and the associated information matrix, and proposed a form of the LM test for practical application. As the test statistic, λ_H^E, is distributed as χ_1^2 under the null hypothesis, linearity would be rejected if λ_H^E exceeded the critical value $\chi_{1,\alpha}^2$ for the chosen level of significance α.[3] For example, at the $\alpha = 5$ per cent level, the null hypothesis would be rejected if $\lambda_H^E > 3.84$.

The usefulness of the Hamilton LM test depends on a set of nuisance parameters that are only identified under the alternative hypothesis. As Hansen (1996) shows, dealing with unidentified nuisance parameters by assuming full knowledge of the parameterised stochastic process that determines the random field may have adverse effects on the power of the test. To take account of this, Dahl and González-Rivera (2003) introduce other LM tests that extend the Hamilton approach. The first, based on the statistic λ_{OP}^E, assumes, like Hamilton's test, knowledge of the covariance matrix, but its behaviour is based on the L_1-norm. The nuisance parameters are still present but now only enter the test in a linear fashion. The second, the λ_{OP}^A test, only assumes that the covariance function is smooth enough to be depicted by a Taylor expansion. The final test is a test of the null hypothesis $H_0 : \boldsymbol{g} = 0$; this $\boldsymbol{g}$-test makes no assumption about either the covariance function or λ. Dahl and González-Rivera (2003) show that in many circumstances, λ_{OP}^A and the $\boldsymbol{g}$-test have better power than other tests of nonlinearity.

The full importance of Hamilton's random field approach is only realised when the parameters λ and $\boldsymbol{g}$ are estimated. In particular, the estimated value of $\boldsymbol{g}$ can be used for inference on the form of the nonlinearity. A highly significant $g_i, i = 1, 2, ..., k$, suggests that the corresponding variable plays an important role in the nonlinearity of the model. Hamilton showed that estimating the unknown parameters $\boldsymbol{\varphi} = \{\alpha_0, \boldsymbol{\alpha}_1, \boldsymbol{g}, \sigma^2, \lambda\}$ can be reduced to maximum likelihood estimation of a reparameterisation of equations (7) and (8):

$$\eta\left(\mathbf{y}, \mathbf{X}; \boldsymbol{g}, \zeta\right) = -\frac{T}{2}\ln(2\pi) - \frac{T}{2}\ln\sigma^2\left(\boldsymbol{g}, \zeta\right) - \frac{1}{2}\ln\left|\mathbf{W}\left(\mathbf{X}; \boldsymbol{g}, \zeta\right)\right| - \frac{T}{2}, \quad (15)$$

and

$$\widetilde{\boldsymbol{\beta}}\left(\boldsymbol{g}, \zeta\right) = \left[\mathbf{X}'\mathbf{W}\left(\mathbf{X}; \boldsymbol{g}, \zeta\right)^{-1}\mathbf{X}\right]^{-1}\left[\mathbf{X}'\mathbf{W}\left(\mathbf{X}; \boldsymbol{g}, \zeta\right)^{-1}\mathbf{y}\right], \quad (16)$$

$$\widetilde{\sigma}^2\left(\boldsymbol{g}, \zeta\right) = \frac{1}{T}\left[\mathbf{y} - \mathbf{X}\widetilde{\boldsymbol{\beta}}\left(\boldsymbol{g}; \zeta\right)\right]'\mathbf{W}\left(\mathbf{X}; \boldsymbol{g}; \zeta\right)^{-1}\left[\mathbf{y} - \mathbf{X}\widetilde{\boldsymbol{\beta}}\left(\boldsymbol{g}; \zeta\right)\right], \quad (17)$$

where $\zeta = \frac{\lambda}{\sigma}$ and $\mathbf{W}\left(\mathbf{X}; \mathbf{g}, \zeta\right) = \zeta^2\mathbf{H}_k + \sigma^2\mathbf{I}_T$. The profile likelihood can be maximised with respect to $(\boldsymbol{g}, \zeta)$ using standard optimisation algorithms, though as Bond, et al. (2005a) point out, care needs to be taken because of computational difficulties. Also, as Hamilton (2005) explains, other computational issues make it is possible for the nonlinearity tests based on λ to be strongly significant but the results of the nonlinear maximisation of the likelihood function to suggest that ζ is insignificant. Once estimates for $\boldsymbol{g}$ and ζ have been obtained equations (16) and (17) can be used to obtain estimates of $\boldsymbol{\beta}$ and σ.

[3] The notation used here for the λ statistic is that of Dahl and González-Rivera (2003). The superscript E shows that full knowledge of the parametric nature of the covariance function is assumed. The alternative is superscript A, which signals that no assumption about the covariance function is assumed. The subscript H shows that the Hessian of the information matrix is used. The alternative is subscript OP, which indicates that the outer product of the score function is used.

5 Methodology

To investigate the usefulness of the FADF test and the Hamilton random field approach in exploring and understanding the issues surrounding PPP, this paper applies the techniques to Irish data. The data used are for Ireland and Germany and Ireland and the United Kingdom. In both cases, the observations are quarterly and run from the first quarter of 1975 to the third quarter of 2003, inclusive, giving a sample size of 115 observations. The specification for the explanatory model used is taken from Wright (1994), namely,

$$s_t = \alpha_0 + \alpha_1 p_t + \alpha_2 p_t^* + \alpha_3 i_t + \alpha_4 i_t^* + \epsilon_t, \tag{18}$$

where, in addition to the variables defined in Section 2, i_t and i_t^* are the domestic and foreign interest rates. The real exchange rate series, $\{q_t\}_{t=1}^{T}$, is constructed using Equation (1).[4]

To place the long memory and random field analysis into context, the standard $I(1)/I(0)$ analysis using the ADF unit root test is conducted. The strategy of Dolado, et al. (1990), to determine whether the ADF regressions have significant constants or trends, is adopted. The lag length for the ADF test is determined using the modified Akaike information criterion (MAIC), which Ng and Perron (2001) showed to be a generally better decision criteria, as it takes account of the persistence found in many series. The alternative KPSS and NG unit root tests are also applied, the latter being generally more powerful against the alternative of fractional integration than the standard ADF; see Kwiatkowski, et al. (1992) and Perron and Ng (1996), respectively. These procedures are implemented using the `Eviews` package.

Following on from this traditional analysis, the issue of fractional integration is investigated. Two approaches to applying the FADF test have emerged in the literature. The first, stemming from Hansen (1999), is to run the FADF regression for various values of $d \in [0, 1)$ and either tabulate or plot the test statistic results before making any inferences; see Heravi and Patterson (2005). The second, suggested by Dolado, et al. (2002), is to obtain a consistent parametric estimate of d and apply the FADF test for this value. It is this second approach that is adopted here. The 'over differenced' *ARFIMA* model, which uses the first differences of the observations on a variable rather than the raw levels observations themselves, is estimated to avoid the problems associated with drift, as recommended by Smith, et al. (1997). Two parametric estimates of d are calculated using the Doornik and Ooms (1999) `ARFIMA` package, namely, the exact maximum likelihood (EML) estimate produced by the algorithm suggested by Sowell (1992),[5] and an approximate maximum likelihood estimator based on the conditional sum of squared naïve residuals, developed by Beran (1995) and referred to by Doornik and Ooms (1999) as a nonlinear least squares (NLS) estimator. The nonparametric estimate of d from the log-periodogram method of Geweke and Porter-Hudak (1983) (GPH) and the semiparametric estimate from the

[4] The short-term interest rates were obtained from `EcoWin`; the remainder of the series were provided by Jonathan H. Wright.

[5] The Sowell algorithm requires that $d < 0.5$, which is another reason for using the 'over-differenced' model.

Gaussian method (GSP) discussed by Robinson and Henry (1998) are also available in `ARFIMA`; these are also calculated. The estimates of d are then used in the FADF test, with the MAIC being used to set the lag length for the test.

Traditional cointegration analysis is then applied to the simple PPP model of Equation (2). Firstly, the Engle and Granger (1987) two-step procedure is used, with the lagged residuals from the levels regression serving as the error correction term. Then the Johansen (1988) *VAR* approach is applied to the data. The effect of applying the Johansen (2002) small sample correction factor is also investigated. The `Eviews` package is used for the Engle-Granger and Johansen analysis, with `RATS` being employed for the calculation of the Johansen correction factor, using Johansen's program.

The analysis then turns to an examination of the possibility of nonlinearity in the data. For the causal models, the standard RESET test is applied, together with the random field based tests described above. Also, for an autoregressive model involving q_t, the now standard STAR tests for nonlinearity are applied. These tests derive from the model

$$q_t = \beta_0 + \sum_{j=1}^{3} \beta_j \ \widetilde{z}_{tj} \ \tau_t^j + u_t^*, \quad t = 1, 2, ..., T, \tag{19}$$

where τ_t is the t^{th} observation on the transition variable, $\widetilde{z}_{tj}, t = 1, 2, 3,$ is the t^{th} observation on the j^{th} explanatory variable, which in the simple autoregressive case is just the j-period lagged value of q_t, and u_t^* is an $iid(0, \sigma^2)$ disturbance. The lag length for the STAR tests is decided by reference to both the Akaike information criterion (AIC) and the Schwarz information criterion (SIC) .

The four standard tests have the following null hypotheses:

$$\begin{aligned} H_0 &: \beta_1 = \beta_2 = \beta_3 = 0 \\ H_{04} &: \beta_3 = 0, \\ H_{03} &: \beta_2 = 0 | \beta_3 = 0, \\ H_{02} &: \beta_1 = 0 | \beta_2 = \beta_3 = 0. \end{aligned}$$

If H_{03} yields the strongest rejection, the LSTAR or ESTAR model is selected. If one of the other hypotheses yields the strongest rejection the LSTAR2 model is used. The STAR analysis is conducted using the `JMulTi` package of Lütkepohl and Krätzig (2004), available at http://www.jmulti.de/.

Finally, the parameters of the random field model are estimated. The random field analysis is carried out using the `Gauss` code provided by Hamilton (2001) at http://weber.ucsd.edu/~jhamilto/. This code includes the Dahl and González-Rivera (2003) tests;[6] it was adapted so as to apply the algorithm switching approach to the numerical optimisation suggested by Bond, et al. (2005a). Specifically, switching between the *Steepest Descent* and *Newton* algorithms was employed. Hamilton's (2001) covariance specification was retained and an initial value of $\zeta = 0.5$ was used.

[6] Code for the Dahl and González-Rivera (2003) tests is also available from Dahl's webpage, namely, http://www.krannert.purdue.edu/faculty/dahlc/.

6 Results

6.1 Univariate analysis

The results of the basic unit root analysis are given in Table 1.[7] In half of the cases, the Dolado, et al. (1990) testing strategy suggested that the existence of a trend in the ADF test regressions, or drift in the series in question, can not be rejected; the associated probabilities given in Table 1 are therefore from the standard normal distribution. In the other half of the cases, the existence of a constant and trend is rejected so the probabilities given are from MacKinnon (1996).

These results generally seem to suggest that most series are $I(1)$. The performance of the KPSS test, which has as it null hypothesis that the series is stationary, is strange for the Ireland-United Kingdom data as the test does not reject this null in three of the six cases. Also, it is interesting that the traditional ADF test rejects the unit root hypothesis for one of the real exchange rates, whereas the 'more powerful' NP test fails to reject for both series.

Table 2 gives the results of the simple fractional integration analysis. For each series, four different estimates of d are given, together with their estimated standard errors and associated FADF test statistic values, where computed. The FADF test is only meaningful, and hence reported, if $d \leqslant 1$, when the probabilities to be applied to the test statistics are the standard normal ones. The results are interesting and would seem to imply that the only series that is likely to be unambiguously fractionally integrated is Irish interest rates. While all the estimates of d for the nominal exchange rate between Ireland and the United Kingdom are less than one, the FADF test fails to reject the null hypothesis of a unit root. For all other series, the estimates of d gave conflicting values, although the suggestion is of a unit root in the Ireland-United Kingdom real exchange rate. The FADF test only gave strong evidence of fractional integration in the case of the Ireland-Germany nominal and real exchange rates when the GPH and GSP estimates of d are used.

6.2 Cointegration analysis

The results of applying the standard Engle-Granger analysis in the context of explanatory model (18) are given in tables 3 and 4. Table 3 reports the findings of the levels analysis and in all cases both the traditional ADF test on residuals (augmented Engle-Granger test) and the Ng-Perron test fail to reject the null hypothesis that the residuals have a unit root. The KPSS test also rejects the null of stationary residuals in all but one case. Therefore, treating the variables as $I(1)$, it seems that cointegration of the nominal exchange rate, price levels and interest rates is overwhelmingly rejected for both the Ireland-United Kingdom and the Ireland-Germany data.

Table 4 gives the results of trying to estimate parsimonious error correction models, using the first lag of the residuals from the corresponding

[7] All tables are in the Appendix.

levels model as the error correction term in each of the two cases. While the coefficients of the error correction terms have the 'right' sign, the t-ratios are small in absolute value, confirming the conclusion about the lack of cointegration. Dropping the insignificant constant terms has a minimal effect on the results.

Table 5 summarises the Johansen analysis of the data, while more detailed results are given in tables 6, 7, 8 and 9. Table 6 shows evidence of one cointegrating vector in the Ireland-Germany case, when interest rates are excluded from the equation. Importantly, this result is overturned by the trace test when Johansen's small sample correction to that test is applied. However, when interest rates are included, one cointegrating vector is suggested whether or not the small sample correction is used, as shown in Table 7. In this case, the trace and maximal eigenvalue tests concur. Tables 8 and 9 present the results for the Ireland-United Kingdom relationship. As with the previous case, the finding of one cointegrating vector in the specification without interest rates is overturned by the adjusted trace test. In contrast, two vectors are suggested when the interest rates are included, and this result is unaffected by the small sample correction factor, which strangely is less than 1.

Taken together, the results so far are rather mixed and indicate that there is little evidence of cointegration in a traditional PPP setting, but that the introduction of interest rates appears to be significant. Overall, as in previous studies, this attempt to place the PPP analysis of Irish data in a cointegrating framework is not entirely satisfactory. We therefore turn to the results from the alternative nonlinear methodologies.

6.3 Nonlinearity tests

Tables 10 and 11 give the results of the various nonlinearity tests. In all tests, the null hypothesis is that the model/series is linear. For the RESET test, both the F and LR variants are given. For the STAR nonlinearity test, an F-test version is used, with F being the test statistic for H_0 and $F4$, $F3$ and $F2$ being, respectively, the test statistics for the hypotheses H_{04}, H_{03} and H_{02}, specified in Section 5. The AIC suggested a lag length of three for the STAR test in the case of the Ireland-Germany exchange rate and a lag length of two for the Ireland-United Kingdom case. The SIC suggested a lag length of one in both cases.

As can be seen from Table 10, the RESET test and the four random field based tests emphatically reject linearity at the 5 per cent significance level in the case of the Ireland-Germany model. For the Ireland-United Kingdom model, however, there is a marked contrast between the findings from the two test approaches, with the RESET test failing to reject linearity but all of the random field tests strongly rejecting it.

Table 11 contains similar, though opposite findings. The RESET test, STAR tests and random field based tests all suggest that the assumption of linearity is adequate for the Ireland-United Kingdom real exchange rate taken on its own; but whereas the random field tests overwhelmingly support linearity of the Ireland-Germany real exchange rate, the STAR test based on the use of three lags gives some indications of nonlinearity and the RESET test rejects linearity very strongly. It is difficult to explain these conflicting outcomes in tables 10 and 11, especially in the absence of information on the relative power of the different types of test.[8]

6.4 Random field estimation

Given that the bulk of the results in Table 10 suggest that the linear equation used in the analysis of PPP is not an appropriate specification, interest focuses on the results of the nonlinear estimation of the random field regression. These are given in Table 12. Convergence was achieved after 36 iterations in the case of both variants of the Ireland-Germany model, and after 42 and 19 iterations in the case of the basic and interest rate augmented Ireland-United Kingdom equations, respectively. Interestingly, in the case of both country pairings, the standard model and the augmented model exhibit nonlinearity with respect to the two price variables, the price coefficients in the nonlinear component of the models being highly significant. However, in the augmented Ireland-Germany model, the German interest rate is nonlinearly significant, while in the Ireland-United Kingdom model it is the Irish interest rate that appears to have a significantly nonlinear influence on the nominal exchange rate. Graphical inspection of cross-plots of the data suggests that a number of regime shifts may be responsible for these findings, though the choice of appropriate specifications and modelling strategies remains problematical, particularly in the Ireland-United Kingdom case. The data do not suggest an obvious approach, nor is there a theoretical framework within which to work.

Most strikingly, perhaps, is the fact that when nonlinearity is modelled by means of a random field, the coefficients on the domestic and foreign prices in the specifications with and without interest rates, are not statistically significantly different from their -1 and 1 values under purchasing power parity theory. This finding contrasts with the findings in the earlier Irish studies by, for example, Thom (1989) and Wright (1994), both of whom report cointegrating vectors, corresponding to the vector of variables s_t, p_t and p_t^*, that are markedly different from (1, -1, 1).

7 Conclusions

This paper has explored the well-known concept of purchasing power parity between Ireland and Germany and Ireland and the United Kingdom, using a number of recent econometric methods concerning fractional integration, smooth transition autoregression, and random field regression. The theoretical background to purchasing power parity has been sketched, as has the particular approach to fractionality offered by the fractional augmented Dickey-Fuller test of Dolado, et al. (2002) and the approach to nonlinear

[8] In particular, no results appear to be available on the power of the RESET test relative to random field based LM tests for nonlinearity. This is a subject of ongoing research and the findings will be presented in a forthcoming paper.

inference suggested by Hamilton (2001). The findings reported have illustrated the potential difficulties inherent in placing the study of purchasing power parity in the $I(1)/I(0)$ econometric framework, difficulties that were implicit in the very mixed results of several of the earlier studies of Irish purchasing power parity that employed the Engle-Granger and Johansen cointegration approaches.

As mentioned in the earlier work, the difficulties might relate to the low power of unit root tests; see Wright (1994, p. 275). We have suggested they might also relate to fractional integration of the processes generating the series used. However, our results have shown that, in the cases examined, this possibility is unlikely and that difficulties can not be overcome solely by moving to a fractional integration framework.

Another possibility is that the processes in question may be stationary but parametrically unstable or nonlinear. As is well known, in such a situation, standard unit root tests are not likely to reject the null hypothesis of a unit root and cointegration analysis may be adopted mistakenly. It is interesting to note that Thom (1989, p. 162) reported some evidence of parameter instability and that Lane and Milesi-Ferretti (2002) chose to view the Irish long-run real exchange rate as time varying; but neither of these studies attempted to grapple with this problem in the PPP framework. Our results provide further strong evidence of nonlinearity. Moreover, if the nonlinearity is modelled using a random field regression, they show, importantly, that the Irish experience vis-à-vis Germany and the United Kingdom accords well with purchasing power parity theory.

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A Appendix

A.1 Tables

Table 1: Unit Root Tests

Variables	ADF	P-value	No. of Lags	KPSS†	NP†
		Ireland & Germany			
Nominal exchange rate	-1.119	0.266	7	Yes	No
Irish price level	-2.155	0.034	4	Yes	No
German price level	-1.933	0.056	2	Yes	No
Irish interest rate	-1.085	0.250‡	2	Yes*	No*
German interest rate	-0.936	0.309‡	1	Yes	No
Real Exchange Rate	-3.543	0.00	2	Yes	No
		Ireland & United Kingdom			
Nominal exchange rate	-1.221	0.203‡	0	No	No
Irish price level	-2.155	0.034	4	Yes	No
UK price level	-1.722	0.088	8	Yes	No
Irish interest rate	-1.085	0.250‡	2	Yes*	No*
UK interest rate	-0.645	0.436‡	10	No	No
Real Exchange Rate	-1.103	0.24‡	2	No	No

†Yes - significant at 5 per cent level. No - not significant at 5 per cent level.
‡Trend and constant not included. MacKinnon (1996) p-values used.
* Not significant at 1 per cent level.

Table 2: Fractional Integration Analysis

Variables	EML	NLS	GPH	GSP
			FADF	
Common Series				
Irish Price Level	1.46 (0.04)	1.50 (0.07)	1.01 (0.11)	0.89 (0.07)
	-	-	-	4.5
Irish Interest Rates	0.79 (0.10)	0.78 (0.10)	0.97 (0.10)	0.80 (0.06)
	-3.22	-3.21	-3.35	-3.23
Ireland & Germany				
Nominal Exchange Rate	1.49 (0.14)	1.89 (0.10)	0.94 (0.11)	0.82 (0.07)
	-	-	-5.48	-5.51
German Price Level	1.46 (0.05)	1.57 (0.09)	1.02 (0.11)	0.92 (0.07)
	-	-	-	2.89
German Interest Rates	0.69 (0.24)	0.65† (0.23)	1.12 (0.11)	1.03 (0.07)
	-1.49	-1.48	-	-
Real Exchange Rate	1.41 (0.08)	1.48 (0.08)	0.98 (0.11)	0.85 (0.07)
	-	-	-5.05	-5.12
Ireland & United Kingdom				
Nominal Exchange Rate	0.95 (0.09)	0.95 (0.09)	0.88 (0.11)	0.91 (0.07)
	-1.60	-1.60	-1.608	-1.60
UK Price Level	1.48 (0.02)	1.55 (0.06)	0.99 (0.11)	0.87 (0.07)
	-	-	5.03	4.69
UK Interest Rates	1.07 (0.09)	1.08 (0.10)	1.00 (0.11)	0.94 (0.07)
	-	-	-	-2.53
Real Exchange Rate	1.07 (0.09)	1.08 (0.09)	1.15 (0.11)	0.97 (0.07)
	-	-	-	-1.09

†Trend and constant not included. McKinnon (1996) p-values used.
- Indicates FADF test not applicable.
Note: standard errors in parentheses.

Table 3: $I(1)/I(0)$ Levels Regression Analysis

Variables	Ireland & Germany		Ireland & United Kingdom	
Constant	2.854 (0.549)	1.804 (0.575)	0.859 (0.108)	0.833 (0.108)
Price Levels				
Irish	−0.568 (0.083)	−0.672 (0.081)	−0.875 (0.111)	−1.029 (0.123)
Foreign	0.007 (0.200)	0.329 (0.203)	0.670 (0.095)	0.825 (0.110)
Interest Rates				
Irish		0.005 (0.002)		0.007 (0.003)
Foreign		0.002 (0.003)		−0.003 (0.003)
Augmented Engle-Granger (critical value)	−2.475 (−3.817)	−2.835 (−4.5398)	−2.653 (−3.8172)	−2.728 (−4.540)
Ng-Perron†	No	No	No	No
KPSS†	No	Yes‡	Yes‡	Yes‡

†Yes - significant at 5 per cent level. No - not significant at 5 per cent level.
‡Significant at 5 per cent level but not the 1 per cent level.
Note: standard errors in parentheses.

Table 4: Error Correction Analysis

Variables	Ireland & Germany		Ireland & United Kingdom	
Constant	−0.004 (−0.003)	−0.004 (−0.003)	0.004 (0.005)	0.001 (0.004)
Δ Price Levels				
Irish	−0.686 (0.157)	−0.667 (0.164)	−1.105 (0.282)	−1.020 (0.284)
Foreign	1.021 (0.428)	0.927 (0.502)	0.831 (0.361)	0.715 (0.357)
Δ Interest Rates				
Irish		0.0004 (0.001)		0.005 (0.001)
Foreign		0.001 (0.004)		0.00006 (0.003)
ECM	−0.108 (0.039)	−0.107 (0.040)	−0.133 (0.049)	−0.124 (0.052)

Note: standard errors in parentheses.

Table 5: Johansen's Cointegration Tests Summary

Test Type	no inpts no trends	rest'd inpts no trends	unrest'd inpts no trends	unrest'd inpts rest'd trends	unrest'd inpts unrest'd trends
			Ireland & Germany excluding interest rates		
Trace	1	1	1	0	0
Max-Eig	1	1	1	0	0
			Ireland & Germany including interest rates		
Trace	2	2	2	1	1
Max-Eig	2	2	1	1	1
			Ireland & United Kingdom excluding interest rates		
Trace	1	1	1	1	1
Max-Eig	1	1	1	1	0
			Ireland & United Kingdom including interest rates		
Trace	2	2	2	2	3
Max-Eig	0	1	1	2	1

Note: 0.05 per cent critical values based on Osterwald-Lenum (1992).

Table 6: Johansen Results for IRELAND & GERMANY excluding Interest Rates

Cointegration Rank Test (Trace)†					
Hypotheses		Trace Statistic	0.05 Critical Value	0.10 Critical Value	Modified 0.05 Critical Value
$r = 0$	$r \geq 1$	39.203	34.870	31.930	45.680
$r \leq 1$	$r \geq 2$	13.347	20.180	17.880	-
$r \leq 2$	$r = 3$	5.903	9.160	7.530	-

Cointegration Rank Test (Maximum Eigenvalue)†				
Hypotheses		Maximum Eigenvalue Statistic	0.05 Critical Value	0.10 Critical Value
$r = 0$	$r = 1$	25.856	22.040	19.860
$r \leq 1$	$r = 2$	7.444	15.870	13.810
$r \leq 2$	$r = 3$	5.903	9.160	7.530

†Cointegration with restricted intercepts and no trends in the *VAR*.
Note: The correction factor is 1.310.

Table 7: Johansen Results for IRELAND & GERMANY including Interest Rates

Cointegration Rank Test (Trace)†					
Hypotheses		Trace Statistic	0.05 Critical Value	0.10 Critical Value	Modified 0.05 Critical Value
$r = 0$	$r \geq 1$	111.587	87.170	82.880	98.328
$r \leq 1$	$r \geq 2$	57.298	63.000	59.160	-
$r \leq 2$	$r \geq 3$	31.448	42.340	39.340	-
$r \leq 3$	$r \geq 4$	15.809	25.770	23.080	-
$r \leq 4$	$r = 5$	6.057	12.390	10.550	-

Cointegration Rank Test (Maximum Eigenvalue)†				
Hypotheses		Maximum Eigenvalue Statistic	0.05 Critical Value	0.10 Critical Value
$r = 0$	$r = 1$	54.290	37.860	35.040
$r \leq 1$	$r = 2$	25.850	31.790	29.130
$r \leq 2$	$r = 3$	15.639	25.420	23.100
$r \leq 3$	$r = 4$	9.751	19.220	17.180
$r \leq 4$	$r = 5$	6.057	12.390	10.550

†Cointegration with unrestricted intercepts and restricted trends in the *VAR*.
Note: The correction factor is 1.128.

Table 8: Johansen Results for Ireland & UK excluding Interest Rates

Cointegration Rank Test (Trace)†					
Hypotheses		Trace Statistic	0.05 Critical Value	0.10 Critical Value	Modified 0.05 Critical Value
$r = 0$	$r \geq 1$	57.532	42.340	39.340	70.030
$r \leq 1$	$r \geq 2$	21.695	25.770	23.080	-
$r \leq 2$	$r = 3$	4.788	12.390	10.550	-

Cointegration Rank Test (Maximum Eigenvalue)†				
Hypotheses		Maximum Eigenvalue Statistic	0.05 Critical Value	0.10 Critical Value
$r = 0$	$r = 1$	35.838	25.420	23.100
$r \leq 1$	$r = 2$	16.907	19.220	17.180
$r \leq 2$	$r = 3$	4.788	12.390	10.550

†Cointegration with unrestricted intercepts and restricted trends in the *VAR*.
Note: The correction factor is 1.654.

Table 9: Johansen Results for IRELAND & UK including Interest Rates

Cointegration Rank Test (Trace)†					
Hypotheses		Trace Statistic	0.05 Critical Value	0.10 Critical Value	Modified 0.05 Critical Value
$r = 0$	$r \geq 1$	127.997	87.170	82.880	85.427
$r \leq 1$	$r \geq 2$	77.194	63.000	59.160	61.740
$r \leq 2$	$r \geq 3$	41.665	42.340	39.340	41.493
$r \leq 3$	$r \geq 4$	21.103	25.770	23.080	-
$r \leq 4$	$r = 5$	4.707	12.390	10.550	-

Cointegration Rank Test (Maximum Eigenvalue)†				
Hypotheses		Maximum Eigenvalue Statistic	0.05 Critical Value	0.10 Critical Value
$r = 0$	$r = 1$	50.803	37.860	35.040
$r \leq 1$	$r = 2$	35.530	31.790	29.130
$r \leq 2$	$r = 3$	20.562	25.420	23.100
$r \leq 3$	$r = 4$	16.395	19.220	17.180
$r \leq 4$	$r = 5$	4.707	12.390	10.550

†Cointegration with unrestricted intercepts and restricted trends in the *VAR*.
Note: The correction factor is 0.980.

Table 10: Nonlinearity Tests - Causal Models

Test	Test Statistic	P-value	Bootstrap p-value	Test Statistic	P-value	Bootstrap p-value
	Ireland & Germany			Ireland & United Kingdom		
Reset						
			excluding interest rates			
F	35.04	0.000		0.948	0.431	
LR	77.646	0.000		3.969	0.414	
			including interest rates			
F	24.474	0.000		0.882	0.477	
LR	60.085	0.000		3.765	0.439	
Random Field						
			excluding interest rates			
Hamilton	575.388	0.000	0.001	648.928	0.000	0.001
Lamba A	324.321	0.000	0.001	151.160	0.000	0.001
Lamba E	233.907	0.000	0.001	233.152	0.000	0.001
$\boldsymbol{g}$-test	11.380	0.044	0.001	104.661	0.000	0.001
			including interest rates			
Hamilton	179.66	0.000	0.001	205.475	0.000	0.001
Lamba A	224.382	0.000	0.001	545.731	0.000	0.001
Lamba E	180.758	0.000	0.001	161.323	0.000	0.001
$\boldsymbol{g}$-test	156.695	0.000	0.001	211.304	0.000	0.001

Table 11: Nonlinearity Tests - Real Exchange Rates

Test	Test Statistic	P-value	Bootstrap p-value	Test Statistic	P-value	Bootstrap p-value
	Ireland & Germany			Ireland & United Kingdom		
Reset						
F	8.136	0.000		1.043	0.376	
LR	23.606	0.000		3.969	0.349	
STR			lag length 1			
F		0.236			0.576	
$F4$		0.379			0.952	
$F3$		0.121			0.169	
$F2$		0.303			0.764	
		lag length 3			lag length 2	
F		0.010			0.207	
$F4$		0.054			0.108	
$F3$		0.010			0.236	
$F2$		0.039			0.591	
Random Field						
Hamilton	2.410	0.121	0.058	0.187	0.665	0.653
Lamba A	4.481	0.923	0.369	6.721	0.751	0.394
Lamba E	0.035	0.852	0.922	1.056	0.304	0.562
$\boldsymbol{g}$-test	4.551	0.871	0.367	2.847	0.970	0.458

Table 12: Hamilton analysis - Ireland, Germany, and UK

	Ireland & Germany			Ireland & United Kingdom	
	Estimates				
Linear					
c	0.332 (1.488)	0.769 (1.121)	c	1.176 (0.751)	0.907 (0.213)
p_t^{Ire}	-0.896 (0.191)	-0.836 (0.152)	p_t^{Ire}	-1.439 (0.308)	-1.093 (0.239)
p_t^{Ger}	0.892 (0.502)	0.724 (0.390)	p_t^{UK}	1.164 (0.320)	0.882 (0.218)
i_t^{Ire}		-0.0004 (0.002)	i_t^{Ire}		0.009 (0.004)
i_t^{Ger}		0.007 (0.005)	i_t^{UK}		-0.009 (0.004)
Nonlinear					
σ	0.019 (0.002)	0.010 (0.004)	σ	0.021 (0.003)	0.009 (0.004)
ζ	3.987 (0.817)	5.859 (2.551)	ζ	9.572 (2.109)	8.148 (4.368)
p_t^{Ire}	4.265 (0.375)	4.609 (1.103)	p_t^{Ire}	0.480 (0.116)	2.777 (1.214)
p_t^{Ger}	11.068 (0.733)	16.971 (3.021)	p_t^{UK}	-1.864 (0.044)	10.454 (1.846)
i_t^{Ire}		-0.032 (0.023)	i_t^{Ire}		0.118 (0.039)
i_t^{Ger}		-0.146 (0.052)	i_t^{UK}		$-2.26E-7$ (0.040)

Note: standard errors in parentheses.