Monetary Theory of Inflation and the LBD in Transactions Technology.

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Abstract

Classical models of inflation, utilising the transactions-based demand for money, predict that monetary policy will be ineffective in changing real variables. In response to this, the New Keynesian sticky-price models assume price-rigidity in order to address the possibility for the existence of real effects of monetary policy. At the same time, both major theories have difficulty in explaining persistency in the money demand of households in the absence of uncertainty. We develop a flexible price model with endogenous transactions-costs driven demand for money that captures the possibility for real effects of monetary policy and accounts for the persistency of money demand. In our model, persistency is derived from transactions technology that assumes the existence of learning-by-doing effects in shopping costs. We proceed to compare the model with the standard monetary model of inflation.

JEL Classification:

Key Words: Inflation, Money Demand, Learning-by-Doing, Transactions Technology, Seigniorage.

Introduction.

Classical models of inflation in the presence of endogenous money demand traditionally assume that money enters the household problem via a cash-in-advance constraint. In this case, fiat money may serve all, or any, of the following three functions. Money balances can be used as a unit of accounting, can be held as a store of value, and finally, money can facilitate transactions in the markets for consumption. The latter function of fiat money is characterised by specifying the technology for transactions that usually (see Sargent and Ljungqvist (2000) for comprehensive discussion) relates time spent shopping to real balances held by the households relative to their consumption needs. In such case, as shown in Sargent and Ljungqvist (2000), money demand depends negatively on interest rate.

Another important feature of the classical models of money demand, is that monetary policy has no real effects in the absence of price frictions. This is known as the quantity theory of money. An expansionary monetary policy that acts to increase money supply is fully absorbed into the price level fluctuations. This, in turn, is contrasted by the New Keynesian models of inflation where due to nominal rigidities in prices and/or wages monetary expansion can have a direct effect on consumption and output.

So far, to the best of my knowledge, there is little theoretical work available that would permit a standard classical model to account for persistency of money demand and inflation observed in the data. This current model intends to fill this gap.

A broader problem of both the New Keynesian and the classical models of inflation relates to the responsiveness of the demand for money to changes in the real interest rates. In a comprehensive analysis of the money demand in Japan and the US, Grivoyannis (1991) observes the empirical facts concerning the demand for real balances, shown below. In general, modern literature on demand for money supports these findings. We discuss these results as stylised facts in the context of our model.

- In the short-run, the sluggish adjustments of money demand in response to changes in the real interest rates cannot be explained by the more volatile underlying fundamentals, such as output, wealth, consumption and inflation. In terms of our model, this implies that we are warranted at modelling the persistence in money demand as a function of the behaviourally stationary parameters (the learning-by-doing efficiency parameter, a, as specified below, provides one of such possibilities).
- Portfolio adjustment costs fail to account for the long-run sluggish adjustments in money demand across the two regimes of high and low transactions costs and across countries. This implies that the markets efficiency in the money market is not sufficient in explaining persistency of the demand for real balances. Thus, we need to look into the household behaviour or consumption markets for a possible explanation.
- Lagged real balances explain on average 95% of the variation in the money demand both within each country and within each period. This result is robust across various specifications of the money demand. This goes to the heart of our argument for, as shown below, persistency in money demand can be modelled on the basis of transaction costs technology.
- The short-term elasticity of money demand is statistically not significant in all countries and across all adjustment costs regimes. As shown in our model below, the presence of the learning-by-doing (LBD) effects of past balances helps explain the lower elasticity of money demand.
- The residuals from structural models are not correlated across countries, which in relation to our model implies that learning-by-doing effects may be country-specific. This is further supported by the fact that many studies (see Grivoyannis (1991) for examples) find that the variation in money demand not captured by the model of underlying fundamentals is country-

specific. In addition, the level of adjustment costs was found to be important to US households, while being statistically insignificant to Japanese households.

- The rate of change in the level of transactions was not an important determinant of the demand for money in the case of the US and Japanese households. In the presence of the LBD effect, changes in transactions levels will enter determination of money demand through the lagged real balances demand. This implies that the effects of the actual transactions levels on demand for money will be distributed across two parameters: the coefficient on the levels of transactions and the coefficient on the lagged demand for money. Grivoyannis (1991) confirms this possibility in his tests.
- The level of adjustment costs was important to US households, while being statistically insignificant to Japanese households. In context of the country-specific LBD effects, this can be supported by the assumption of the diminishing returns to learning. Specifically, in the subsample for the years 1950-1973, Japan had less developed retail networks than the US. As a result, efficiency improvements to the Japanese retail transactions technology may have dominated the adjustment costs effect over the sample period. As the Japanese networks converged in efficiency toward those found in the USA, the later subsample shows that in both countries, the level of transactions became less important.
- Inflation fails to account for the variation in money demand in all cases, with the exception of demand deposits by the firms in Japan. This means that household balances in both countries are not sensitive to the inflation rate. The result applies in both, the high inflation environment of the 1974-1983 period, and the lower inflation environment prior to 1974. Once again, these results are broadly consistent with our findings that the presence of the LBD effects in transactions technology acts to reduce the money demand elasticity with respect to the real rates of return.

Auray and Feve (2002) provide a discussion of the empirical findings in the testing of the classical and new Keynesian models of inflation in relation of the models to the Taylor rules for monetary policy. They find that the estimated flexible price models are closer to the empirical data than the sticky price models in matching the Taylor rule estimates from the actual data. In their discussion they identify three main stylised facts. Following the contraction in money supply, (a) there is a persistent decline in real GDP; (b) prices almost do not respond to changes in money supply in the short run; (c) nominal interest rate rises. Points (a) and (b) are known in the literature as the monetary transmission mechanism, while (a) and (c) define the liquidity effect.

These stylised facts combine to provide the necessary requirement for any structural model of money demand vis-à-vis monetary policy analysis. In the standard flexible price models, following a monetary expansion, the households attempt to reduce the negative effects of an inflation tax by switching away from consumption in favour of leisure. As a result, the output falls. As households postpone consumption, savings, and the nominal interest rate rise. These effects are discussed in Lucas and Stokey (1983) and other works. Since these dynamics preclude any possibility for either a monetary transmission mechanism or the liquidity effect, monetary theory developed two main responses to the flexible price models shortcomings. The first approach was to introduce price stickiness. In the models with rigid prices, following monetary expansion, output rises, while prices are fixed in the short run. This allows for capturing the transmission mechanism, but not the liquidity effect. The second approach was to assume limited participation in the financial markets, which allowed for the liquidity effect.

As Auray and Feve (2002) show, flexible price models do better in estimating the relationship between the nominal interest rate, the expected inflation and the output gap. Hence, we are warranted in looking into the possibility for extending the flexible price model to account for both the liquidity effect and the monetary transmission mechanism. As shown below, our model allows for precisely this possibility.

Amplifying the Auray and Feve (2002) results, J. Linde (2003) shows that in the case of the utility function being additively-separable in leisure and consumption, the flexible price models yield the super-neutrality result for changes in money supply. Any temporary shock to the growth rate in money supply will have no transitive effects on economic dynamics. Furthermore, for permanent changes in the money growth rate, the steady state quantities of the real variables will be unaffected by changes in the money supply. Again, these results are shown not to hold in the case of the model specification provided below that allows for real effects of monetary policy in the presence of the LBD effects even in the case of additively-separable preferences.

In Part 1, we take a standard classical model of money demand and introduce the possibility of endogenous persistency in the demand for the real balances. Due to the presence of the LBD effects in transactions technology, agents demand for money in each period is determined, in part, by the history of transactions carried out in the past. As a result, within any given period, the real balances held by the agents are less responsive to the changes in the real return to money, given by the inverse of the inflation rate. Thus, lower real rates of return are capable of generating a higher demand for money, given by the inverse of the inflation rate. In the presence of learning effects, an increase in the money supply has a contemporaneous effect of raising consumption. In addition, an increase in the real rate of return to money relative to the rate of return on bonds is shown to have an indeterminate effect on the overall demand for money. Finally, in the case of a specific form assumption for utility function and transactions technology we show that stationary seigniorage costs of money creation are uniformly lower in the presence of the LBD effects, than in standard models, whenever the real rate of return to money balances is below 1. This implies that in the short run, the cost of saturating the economy with money balances overshoots its long-run level. At the same time, the long-run equilibrium cost of the

Friedman rule is below the equilibrium cost in the standard models. Furthermore, the model implies that the overall costs of inflation will be lower in the presence of the LBD effect.

Following the general discussion in Part 1, we simplify the model to a two-period model. We first proceed to show that the results obtained in the more general setting in Part 1 continue to hold in a more stylised setting. We also show that unlike in the traditional model, the intertemporal effects of the money supply expansion in period 1 are not absorbed into higher prices alone. Instead, the LBD effect has real implications for consumption in periods 1 and 2.

Part 1. A General Model of Inflation in the Presence of LBD in Transactions Technology.

Let $\{c_t, g_t\}_{t=0}^{\infty}$ be consumption and government expenditure streams. Then within the period exogenous income constraint is given by:

$$y = c_t + g_t \tag{1}$$

By (1), y is exogenous constant income per period in terms of the real goods that can be divided between consumption and government expenditure.

Preferences.

Households take income as exogenously given and maximise

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \tag{2}$$

by choosing the stream of consumption and leisure $\{c_t, l_t\}_{t=0}^{\infty} \subset \Re_+^2$ so that both consumption and leisure are restricted to be non-negative. As standard, we assume that in (2) the instantaneous utility function is well behaved, i.e.

$U_c, U_l > 0$, $U_{cc}, U_{ll} < 0$, $U_{cl} \geq 0$

Transactions Technology.

Let s_t denote time spent on conducting transactions in the market for consumption goods. Likewise, let z_t be the time spent transacting in the market for government goods. Assume that the government goods are distributed centrally, so that the transaction time allocated to shopping for publicly provided goods is exogenous to the agent's choice. We can think of z_t as the time spent by the voters in political voting and/or public education, as well as time allocated to charitable activities, participation in the productive social networks, such as food banks, adult education, etc. It can also refer to the more direct costs of public consumption, such as time required to arrive to the state-owned museum or theatre, queuing time, or waiting lists, in the state-run hospital, and so forth.

Private transactions are governed by the following technology:

$$s_t = H\left(c_t, \frac{m_{t+1}}{p_t}, \frac{m_t}{p_t}\right) \tag{3}$$

where we assume that:

- $H_1 \geq 0$, so that a higher volume of consumption requires more shopping time;
- $H_{11} \leq 0$, so that the marginal cost of consumption in terms of time spent on transacting in the private markets is a non-increasing function of the volume of private consumption;

- $H_2 \leq 0$, which implies that a higher holding of cash on hands carried over to the next period will economise on the transactions time in private markets in the present period (note: this corresponds to the timing assumption found in the standard monetary models of inflation);
- $H_{22} \geq 0$: as cash on hands to be carried over into the next period increases, the marginal benefits of the cash balances in terms of time savings in the present period will decline;
- $H_{21} \leq 0$, so that an increase in private consumption today, *ceteris paribus,* will lower the marginal benefits of the real balances to be carried over to the next period. This is equivalent to the statement, found in the standard models, that the current consumption costs dominate the effect of the future period real balances;
- $H_3 \leq 0$, which is equivalent to saying that the current real spending on private consumption reduces the time-cost of consumption. It is assumed hereinafter, that $H_1 > \text{mod}\, H_3$: the current period consumption dominates the savings of time arising from the current period real balances;
- $H_{33} \geq 0$, so that the transactions technology exhibits the diminishing returns to savings arising from the current period real balances held by the agents;
- $H_{31} \leq 0$, implying that consumption costs in the current period dominate the current period real balances effect on the time savings;
- $H_{32} \geq 0$, so that the marginal benefit of learning from using the real balances today are non-increasing in the real balances to be carried over into the next period.

As an example of the specific form for the transactions technology, we can assume that

$$H\left(c_t, \frac{m_{t+1}}{p_t}, \frac{m_t}{p_t}\right) = \frac{\varepsilon_t}{\dfrac{m_{t+1}}{p_t} + a\dfrac{m_t}{p_t}} c_t \tag{4}$$

Equation (4) follows from a direct extension of the Sargent and Ljungqvist (2000) specification to incorporate the LBD effect of the past real balances. Here ε_t denotes the time cost of each trip to the bank in the private transactions markets. We define, as in Sargent and Ljungqvist (2000), the number of trips to the bank as

$$\frac{p_t c_t}{m_{t+1} + a m_t} \tag{5}$$

where if $a=0$ we attain the specification given in the Sargent and Ljungqvist (2000) model. For any $a \in \Re_{++}$, the current period real expenditure acts to reduce the future time of transactions in the private markets, holding consumption constant. This is precisely the LBD effect of the real balances.

Proposition 1. For any $a>0$, the current period real balances act to reduce the required number of trips to the bank relative to the benchmark model.

Proof: from the definition of the number of trips given by (5) above.

Overall our specification is consistent, in terms of the presence of the contemporaneous demand for money in (4), with the Baumol-Tobin class of the transactions demand models of money. In these models, inflation effects the demand for money only through its effects on the interest rates. This is confirmed below in equilibrium equation (21.a). However, in addition, our model accounts for the Friedman effect as well. Friedman (1956) argued that consumption and leisure may act as substitutes for money. In this case, the expected inflation may result in a shift away from money and in favour of physical assets. In the absence of changes in the physical asset holdings, this implies that there is a possibility for a trade-off between the real variables of choice and the money demand in response to inflation. In so far as the real variables are captured by the within-period current real expenditure, m_t / p_t, our model supports Friedman's assertion (see equation (21)). The results of part 2 confirm this in a more explicit specification and in greater detail.

Household Time Constraint.

Households are endowed with a unit of time which they can allocate to leisure, conducting private transactions, or to the consumption of public goods, so that

$$l_t + s_t + z_t = 1 \tag{6}$$

In what follows, we assume that $z_t = z$. However, it may be of interest to consider the possibility of $z_t = z(\tau_t)$, where $z(\tau_t)$ is a decreasing function of taxes, τ_t. This assumption will allow us to extend the model to include the interaction between the transactions costs in the public markets and the costs of transacting in private markets. However, for brevity reasons, we shall leave this possibility outside the scope of the present paper.

Household Budget Constraint.

The within-period budget constraint specifies that the household total resources equal the total expenditure within each period. The total resources of the household's consist of the exogenous income endowment, y, net of taxes, τ_t, plus the real balances and the real bond holdings carried over from the previous period, $\frac{m_t}{p_t} + b_t$. These can be spent on consumption today, c_t, or saved for the next period in the form of cash on hands, m_{t+1}, or bonds, b_{t+1}. Thus,

$$y - \tau_t + b_t + \frac{m_t}{p_t} = c_t + \frac{b_{t+1}}{R_t} + \frac{m_{t+1}}{p_t} \tag{7}$$

Optimisation Program.

Households choose the streams of private consumption, leisure, bonds and money balances, $\{c_t, l_t, b_{t+1}, m_{t+1}\}_{t=0}^{\infty}$ so as to maximise their life-time utility (2) subject to the budget constraint (6) and the time endowment constraint (5) with the transactions technology given by (3). Thus

$$\{c_t, l_t, b_{t+1}, m_{t+1}\}_{t=0}^{\infty} = \arg\max \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} U(c_t, l_t) + \lambda_t \left[y_t - \tau_t + b_t + \dfrac{m_t}{p_t} - c_t - \dfrac{b_{t+1}}{R_t} - \dfrac{m_{t+1}}{p_t} \right] + \\ + \eta_t \left[1 - l_l - z_t - H\left(c_t, \dfrac{m_{t+1}}{p_t}, \dfrac{m_t}{p_t} \right) \right] \end{array} \right\} \quad (8)$$

The first order conditions for optimisation with respect to c_t, l_t, b_{t+1}, and m_{t+1} are:

$$U_{1,t} = \lambda_t + \eta_t H_{1,t} \quad (9)$$

$$U_{2,t} = \eta_t \quad (10)$$

$$\lambda_{t+1} = \frac{\lambda_t}{\beta R_t} \quad (11)$$

$$\beta \frac{\lambda_{t+1}}{p_{t+1}} - \frac{\lambda_t}{p_t} = \frac{H_{2,t}}{p_t} \eta_t + \underbrace{\beta \frac{H_{3,t+1}}{p_{t+1}} \eta_{t+1}}_{LBD\ effect} \quad (12)$$

Equation (10) links the marginal value of leisure in current period to the contemporaneous consumption and, via the time constraint (5), to the time costs of private transactions, s_t, and the public consumption time requirements, z_t. Since, by (3) and (4), time required for private consumption is dependent on current and past real balances, the link between money demand in the past, through LBD effect, is extended to the marginal utility of leisure as well. This is important in the context of our discussion in Part 3 below.

By (9) and (10),

$$\lambda_t = U_{1,t} - U_{2,t}H_{1,t} \tag{13}$$

Equation (13) implies that the shadow value of income equals the marginal utility of consumption within the period, less the marginal utility costs of private transactions. This result confirms Sargent and Ljungqvist (2000).

Also, by (13),

$$\frac{d\lambda_t}{d(m_t / p_t)} = \frac{d\lambda_t^{sar}}{d(m_t / p_t)} - \underbrace{U_{2,t}H_{13,t}}_{(-)\ LBD\ effect} > \frac{d\lambda_t^{sar}}{d(m_t / p_t)}$$

Then in the presence of the LBD effect in private transactions yields the following results:

(i) The shadow value of income is increasing in the LBD effect, since an increase in income results in the higher within period real balances held by the agents. As in Sargent and Ljungqvist (2000), this generates greater savings of time within the period. However, in addition to this direct effect, the higher current real balances yield future utility gains due to the LBD effect.

(ii) Holding leisure demand and consumption constant within the current period, the shadow value of income in the presence of the LBD effect will be increasing in the current period real balances.

The second point (ii) is worth considering more closely. Suppose the endowed income increases. This generates two sources of marginal utility gains. First, as income rises consumption will tend to increase, and the future period real balances will follow. This is the direct Sargent and Ljungqvist (2000) effect. However, as income rises, the LBD effect implies that the future consumption and leisure demand will increase as well. This can be better seen in the following interpretation of the preceding equation. By (13),

$$\frac{d\lambda_t}{d\left(\frac{m_{t+1}}{p_t}\right)} = \underbrace{-U_{2,t}H_{12,t}}_{(+)\ \text{Sargent Effect}} \underbrace{-U_{2,t}H_{13,t}d\left(\frac{m_t}{m_{t+1}}\right)}_{(-)\ \text{LBD Effect}}$$

The second term in this equation restates point (ii) made above. The left-hand side of the above equation is the effect of the monetary policy shock on the marginal utility of income.

Assuming that money supply grows at some gross rate $1+\mu_{t+1}=\frac{M_{t+1}}{M_t}$, for a representative agent, we can re-write the above equation as:

$$\frac{d\lambda_t}{d\left(\frac{m_{t+1}}{p_t}\right)} = \underbrace{-U_{2,t}H_{12,t}}_{(+)\text{ Sargent Effect}} + \underbrace{U_{2,t}H_{13,t}\left(1+\mu_{t+1}\right)^{-2}d\mu_{t+1}}_{(-)\text{ LBD Effect}}$$

Clearly, if the money supply follows a constant growth path, so that $\mu_t=\mu$ for all *t*, the marginal utility of income depends on the real money balances to the extent determined by Sargent and Ljungqvist (2000) effect. Thus, once the economy reaches the steady state, money demand behaves in a standard fashion and the LBD effect no longer applies. However, the level of the steady sate demand will be determined by a combination of both effects.

By (11) and (13), we define the real rate of return to bonds as

$$R_t = \frac{U_{1,t}-U_{2,t}H_{1,t}}{\beta\left[U_{1,t+1}-U_{2,t+1}H_{1,t+1}\right]} \tag{14}$$

In general, by (14), holding the marginal utility of income constant, the LBD in transactions has an indeterminate effect on the real rate of return. However, whenever $\beta R_t>1$, greater efficiency of transactions learning technology will have unambiguous effect of raising the real rate of return to money relative to the rate of return to bonds.

Proposition 2. If $\beta R_t>1$, so that holding the real balances into the next period is costly, the permanently higher efficiency in the LBD in transactions ($a\uparrow$ permanently), will be associated with a lower real rate of return.

Proof. See Appendix 1 below.

Intuitively, the LBD in transactions acts to increase the attractiveness of holding real balances over and above the standard rate of time savings. Thus, if bond markets are to compete for the households funds, the real rate of return to bonds must compensate the households for the losses of savings in time. These losses arise as the households are trading away from holding the real balances and in favour of bonds. However for $R_{m,t} = p_t / p_{t+1}$, in the absence of inflation (i.e. in the stationary long run equilibrium), $\beta R_t > 1$ implies that the real bonds dominate the real balances in the present value terms. Thus, in order to maintain the present consumption, in response to the rise in the efficiency of the LBD technology, the interest rate must fall. This will stimulate households investment in the transactions learning.

On the other hand, for $\beta R_t < 1$ the real bonds are dominated in return by rising efficiency of the LBD in transactions. In order to attract households into bond markets, the real rate of return must compensate them for the aforementioned trade-off. The real interest rate will rise in this case in response to an increase in the LBD efficiency.

Both results can be seen clearly from taking a derivative of (14) with respect to *a*:

$$\frac{dR_t}{da} = \frac{-U_{2,t}\lambda_{t+1}\frac{dH_{1t}}{da} + U_{2,t+1}\lambda_t \frac{dH_{1,t+1}}{da}}{\beta\left[U_{1,t+1} - U_{2,t+1}H_{1,t+1}\right]^2} >,< 0$$

if and only if

$$\frac{U_{2,t}}{U_{2,t+1}}\frac{dH_{1,t}}{dH_{1,t+1}} >,< R_t\beta$$

Which under the assumption of the constant marginal utility of income holds if and only if

$\beta R_t <,> 1$

These effects are more transparent in the case of the one-time increase in the efficiency of transactions. From above, holding both the marginal utility of income, and the marginal cost of consumption in the future periods constant ($dH_{1,t+1} = 0$), a one time increase in *a* in the current period will unambiguously result in an increase in the real rate of return.

In contrast to the real rate of return on bonds, we define the real return to the money balances as:

$R_{m,t} = p_t / p_{t+1}$

Then by (11) and (12) above,

$$\lambda_t \left[\frac{R_{m,t}}{R_t} - 1 \right] = \eta_t H_{2,t} + \eta_{t+1} R_{m,t} \beta H_{3,t+1} \qquad (15)$$

The no Ponzi game condition requires that $R_{m,t} < R_t$. Then the RHS of (15) can be decomposed into two parts. The first part, $\eta_t H_{2,t} < 0$ is the real marginal value of time spent in the private transactions markets in terms of foregone immediate consumption. This is the standard component of the monetary models of inflation. The second element, $\eta_{t+1} R_{m,t} \beta H_{3,t+1}$ is the discounted real return to the LBD.

Substitute (10) and (13) into (15) and divide through by $U_{2,t}$:

$$\left[1 - \frac{R_{m,t}}{R_t} \right] \left(H_{1,t} - \frac{U_{1,t}}{U_{2,t}} \right) = H_{2,t} + \underbrace{R_{m,t} \beta H_{3,t+1} \frac{U_{2,t+1}}{U_{2,t}}}_{(-)\text{ LBD effect}} \qquad (16)$$

As can be clearly seen from (16), the LBD technology in transactions has a negative effect on the $R_t / R_{m,t}$ by increasing the attractiveness of real balance holdings. The LBD effect here is captured by the real time savings in period *t+1* achieved by learning from period *t* cash balances.

These savings are expressed in terms of the marginal utility gains in leisure time freed up by lower transactions costs in period *t+1* relative to period *t*.

Proposition 3. In the presence of the LBD technology in transactions, an increase in the real money supply will be associated with an increase in consumption today. Relative to the standard models, $\frac{dc_t}{d(m_{t+1}/p_t)} <,> \frac{dc_t^{sar}}{d(m_{t+1}/p_t)}$ **if and only if** $H_{33} <,> \operatorname{mod} H_3$**.**

Proof: see Appendix 1.

In the specific case of the transactions technology given by (4),

$$\frac{dc_t}{d(m_{t+1}/p_t)} < \frac{dc_t^{sar}}{d(m_{t+1}/p_t)}.$$

Similarly, the effect of the relative real rate of return to money, $R_{m,t}/R_t$ on future money balances carried over by the households will depend on the size of the first order effect of the LBD technology relative to the second order effect, $\operatorname{mod} H_{33,t}/H_{3,t}$.

Proposition 4. An increase in the real rate of return to money balances relative to the real rate of return to bonds will have a positive (negative) effect on the money balances carried over by the households into the next period whenever $1 <,> -\frac{H_{33,t+1}}{H_{3,t+1}} \frac{m_{t+1}}{p_t} R_{m,t}$**.**

Proof: follows total differentiation of (16), as shown in the Appendix 1.

Note that in Proposition 4 we omit the possibility that for a large value of parameter *a*, there exists a possibility for the degenerate case of $\frac{d(m_{t+1}/p_t)}{d(R_{m,t}/R_t)}<0$.

In the special case of (4): $0<\frac{d(m_{t+1}/p_t)}{d(R_{m,t}/R_t)}<\frac{d(m_{t+1}^{Sar}/p_t)}{d(R_{m,t}/R_t)}$.

In general, we can write:

$$d(m_{t+1}/p_t)=\underbrace{\frac{A_2}{A_1}}_{(+)}d(R_{m.t}/R_t)-\underbrace{\frac{A_3}{A_1}}_{(+)}dc_t+\underbrace{\frac{A_4}{A_1}}_{(+)}dR^{-1}{}_{m.t}+\underbrace{\frac{A_5}{A_1}}_{(-)}d(m_t/p_t)-$$
$$-\underbrace{\frac{A_6}{A_1}}_{(+)}dc^E{}_{t+1}-\underbrace{\frac{A_7}{A_1}}_{(-)}d(m^E{}_{t+2}/p^E{}_{t+1})$$

Where

$$A_1=\underbrace{\frac{U_{2,t+1}}{U_{2,t}}\beta H_{33,t+1}}_{(+)\text{ LBD Effect}}+\underbrace{H_{22,t}-H_{12,t}}_{(+)\text{ Sargent}}>A_1^{Sar}>0$$

$$A_2=\underbrace{\frac{U_{1,t}}{U_{2,t}}d\left(\frac{R_{m,t}}{R_t}\right)}_{(+)\text{ Sargent}}-\underbrace{\frac{U_{2,t+1}}{U_{2,t}}\beta H_{3,t+1}}_{(-)\text{ LBD Effect}}dR_{m,t}>A_2^{Sar}>0$$

$$A_3^{Sar}<A_3=\overbrace{\left(1-\frac{R_{m,t}}{R_t}\right)\underbrace{\frac{U_{11,t}}{U_{2,t}}}_{(-)}+\underbrace{\frac{U_{12,t}U_{1,t}}{U_{2,t}}}_{(+)}+\underbrace{H_{11,t}}_{(+)}+\underbrace{H_{12,t}}_{(-)}}^{(-),\text{ Sargent}}-\beta R_{m,t}\underbrace{H_{3,t}\frac{U_{2,t+1}}{U_{2,t}}}_{(-)\text{ LBD Effect}}<0$$

$$A_4=\frac{U_{2,t+1}}{U_{2,t}}\beta R_{m,t}H_{33,t+1}\frac{m_{t+1}}{p_t}>A_4^{Sar}=0$$

$$A_5=H_{13,t}-H_{23,t}<A_5^{Sar}=0$$

$$A_6=\left(\frac{U_{21,t}}{U_{2,t}}+\frac{U_{2,t+1}}{U_{2,t}}\right)\beta R_{m,t}H_{3,t+1}<0=A_6^{Sar}$$

$$A_7 = \frac{U_{2,t+1}}{U_{2,t}} \beta R_{m,t} H_{32,t+1} > 0 = A_7^{Sar}$$

This allows us to re-write the demand for real balances to be carried over into the next period as as a function of four variables:

$$\frac{m_{t+1}}{p_t} = F\left(\underbrace{c_t}_{\substack{(+),\text{ higher} \\ \text{than Sargent}}}, \underbrace{c_{t+1}^E}_{\substack{(+),\text{ zero} \\ \text{in Sargent}}}, \underbrace{\frac{R_{m,t}}{R_t}}_{\substack{(+),\text{ lower} \\ \text{than Sargent}}}, \underbrace{\frac{m_t}{p_t}}_{\substack{(-),\text{ zero} \\ \text{in Sargent}}} \right) \tag{17}$$

In equation (17):

- The first term captures the dependency between the current period consumption and the next period real balances. A higher level of private consumption today, holding constant the money balances available today (since these are determined in previous period) implies higher spending out of bonds. Thus bonds holdings fall. In addition, higher spending today implies anticipated higher spending in the next period, so that the households will increase their real balances to be carried over into the next period. However, since higher current spending implies the LBD effect that acts to increase future period consumption, the households will demand higher real balances in the next period. Thus our effect, while preserving the general sign, is amplified relative to the standard models.
- The second term deals with the effect of the anticipated private consumption in the next period on the demand for real balances to be carried over into tomorrow from today. In Sargent and Ljungqvist (2000), this term is zero since the households preferences for money are fully separable across time. In our model, due to the LBD effects, households must anticipate the future consumption-driven demand for money. As a result, when households anticipate higher consumption in the future, they will increase their money balances. This is the second order, or indirect, effect of the LBD on money demand.

- The third term accounts for the responsiveness of the real balances to be carried over into the future period to a change in the real rates of return. As in the standard models, the general direction of this relationship implies that an increase in the real rate of return to money relative to the bonds, will result in higher rate of money balance holdings to be carried over into the next period. However, in our model, the LBD effects make the next period money demand less responsive to changes in the real rates of return. This is warranted intuitively by the fact that LBD raises the internal rate of return to money above the standard model $R_{m,t}$.
- The final term in (17) captures the direct effect of the LBD on money demand. If the current period real expenditure out of money balances, m_t / p_t, were to increase, the LBD effect of such spending will tend to reduce future demand for money balances, when c_{t+1} is held constant. This follows directly from the effect of the current period real balances on the time cost of private transactions in the following period, i.e. from the LBD effect of current spending. Holding consumption in the next period fixed, learning gains from a higher spending out of money balances today will mean that the shopping time required to support the fixed level of expected future consumption will fall. The result is to lower the households demand for money balances to be carried over.

In what follows we shall assume that the direct effect of real balances on savings of time in private transactions market is stronger than the LBD effect, so that

$$\operatorname{mod} H_{2,t} > \operatorname{mod} H_{3,t}$$

Under this assumption, Proposition 1 implies that, holding other variables fixed, an increase in the technological efficiency of the LBD mechanism, or in terms of (4), an increase in *a*, will result in a lower demand for real balances to be carried over into the future, m_{t+1} / p_t.

Government Budget Constraint.

$$g_t = \tau_t + \frac{B_{t+1}}{R_t} - B_t + \frac{M_{t+1} - M_t}{P_t} \tag{18}$$

Note, that in the context of exogenously determined time costs of public transactions, we can allow for a link between z_t and τ_t, as was mentioned earlier. For example, $z(\tau_t)$ such that $z'(.) < 0$ can correspond to the case where a higher level of taxation is related to a higher efficiency of public spending infrastructure, which in turn acts to lower the time costs of public consumption. Both z_t and τ_t are exogenous to the household optimisation and thus the results attained so far will continue to hold.

Equilibrium.

Definition 1. For exogenously given $\{z_t, g_t, \tau_t\}_{t=0}^{\infty}$, and for the given levels of $B_0 = b_0$ and $M_0 = m_0$, an equilibrium is a price system $\{R_t, p_t\}_{t=0}^{\infty}$, private consumption sequence $\{c_t\}_{t=0}^{\infty}$ together with a sequence of government bonds and money supply $\{B_{t+1}, M_{t+1}\}_{t=0}^{\infty}$ such that the following are true:

i) given $\bar{g}$, $\bar{\tau}$ and $\bar{z}$, households optimisation yields $b_t = B_t$ and $m_t = M_t$;

ii) government budget constraint (18) is satisfied for all $t \geq 0$;

iii) $c_t + g_t = y_t$.

Note that if public consumption time is linked to the level of taxation, as mentioned above, $\bar{z}$ is no longer pre-determined, but instead is chosen by the households in response to an exogenously given level of $\bar{\tau}$. In such a case, the forth condition that defines equilibrium must be added to ensure that the choices of public consumption and private transactions time satisfy the time endowment constraint (6) for a given level of taxes.

To distinguish between the short-run (*SR*) and the long-run (*LR*) variables, we define:

$$\begin{aligned} &SR: \quad \{\tau_0 \neq \tau, B_0 \neq B\} \\ &LR: \quad \{g_t = g, B_t = B, \tau_t = \tau\}, \quad \forall t \geq 0 \end{aligned} \tag{19}$$

Thus, the second set in (19) describes the conditions for the long run equilibrium. The system attains the long run equilibrium whenever:

$$\begin{aligned} &R_{m,t} \equiv \frac{P_t}{P_{t+1}} = R_m \quad \forall t \geq 0 \\ &R_t = R \quad \forall t \geq 0 \\ &c_t = c \quad \forall t \geq 0 \\ &s_t = s \quad \forall t \geq 0 \end{aligned} \tag{20}$$

Substitute (20) into (17) and (14) above to get:

$$\begin{aligned} &R\beta = 1 \\ &\frac{m_{t+1}}{p_t} = F\left(c, R_m / R, m / p\right) = f\left(R_m, m_t\right) \end{aligned} \tag{21}$$

Equation (21) describes the stationary equilibrium in the money markets.

By (21) and (17) it follows that

$$0 < f_1 < f_1^{Sar}$$

$$f_2 < 0 = f_2^{Sar}$$

$$f_a < 0 = f_a^{Sar}$$

The last two inequalities arise solely due to the effects of the LBD on money demand.

Using the equilibrium condition (i) in Definition 1 above, we can re-write (21) as:

$$\beta R = 1$$

$$\frac{m_{t+1}}{p_t} = f(R_m) \tag{21.A}$$

Where, $0 < f' < f'_{Sar}$.

Note that (21.A) assumes that $\mathrm{mod}\, f_1 > \mathrm{mod}\, f_2$, so that the direct effect of R_m on money demand is stronger than the effect of LBD.

Equation (21.A) implies that the presence of the LBD effects in transactions technology acts to alleviate the standard problem of capturing the liquidity effect found in the traditional flexible-price models. The positive correlation between the real return to money holdings and the demand for real balances is reduced, while the liquidity effect vis-à-vis the nominal interest rate is fully consistent in our model with the stylised facts discussed in the Introduction.

Substitution of (19), (20) and (21.A) into the government budget constraint (18) yields

$$g - \tau + \frac{rB}{R} = r_m f(R_m) \tag{22}$$

where $r = R - 1$ and $r_m = R_m - 1$.

In (22), due to the presence of LBD, in the long run stationary equilibrium:

- the direct effect of LBD: $f(r_m) < f_{Sar}(r_m)$ for all r_m;
- the indirect effect of LBD: $f'(r_m) < f'_{Sar}(r_m)$
- the efficiency-in-technology effect: $f_a(r_m) < 0 = f_{a,Sar}(r_m)$.

The same holds for the specific form of technology assumed in (4). Appendix 1 establishes the details of this result.

At the initial date,

$$\frac{M_0}{p_0} = f(r_m) - (g + B_0 - \tau_0) + \frac{B}{R} \tag{23}$$

Restrict (23) so that M_0 / p_0 in our case is the same as in Sargent and Ljungqvist (2000). Clearly, in the presence of the LBD mechanism,

$$f(r_m) = \left.\frac{M_1}{p_0}\right|_{our} < f_{Sar}(r_m) = \left.\frac{M_1}{p_0}\right|_{\text{Sargent}}.$$

This occurs due to an anticipated effect of the LBD mechanism on time-efficiency of the future periods private transactions. Hence, in the initial state,

$$\left[f(r_m) - f_{Sar}(r_m)\right] = \left.\frac{B}{R}\right|_{\text{Sargent}} - \left.\frac{B}{R}\right|_{\text{our}} < 0.$$

Controlling for government expenditure, initial outstanding volume of real bonds, initial tax revenues and the equilibrium real interest rate,

$$B > B_{\text{Sargent}} \tag{24}$$

Hence, in our model, the steady state savings rate will be higher than in the standard flexible-price models. This result is driven by the fact that in the presence of the LBD effects, savings from higher efficiency in technology generated by the learning effects of past consumption are absorbed into both higher consumption and higher real wealth in the future.

Specific Solution.

Assume that the CRRA instantaneous utility function is additively separable in consumption and leisure components:

$$U(c_t, l_t) = \frac{c_t^{1-\delta}}{1-\delta} + \frac{l_t^{1-\alpha}}{1-\alpha} \tag{25}$$

while the transactions technology is given by (4) under the assumption that $\varepsilon = 1$. Then using (20) and (16) in (21) we attain:

$$\left(1 - \frac{R_m}{R}\right)\left[\frac{1}{f(r_m)(1 + aR_m)} - \frac{c^{-\delta}}{l^{-\alpha}}\right] = -\frac{(1 + a\beta R_m)}{f(r_m)^2(1 + aR_m)^2} c \tag{26}$$

We want to determine the relationship:

$$g(r_m) = f(r_m)(1 - R_m) \tag{27}$$

which captures the stationary seigniorage, and the rate of return to real balances, R_m.

Rewriting (26)

$$\left(\frac{R - R_m}{R}\right)\left[\frac{1 - R_m}{g(R_m)(1 + aR_m)} - \frac{c^{-\delta}}{l^{-\alpha}}\right] = -\frac{(1 - R_m)^2(1 + a\beta R_m)}{g(R_m)^2(1 + aR_m)^2} c \tag{28}$$

In (28) we get the relationship that implicitly defines $g(r_m)$ in (27).

As shown in the Appendix 1,

- $g(R_m) <,> g_{Sar}(R_m)$, whenever $R_m <,> 1$;

- The horizontal intercept for (27) is $R_m^0 = 1$, same as in the standard model;

- The vertical intercept for (27) is $\left[\frac{R - R_m}{R}\right]\left[\frac{1}{f(0)} - \frac{c^{-\delta}}{l^{-\alpha}}\right] = -\frac{c}{f^2(0)}$, which is independent of *a* and equals to the vertical intercept for the standard flexible-price model;

- The function extremum is to the left of that in the standard model.

- As $a \uparrow 1$, the extremum moves to the vertical intercept and the seigniorage function is monotone decreasing in R_m.

- As $a \downarrow 0$, the seigniorage function coincides with the standard model result shown in Sargent and Ljungqvist (2000).

The stationary seigniorage relationship is graphically shown in Figure 1 provided below following the Appendix 2.

As shown in Figure 1, for any $a \in (0,1]$ and for any $R_M < 1$, the LBD effect acts to decrease the seigniorage available from the monetary expansion. The reason for this is that in equilibrium, in the presence of the LBD effect, the households are less responsive to changes in the real rate of return to money balances. As the real return to money increases (either due to a fall in the inflation rate, or due to a monetary contraction), at the moment of impact, households are willing to hold more real balances. They do so in order to take advantage of the direct effect of rising real return to money, and of the LBD effect that reinforces it. However, over time as the gains from learning accumulate, for a given level of private consumption agents will demand less cash, since the LBD effect reduces their transactions costs. At the same time, for any $R_M < 1 < R_t$, bonds strictly dominate money as the store of value instrument. Hence in equilibrium, the money demand will be lower in the presence of the LBD effect. Likewise, in equilibrium, the LBD effect implies a lower responsiveness of the real balances to changes in the real interest rates.

However, for $1 < R_M < R$, the benefits of LBD reinforce the time savings from the use of real balances. This acts to increase households' demand for money over and above the standard model. Thus the seigniorage losses due to a monetary expansion are lower in the model with learning than in the Sargent and Ljungqvist (2000) standard flexible-price model.

From Figure 1, we determine the initial real value money balances as a function of the initial price level in the following way. As in Sargent and Ljungqvist (2000), the LHS of equation (22) gives the exogenously determined stationary gross of interest government deficit. Once the specific level of deficit is chosen, the RHS of (22) defines R_M through (27) and (28). At the initial date, in the absence of outstanding debt:

$$m_0 = f(r_M) - r_{M,0} f(r_{M,0}) + B \tag{29}$$

where $f(r_M) = M_1 / p_0$, while $r_{M,0} f(r_{M,0})$ is given by (22).

Thus, for any $R_M < 1$, the equilibrium price level will be lower in the model with learning in transactions. The opposite will hold in the case, where $1 < R_M < R$. The intuition follows the same lines as in the arguments presented above.

As in Sargent and Ljungqvist (2000) it is straightforward to show that the following classical results for monetary models of inflation continue to hold in the presence of the LBD effect:

- The unpleasant monetarist arithmetic of Sargent and Wallace (1981) continues to apply. The only difference in the presence of LBD arises due to the differences in the effects of bond emission on the RHS of (23). In the standard model, as *B* rises, RHS of (23) falls due to an indirect effect of rising R_M on the budget deficit. At the same time, RHS (23) rises due to the direct effect of an increase in *B*. In our model, the second effect is equivalent to Sargent and Ljungqvist (2000), while the first effect is smaller in magnitude due to the ameliorating effects of the LBD technology on the responsiveness of private money demand to changes in the R_M. Hence, in our model, the LBD mechanism strengthens the result shown in Sargent and Wallace's (1981).

- With respect to the 'Laffer-curve', the preferred real return to money balances in our model is lower than in the standard flexible-price models, so that $R^*_{M,L} < R^*_{M,Sar,L}$. Hence, an increase in the stationary gross of interest government budget deficit causes a decrease in the rate of return to money that is smaller in the case of the LBD effects model than in the standard model. Hence, sustained deficits will cause a lower increase in the inflation rate in our model than in the standard model. This is an attractive feature of the LBD model since it helps explaining a combination of sustained low inflation and aggressive monetary policy in the context of the US budget deficits of the past.

- The exchange rate indeterminacy result of Karaken and Wallace (1981) continues to hold in the presence of the LBD effect. However, in the presence of LBD effect, the exchange rate indeterminacy is amplified. In the specific case of (4), the exchange rate in period *t+1* is in part driven by the exchange rate in *t*, so that the price of foreign currency remains sticky. As the result of this, in contrast to the standard flexible price models, our model implies that the exchange rate is divorced from the underlying fundamentals for a longer period of time.

- Woodford's (1995) result holds in our model with timing modifications similar to those in the case of the exchange rate indeterminacy as well.

However, the presence of the LBD effect in transactions technology yields a different result with respect to the social value of real balances. In our model, the social value of real balances is higher due to the savings of time arising from the learning effect of current spending. Hence, in the presence of the LBD effect, the households continue to prefer higher rates of return to money, R_M, but are now willing to hold more real balances relative to Sargent and Ljungqvist (2000) at any given level of R_M along the adjustment path to the new steady state. From (15), the government can attain optimal allocation of money by setting $R_M = R$ as in Sargent and Ljungqvist (2000). However, to achieve the same level of money that saturates economy with real balances, $H_2 = 0$ for some $M_{t+1} / p_t \geq \omega(c)$ must hold. Since $H_{23} > 0$, then, in a stationary equilibrium, the saturation result requires that the real balances are lower in the presence of LBD, than in the standard flexible-price model:

$$m_{t+1} / p_t \big|^{*}_{LBD} < m_{t+1} / p_t \big|^{*}_{\text{Sargent}} . \tag{30}$$

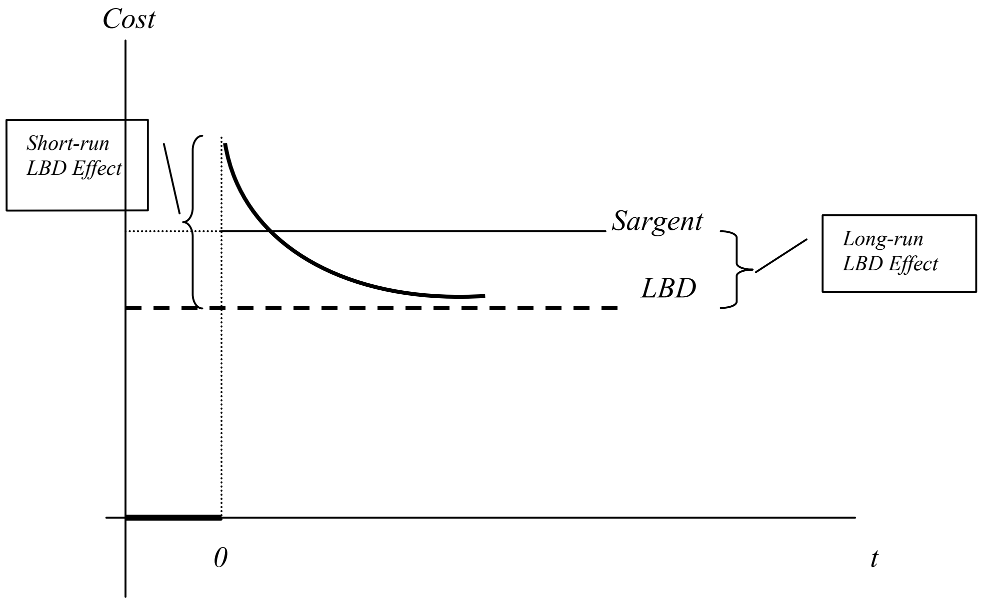
By (30), in the presence of LBD, a saturation policy will be less costly to the government.

Before reaching equilibrium, due to higher demand for the real balances by the households interested in deriving higher learning benefits from transactions LBD,

$$m_{t+1} / p_t \big|_{LBD,t} > m_{t+1} / p_t \big|_{\text{Sargent,t}} , \tag{31}$$

Thus, the cost of saturation result, along the adjustment path to the new stationary equilibrium, can be graphically depicted as shown in Figure 2 below.

Figure 2. Cost of Saturating with Real Balances (Friedman Rule).



Part 2. A Specific Model of Money Demand in the Presence of LBD in Transactions.

In the following we want to develop a simple, specific model of money demand that would illustrate the above results in a clearer functional form. With this in mind we consider a model, in which the households are living for two periods, and choosing consumption, C_t, and cash balances, M_t, for all $t=1,2,$ so as to maximise the life-time utility:

$$V = U(C_1, m_1) + \beta U(C_2, am_1, m_2) \tag{32}$$

where $m_t = M_t / P_t$ as consistent with Brock (1975), and $a \in [0,1]$ is the LBD efficiency parameter, as discussed in Part 1 above.

For simplicity, assume there are no bonds, no endogenous production and no economic growth. This assumption, as argued below, implies that the effects of monetary policy on consumption are not distorted by the simultaneous trade-off between consumption and the real bonds. Then for the two periods of life, the budget constraints faced by the households are:

$$P_1Y + M_0 + P_1T_1 = P_1C_1 + M_1 \quad (33)$$

$$P_2Y + M_1 + P_2T_2 = P_2C_2 + M_2 \quad (34)$$

Note that in the above the timing convention implied by (33)-(34) is slightly different from Model 1 above. Here, $m_t = M_t / P_t$ are the end of period *t* real balances to be carried over to period *t+1*, while T_t refers to the lump-sum currency transfers by the government, defined in (36) below.

Assume that the money supply follows a constant growth process:

$$\frac{M_t - M_{t-1}}{M_{t-1}} = \mu_t = \mu \quad (35)$$

Then the seigniorage revenue rebated by the government, in the absence of bonds, forms the lump-sum transfers, so that

$$P_tT_t = M_t - M_{t-1} \quad (36)$$

The first order conditions for maximising (32) subject to (33)-(34) are:

$$U_C^1(C_1, m_1) = \lambda_1 P_1 \quad (37.a)$$

$$\beta U_C^2(C_2, am_1, m_2) = \lambda_2 P_2 \quad (37.b)$$

$$U_{M_1}^1(C_1, m_1) + a\beta U_{M_1}^2(C_2, am_1, m_2) = P_1(\lambda_1 - \lambda_2) \quad (37.c)$$

$$\beta U_{M_2}^2(C_2, am_1, m_2) = \lambda_2 P_2 \quad (37.d)$$

By (37.a), (37.c) and (37.d):

$$U_{C_1}^1(C_1, m_1) = a\beta U_{M_1}^2(C_2, am_1, m_2) + \frac{P_1}{P_2}\beta U_{M_2}^2(C_2, am_1, m_2) + U_{M_1}^1(C_1, m_1) \quad (38)$$

The LHS of (38) is the marginal utility of consumption in period 1. The first term on the RHS of (38) is the marginal utility of holding 1 unit of currency in period 1 due to savings of transactions time in period 2, i.e. the LBD effect of money balances held in period 1. The second term is the marginal utility of a unit of currency held in period 2 that arises due to savings of time in transactions carried out in period 2, i.e. a standard effect of money balances held in period 2. Thus the second term can be viewed as the marginal utility of the money balances due to the store-of-value function of the fiat currency. Finally, the third term captures the direct effect of the money balances in period 1 on the marginal utility.

By (37.b) and (37.d):

$$U_C^2\left(C_2, am_1, m_2\right) = U_m^2\left(C_2, am_1, m_2\right) \tag{39}$$

Equation (39) arises, since neither LBD nor the store-of-value functions of the real balances apply in the last period of life. Thus, in the second period of life, the marginal utility of consumption equals to the marginal utility of the contemporaneous real expenditure.

In equilibrium, in the absence of storage technology, goods markets must clear in each period, so that

$$Y = C_1 = C_2 \tag{40}$$

Multiply (38) through by M_1 and apply (35) and (40) to get:

$$m_1\left\{U_C^1\left(Y, m_1\right) - U_M^1\left(Y, m_1\right) - a\beta U_M^2\left(Y, am_1, m_2\right)\right\} = \beta \frac{m_2}{\left(1+\mu\right)} U_C^2\left(Y, am_1, m_2\right) \tag{41}$$

Similarly, from (39):

$$U_C^2\left(Y, am_1, m_2\right) = U_M^2\left(Y, am_1, m_2\right) \tag{42}$$

Next we assume that V is additively separable, so that

$$\upsilon(C_t, am_{t-1}, m_t) = U(C_t) + V(am_{t-1}, m_t) \quad (43)$$

Then (41) and (42) can be re-written as:

$$m_1 \{U_Y^1(Y) - V_1^1(m_1) - a\beta V_1^2(am_1, m_2)\} = \beta \frac{m_2}{(1+\mu)} U_Y^2(Y) \quad (44)$$

$$U_Y^2(Y) = V_2^2(am_1, m_2) \quad (45)$$

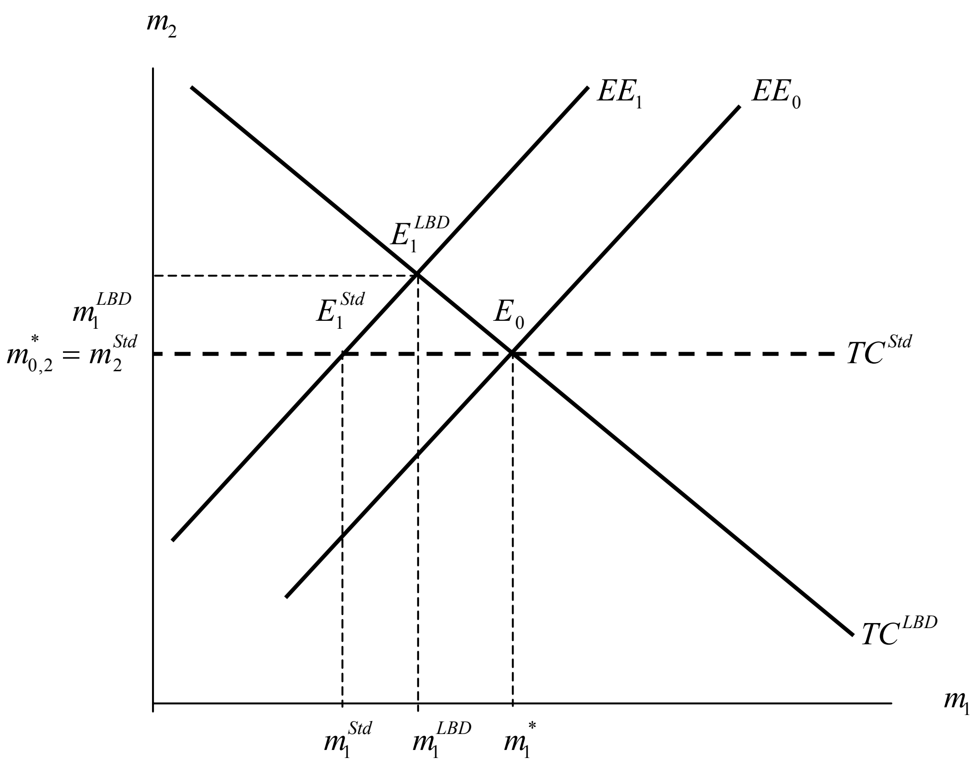
where, $V_j^i = \frac{\partial V^i(x_1, x_2, x_3)}{\partial x_j}$ is the partial derivative of the function in period *i,* with respect to its *j*-th argument.

In the absence of benevolence motives, agents will hold zero cash balances beyond period 2. Then, plotting (45) in the (m_1, m_2) space, we obtain, as shown in the Appendix 2, the *TC* curve in Figure 3 below, where *TC* stands for the Terminal Condition. Likewise, equation (44) in the (m_1, m_2) space yields *EE* curve, corresponding to the Dynamic (Euler equation-like) condition. Now, assuming that the LBD effect is smaller than the direct effect of the real balances on the marginal utility, Figure 3 follows.

As shown in Figure 3, an increase in the growth rate in money supply, μ will shift the *EE* curve up. The equilibrium, in the presence of LBD effects, is characterised by the first period demand for real balances in excess of that found in the standard flexible-price model. It is also important that the new equilibrium will be associated with a higher demand for the second period balances than in the standard model. These results are driven by the real effects of the money supply expansion, as outlined below.

As the growth rate of money supply increases, the equilibrium shifts from point E_0 to E_1^{LBD} in the presence of LBD effects. In the absence of LBD effects, standard model equilibrium moves to E_1^{Std} which corresponds to a lower demand for money in both periods 1 and 2. In standard

Figure 3. Money Markets Equilibrium.



model, as money growth rate increases, households foresee lower need for carrying over the real balances from period 1 to period 2. As a result, the real balances in period 1, m_1, fall to m_1^{STD}.

However, as money balances rise in period 2 in nominal terms, the consumers bid prices up in periods 1 and 2. This bidding up of prices implies that the real balances in period 1 fall even further. At the same time, higher prices in period 2 and the fact that real income does not change as the result of the higher money supply growth imply that in period 2, the real balances remain the same in the benchmark case of no LBD. Hence, an increase in the growth rate of money supply will not have any real effects in the standard model. Instead higher nominal balances in period 2 will be absorbed into higher prices in periods 1 and 2.

In relation to Model 1, these effects correspond to the standard effects shown by Sargent and Ljungqvist (2000). However, in Model 2 due to the absence of bonds and leisure in the household optimisation, the real effects of money supply increase are absorbed solely into consumption. As a result, the interest rate effects are captured by the synthetic rate of return expressed in the marginal utility terms:

$$\frac{\lambda_1}{\lambda_2} = \frac{P_2 U_C^1}{\beta P_1 U_C^2}$$

Thus an increase in the marginal utility of income in period 1 will be associated with an increase in prices across the periods, or a decrease in the marginal utility of consumption in the future. This captures the effects of either changing the real rate of return to money, $R_{m,t}$, or changing the real rate of return to bonds, R_t, as in equation (14) in Model 1.

In the presence of LBD effects, similar responses take place. However, as nominal balances increase in period 2 due to a higher money supply, demand for current balances in period 1 falls by less than in the standard model, so that $m_1^{LBD} - m_1^{STD} > 0$. This wedge between the standard model demand for real balances and the LBD effects-driven demand is caused by incentives to the households to take advantage of learning benefits from higher spending in period 1. As learning is generated from $m_1^{LBD} - m_1^{STD} > 0$, agents see the real cost of consumption fall in terms of the private markets transactions time in period 2 due to the LBD effect. As a result, agents will increase their demand for money in period 2 in real terms, so that $m_2^{LBD} > m_2^{STD} = m_2$. Furthermore, due to the same reasons, real consumption will increase in period 2 requiring higher money balances in real terms.

To summarise, the presence of the LBD effects reduces the responsiveness of current demand for the real balances to the changes in the money supply rule. Likewise, the extent to which an increase in money supply growth rate bids up both current and future prices is altered in the presence of the LBD effects. In period 1, prices are bid-up less due to the LBD effect since a part of the money demand in period 1 is driven by the motive to learn. In period 2, the real balances are more responsive to changes in the money growth rate than in the standard model, while the price level is less responsive. Thus in period 2, households realise their gains from learning in period 1 and demand higher real balances in order to finance greater consumption out of the cost-savings generated by the transactions learning technology.

In addition, it is worth mentioning that the reduced elasticity of money demand with respect to changes in the real rate of return makes our model consistent with the stylised fact concerning the negative correlation between the money supply and the velocity of money. This feature is commonly found in the fixed price models, but not in the standard flexible-price universe. Let *v* denote the velocity of money, then, as shown in Alvarez, Atkeson and Edmond (2003), we can define

$$v_t = P_t c_t / M_t$$

In our model, monetary expansion raises money supply, consumption and prices. However, as shown in Figure 3 above, the real balances rise in the period 2. At the same time, consumption rises by more than the rise in the real balances, as households capitalise on the learning effects from the period 1. Therefore the velocity of money rises. In classical models, this effect is nil, since the price level adjust exactly to offset any impact of an increase in money supply on the real balances.

Conclusions.

Classical models of inflation in the presence of endogenous money demand traditionally assume that money enters the household problem via the cash-in-advance constraint. In this case, fiat money may serve to facilitate transactions in the markets for consumption. The latter function of the fiat money is characterised by specifying a technology for transactions that relates time spent shopping in the private markets to the real balances held by the households relative to their consumption needs. In such a case, as shown in Sargent and Ljungqvist (2000), money demand depends positively on the rate of return to money, and negatively to the real rate of return to bonds.

Another important feature of the classical models of money demand is that monetary policy has no real effects in the absence of price or wage frictions. An expansionary monetary policy that acts to increase money supply is simply fully absorbed into the price level fluctuations. An alternative view is provided by the New Keynesian models, where, due to nominal rigidities in prices and/or wages, a monetary expansion can have a direct effect on consumption and output.

So far, to the best of my knowledge there is little theoretical work available that would permit a standard classical model to account for the persistency of money demand and inflation observed in the data without resorting to either price or wage stickiness assumption. The current model attempts to fill this gap.

The general problem of the New Keynesian synthesis is that price and wage stickiness are difficult to reconcile with the empirical observations. Furthermore, such assumptions, relevant in the world of underdeveloped retail markets and unionised labour force, become less and less intuitive in the world of increasing labour and wage mobility, and lower menu costs afforded by the modern retail markets. Grivoyannis (1991) makes these points clear in his conclusions summarised in the Introduction above.

However, as shown in our model, households' optimisation is based on the intuitively simple concept of learning-by-doing in the transactions markets, and provides a simple alternative to the price and/or wage stickiness assumptions. In so far as the agents may learn new transactions pathways by engaging in consumption, the future time costs of consumption may be negatively related to the present level of expenditure. In such a case, lagged real balances may have a significant impact on the demand for money in the future. This point coincides with the results of many empirical findings.

In Part 1 we take a standard classical model of money demand and introduce the possibility of endogenous persistency in the demand for the real balances. Due to the presence of the learning-by-doing effects in transactions technology, the agents' demand for money in each period is determined, in part, by the history of transactions carried out in the past. This is what we call the LBD effect in transactions technology. Thus, within any given period, the real balances held by the agents are less responsive to the changes in the real return to money, the inflation rate. As a result of the LBD effects in transactions:

- Lower real rates of return are capable of generating a higher demand for money, with the standard models effect of real interest rate being lower in our model for all variables of choice.
- Due to the learning effects, an increase in the money supply has the effect of raising consumption both in the current period and in the future.
- An increase in the real rate of return to money, relative to the rate of return on bonds, is shown to have a smaller effect on the overall demand for money.
- In the case of a specific form assumption for the utility function and transactions technology, we show that the stationary seigniorage costs of money creation are uniformly lower in the presence of LBD effects, than in the standard flexible-price models.
- The preceding point implies that in the short-run, the cost of saturating the economy with money balances overshoots its long-run level.

- At the same time, the long-run equilibrium cost of the Friedman rule is below the equilibrium cost in standard models.
- Learning-by-doing has an effect of strengthening the unpleasant monetarist arithmetic result of Sargent and Wallace (1981).
- In the presence of LBD, sustained fiscal deficits will be associated with lower inflation relative to the standard models.
- The exchange rate indeterminacy result of Karaken and Wallace (1981) is extended by the LBD effects to a longer period of adjustment.

Following a general discussion in Part 1, we simplify the model to a two-period setting without bond markets. We first proceed to show that the results obtained in the more general setting in Part 1 continue to hold in a more stylised setting. We also show that unlike in the traditional flexible-price models, the intertemporal effects of the money supply expansion in period 1 are not absorbed into higher prices alone. Instead, the LBD effect has real implications for consumption choices. These assumptions generate Model 2 above.

In the case of an exogenous increase in the money supply growth rate, the LBD effect in transactions acts to bid up prices in period 1 (period of change in money supply growth rate). However, due to the lower responsiveness of money demand to the changes in inflation rate, caused by the LBD effect, the real balances in period 1 fall less in our model than in standard models of inflation. In period 2, the households realise the gains in time-savings generated by the LBD in period 1. The demand for real balances in period 2 rises above the pre-shock level allowing for a greater consumption arising from the lower cost of consumption in period 1. Thus the LBD effect in the transactions technology can be identified as the source of the real effects of monetary policy in classical setting with flexible prices.

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